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DIPOLE TORUS MODE IN NUCLEAR FLUID-DYNAMICS

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Дипольная тороидная мода в ядерной флюид-динамике

В рамках ядерной флюид-динамики изучается дипольный электрический отклик, связанный с поперечными полоидальными колебаниями потока нуклонов. Данный отклик ассоциируется с возбуждением в ядерном объеме сферического вихря Хилла — тороидоподобной токовой структуры, гармонически осциллирующей во времени; форма ядра остается сферической. Предсказываемая моделью искаженной ферми-поверхности энергия дается оценкой $E(1^-_{\text{тор}}) = 65 - 85A^{-1/3}$ МэВ, т.е. данное возбуждение может быть экспериментально идентифицировано как $2\hbar\omega$ изоскалярный дипольный резонанс. Представлены аналитические выражения электрического формфактора и токовой переходной плотности для данного резонанса. Вклад тороидного дипольного возбуждения в сечение фотопоглощения, рассчитанный в плоско-волновом борновском приближении, равен $\sigma_\gamma(1^-_{\text{тор}}) = 10^{-3} - 10^{-4} Z^2/A$ МэВ фм^2. В работе кратко обсуждаются предложения для будущих экспериментов по поиску дипольного тороидного отклика.

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Dipole Torus Mode in Nuclear Fluid-Dynamics

The nuclear fluid-dynamics is applied to the study of a dipole electric response related to the transverse poloidal oscillations of a particle flow. This response is associated with excitation in the nucleus volume of the spherical Hill vortex — toruslike current structure harmonically oscillating in time; the nucleus shape remains to be spherical. The energy evaluated within the distorted Fermi-surface model is predicted to be $E(1^-_{\text{torus}}) = 65 - 85A^{-1/3}$ MeV, i.e. the torus excitation is expected to be identified experimentally as $2\hbar\omega$ isoscalar dipole resonance. An analytic form is derived for the electric form factor and numerical calculations of the transition current density are presented. The contribution to the cross section of photoabsorption computed in PWBA is estimated to be $\sigma_\gamma(1^-_{\text{тор}}) = 10^{-3} - 10^{-4} Z^2/A$ MeV fm^2. Some hints for futher experiments are suggested.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1 Introduction

The present paper is devoted to the problem of isoscalar electric dipole response in heavy spherical nuclei which nowadays is a subject of intensive discussions in nuclear physics of giant resonances. According to a recent review by van der Woude [1], the strength of $1^-, T = 0$ excitation is clustered in the nuclear spectrum as follows. The low-energy $1\hbar\omega$ dipole resonance (LEDR) located, as is presumed, at energy 30-50 $A^{-1/3}$ MeV exhausts of the order of 10-20% of the isoscalar sum rule [2] computed with the function $r^3 Y_{1\mu}$. Theoretical discussion of this mode can be found in [3, 4]. The last data on LEDR have been reported in [5]. The high-energy $3\hbar\omega\, 1^-, T = 0$ resonance (HEDR) with energy 120-130 $A^{-1/3}$ MeV depletes about 40-50% of the above mentioned sum rule. This resonance has been discussed by many authors in both microscopic and macroscopic approaches. The macroscopic interpretation of HEDR as the dipole squeezing mode, caused by longitudinal dipole compression oscillations of nucleons, is generally accepted by now (see, for instance, [3, 6, 7]). So, we see that the data available exhibit the lack of approximately 30-50% of $1^-, T = 0$ strength. In view of this, it seems to be reasonable to explore new mechanisms of dipole nuclear response which could make up this deficiency. One of such mechanisms is discussed below.

Our analysis is based on similarity of the fluid-dynamical equations governing the nuclear vibrations and the electrodynamical equations for oscillations of an electric current in the metal sphere. From the electromagnetic theory it is known that the current motions of a dipole symmetry may be generated in conducting sphere by a magnetic field (see [8] Sec.9.24): having penetrated into the sphere the magnetic field induces the current which is described by the poloidal dipole field with a torus-like structure of stream lines (Fig.1). The physical arguments underlying this mechanism of dipole distribution of current has been considered by Elsasser (see for details ref.[9] p.206 and ref.[10] p.29) in connection with the problem of terrestrial magnetism (the model of Earth's core).

One of the principal findings of the nuclear fluid-dynamics (NFD) is that linear perturbations of a nucleus may also result in transverse (essentially rotational) oscillations of a nucleon flow like those for electric current in a metallic particle. Note that in the standard hydrodynamical model of liquid drop (LDM) the collective modes are interpreted only in terms of longitudinal (irrotational) motions. From continuum theory we know that both the transverse and longitudinal oscillations are an inherent property of perfectly elastic medium (see, for example, [11]). In this conjunction it should be stressed that the NFD has brought new insight into the fundamental macroscopic properties of nuclear matter to be a quantum elastic-like solid (discovered by Bertsch in ref.[12]). In particular, the nuclear fluid-dynamics states that isoscalar resonances may be interpreted as a dynamical manifestation of quantum elasticity. In Sect.2, the fluid-dynamical method is outlined to make more consistent the analysis of the current-dependent mechanism of transverse dipole response.

In Sect.3, the velocity field of a transverse flow pattern is derived by making use of the condition that the nucleus center of mass is at rest. It is shown that the obtained poloidal dipole field of velocity corresponds to the spherical vortex of Hill. The latter is pictured by the torus-like distribution of stream lines. Therefore, the resonance related to such a current response is referred to as the dipole torus mode. It is interesting to note that recent microscopic investigations of nuclear dipole response performed in [13-15] have revealed just the same behavior of the particle flow. The macroscopic analysis of such a flow pattern can be found in [16-18]. Thereby, our purpose is mainly to supplement these investigations.

In Sect.4, the fluid-dynamical approach is applied to compute the form factor and transition current density which are derived in the plane wave Born approximation. The concluding remarks concerning the experimental search for the dipole torus excitation are presented in Sect.5.

2 Governing fluid-dynamical equations

The mathematical treatment of the NFD is based on the "thirteenmoments approximation". This terminology, borrowed from the plasma physics, originates from the fact that macroscopic variables of nuclear collective motions such as the bulk density ρ, three components of the mean velocity V_i and nine components of the strain tensor P_{ij} are introduced as zero, first and second p-moments of quantum distribution function. The relevant equations for these variables are obtained from the kinetic equation which in turn can be derived from the many-particle Schrhödinger equation (see for details [19-21]). In the linear approximation, the equations governing small oscillations of incompressible nuclear continuum (in the collisionless regime of single-particle Fermi-motion) are written as follows [22-24]

$$\frac{\partial \delta V_i}{\partial x_i} = 0, \tag{2.1}$$

$$\rho_o \frac{\partial \delta V_i}{\partial t} + \frac{\partial \delta P_{ij}}{\partial x_j} = 0, \tag{2.2}$$

$$\frac{\partial \delta P_{ij}}{\partial t} + P_o \left(\frac{\partial \delta V_i}{\partial x_j} + \frac{\partial \delta V_j}{\partial x_i} \right) = 0. \tag{2.3}$$

Here δV_i is the perturbed velocity of collective flow and δP_{ij} stands for small fluctuations in the strain tensor. By ρ_o we denote the ground state density and P_o is the pressure (diagonal part of the equilibrium strain tensor). They are the input parameters of the method and may be taken from microscopic self-consistent calculations. In particular, P_o related to the ground state energy $\mathcal{E}_o$ by means of the equation of state, may be parametrized in the form

$$P_o = \frac{2}{3}\mathcal{E}_o = \frac{\rho_0 < v^2 >}{3}, \tag{2.4}$$

where $< v^2 >$ stands for the mean square velocity of the single-particle Fermi-motion of nucleons setting in the self-consistent mean field. Eqs.(2.1) and (2.2) are the same as in the LDM: first of them is the equation of continuity and the second one is the Euler equation. The linear evolution of elastic stresses is described by eq.(2.3). A nucleus in the ground state, in such an approach, is modelled by an elastic-like globe of spin - and isospin saturated incompressible Fermi-continuum. The resonance excitations are specified by the form of collective velocity.

In what follows it is convenient to represent the velocity departures in the form

$$\delta V_i(\mathbf{r}, t) = a_i^\lambda(\mathbf{r})\dot{\alpha}_\lambda(t), \tag{2.5}$$

where a_i^λ is the field of instantaneous displacements and α^λ is the time-dependent amplitude of harmonic oscillations ($\alpha_\lambda \sim \sin\omega_\lambda t$). The latter may be interpreted as a collective variable in accord with the Bohr and Mottelson treatment of normal nuclear vibrations.

It can be verified that eqs.(2.1)-(2.3) are reduced to standard wave's equation for field of collective velocity $\delta\mathbf{V}$ and further to the Helmholtz equation for standing spherical waves [4]. In the long wavelength approximation, which is justified by numerous calculations in the nuclear physics, the latter equation is transformed into the equation of Laplace for the solenoidal field of displacements

$$\Delta\, \mathbf{a}^\lambda = 0, \quad \mathrm{div}\, \mathbf{a}^\lambda = 0. \tag{2.6}$$

In the frame with a fixed polar axis, the last equation has two independent solutions - the poloidal (following the terminology introduced by Elsasser [25] and accepted in physics of plasma and astrophysics)

$$\mathbf{a}_p^\lambda(\mathbf{r}) = N_p^\lambda \mathrm{rot\,rot}\,\mathbf{r}\, r^\lambda P_\lambda(\cos\theta) = -N_p^\lambda(\lambda+1)\mathrm{grad}\, r^\lambda P_\lambda(\cos\theta), \tag{2.7}$$

and the toroidal

$$a_t^\lambda(r) = N_t^\lambda \text{rot}\, r\, r^\lambda P_\lambda(\cos\theta). \quad (2.8)$$

As it has been pointed out above, the resonance excitations are classified in the NFD by the form of perturbed mean velocity (2.5). The poloidal solution describes the harmonic distortions of nuclear shape. These motions are responsible for electric resonances, since a_p^λ is the truth vector field of normal parity $\pi = (-1)^\lambda$. The toroidal solution describes torsion-like motions and corresponds to magnetic resonances, since a_t^λ is the pseudovector field of abnormal parity $\pi = (-1)^{\lambda+1}$.

In refs.[22-24], it is shown that eqs.(2.1)-(2.3) may be reduced to the standard Hamiltonian of normal vibrations

$$H = \frac{B_\lambda \dot{\alpha_\lambda}^2}{2} + \frac{C_\lambda \alpha_\lambda^2}{2}, \quad (2.9)$$

where the inertia B_λ and stiffness C_λ are given by

$$B_\lambda = \int \rho_0 a_i^\lambda a_i^\lambda d\tau, \qquad C_\lambda = \int P_o \frac{\partial a_i^\lambda}{\partial x_j}\left(\frac{\partial a_i^\lambda}{\partial x_j} + \frac{\partial a_j^\lambda}{\partial x_i}\right) d\tau. \quad (2.10)$$

The eigenfrequencies of resonance excitations are defined as $\omega_\lambda^2 = C_\lambda / B_\lambda$.

The fluid-dynamical interpretation of restoring force of normal nuclear vibrations is linked with the elastic properties of single-particle orbits. According to "scaling hypothesis", it is assumed that external perturbations (with energies of giant resonances $\sim 6 < E_x < 20 MeV$) give rise to the coherent distortions of one-nucleon orbits or, in other words, evoke local deformations of the shape of an equilibrium Fermi-distribution (the distorted Fermi-surface model). In the nucleus volume, these distortions are associated with a harmonically fluctuating field of elastic strains

$$\delta P_{ij} = -P_o \left(\frac{\partial a_i^\lambda}{\partial x_j} + \frac{\partial a_j^\lambda}{\partial x_i}\right) \alpha_\lambda. \quad (2.11)$$

These strains strive to restore the initial equilibrium shape of Fermi-sphere and, hence, return a nucleus to the ground state. The tensor of elastic deformations (2.11), obtained from eq.(2.3) after substitution (2.5), possesses the quadrupole symmetry like the tensor of quadrupole moment: the trace of δP_{ij} is zero owing to assumption of incompressibility. Thus, the coherent elasticity of single-particle orbits (quadrupole deformations of Fermi-sphere) is the main physical factor determining the quantum nature of restoring force for normal vibrations resulting in giant resonances (quantum Hooke's law). On this reason the nuclear fluid-dynamics may be considered as an example of quantum theory of elasticity.

Based on this scheme, in [26] it has first been found that the energies of isoscalar quadrupole resonances are well reproduced throughout the periodic table if one puts the displacement field in the form: $a_x = -x$, $a_y = -y$, $a_z = 2z$. It is easy to check that this field is a particular case of general poloidal solution (2.7). The inertia and stiffness computed with the field (2.7) are

$$B_\lambda = (N_p^\lambda)^2 M\lambda < r^{2\lambda-2} >, \quad C_\lambda = (N_p^\lambda)^2 \frac{2}{3}\lambda(\lambda-1)(2\lambda-1) M < v^2 >< r^{2\lambda-4} >, \quad (2.12)$$

where M is the nucleus mass. For the energy we obtain

$$E(E\lambda) = \hbar \left[\frac{2}{3}(2\lambda-1)(\lambda-1) < v^2 > \frac{< r^{2\lambda-4} >}{< r^{2\lambda-2} >}\right]^{1/2}. \quad (2.13)$$

Note that arbitrary constants N_p^λ disappear in the final formula for energy. Using the sharp edge approximation for the nucleus surface and Fermi-gas estimate for equilibrium pressure, the spectrum of electric isoscalar resonances (2.13) is replaced by

$$E(E\lambda) = \hbar\omega_F \left[\frac{2(2\lambda+1)(\lambda-1)}{5}\right]^{1/2}, \quad (2.14)$$

The latter equation has first been derived by Nix and Sierk [22] (see also [27-30]) within the above outlined approach. Note that the poloidal field of velocity exactly coincides with velocity of irrotational flow which is considered in the standard LDM. Thereby, the fluid-dynamical treatment of electric isoscalar resonances repeats the well known Tassie interpretation of electric collective excitations accepted in the LDM. However, the main difference between fluid-dynamical and hydrodynamical approaches is the treatment of restoring forces of normal vibrations.

The collective motions describing by the toroidal field (2.8) are unique to the NFD and does not appear in the LDM. Based on the above given approach Holzwart and Eckart [31] have envisaged the 2^- twist resonance (see also [20, 32]) which is interpreted in terms of shear oscillations accompanied by the field of displacements of the form $a_x = -yz, a_y = xz, a_z = 0$. This field is a particular quadrupole solution of the general toroidal one given by (2.8). We stress again that the magnetic excitations, caused by the transverse oscillations of nucleons, may be excited essentially owing to the elastic properties of nuclear matter. The characteristics of twist resonances may be represented as follows

$$B_\lambda = (N_t^\lambda)^2 \frac{\lambda(\lambda+1)}{(2\lambda+1)} M < r^{2\lambda} >, \quad C_\lambda = (N_t^\lambda)^2 \frac{1}{3}\lambda(\lambda^2-1) M < v^2 >< r^{2\lambda-2} >, \quad (2.15)$$

$$E(M\lambda) = \hbar \left[\frac{1}{3}(2\lambda+1)(\lambda-1) < v^2 > \frac{< r^{2\lambda-2} >}{< r^{2\lambda} >}\right]^{1/2}. \quad (2.16)$$

The model with a sharp edge yields [23]

$$E(M\lambda) = \hbar\omega_F \left[\frac{(2\lambda+3)(\lambda-1)}{5}\right]^{1/2}. \qquad (2.17)$$

Both electric and magnetic resonances predicted by NFD have a volume origin, i.e. all the nucleons of a nucleus are involved in the collective motions (see ref.[30]). This fact actually means that the giant resonances are mostly determined by the saturation properties of nuclear matter.

It is interesting to note that the "Fermi-frequency" ω_F occurs to be equal to the oscillator frequency ω of the single-particle shell model for a spherical nucleus. Indeed, let us consider the semiclassical limit of the shell model with the oscillator Hamiltonian (ignoring the spin-orbit coupling). Making use of, first, quantum and then statistical averaging of this Hamiltonian and applying the "virial theorem" (according to which $< T_{kin} >=< U_{pot} >$ for oscillator) we find $< v^2 >= \omega^2 < r^2 >$. In a simplified model $< v^2 >= 3/5 v_F^2$ and $< r^2 >= 3/5R^2$. So the virial theorem yields $\omega = \omega_F = v_F/R$. The latter identity can also be found in ref.[26] (compare the fluid-dynamical calculation of the giant quadrupole frequency $\omega(GQR) = \sqrt{2}\omega_F$ with the result of the self-consistent oscillator model $\omega(GQR) = \sqrt{2}\omega$, eqs.(51) and (58) in [26], respectively).

From the above presented equations for energies it follows that dipole excitations of both parities are unstable (the frequency is zero). The purpose of the next section is to analyze the question why the dipole electric mode occurs to be excluded.

3 Hill vortex as a macroscopic mechanism of the dipole torus mode

The long wavelength dipole oscillations with the poloidal field

$$\mathbf{a}_p^1(\mathbf{r}) = N_p^1 \mathrm{rot\,rot}\, \mathbf{r}\, r\, P_1(\cos\theta) = -2N_p^1 \nabla\, r\, P_1(\cos\theta), \qquad (3.1)$$

contribute only to the mass parameter B_1 (kinetic energy), whereas the parameter of rigidity C_1 is canceled. As a result, a nucleus is moved as a whole without changing its intrinsic state. This conclusion is due to the fact that we have used the approximation of long wavelength oscillations. Thereby, in order to investigate the dipole transverse response one has to go beyond the long wavelength limit and include higher terms into the radial parts of the function χ^1 entering into the poloidal dipole field

$$\mathbf{a}_p^1(\mathbf{r}) = N_p^1 \mathrm{rot\,rot}\, \mathbf{r}\, \chi^1(r,\theta). \qquad (3.2)$$

Our further consideration is relied on the microscopic analysis of a nucleon flow under a dipole response performed in refs.[13, 14] within the self-consistent Hartree-Fock and Random Phase Approximations. In these articles, the probe response operator has been taken in the form $r^3 Y_{1\mu}$ and then was corrected for the center of mass motion. Following the prescription of [13, 14], we explore the trial function χ^1 of the same form

$$\chi^1(r,\theta) = r^3 P_1(\cos\theta), \qquad (3.3)$$

i.e. the radial part of χ^1 is taken in the next order beyond the long wavelength approximation. To establish an explicit expression for the displacement field corrected for the center of mass motion we insert $\mathbf{a}_p^1(\mathbf{r})$, with χ^1 given by (3.3), into the condition for the center of mass $\mathbf{R}_{c.m.}$ to be at rest

$$\Delta\mathbf{R}_{c.m.} = \frac{\int \rho \mathbf{a}_p^1 \tau}{\int \rho d\tau} = 0. \qquad (3.4)$$

This procedure yields

$$\mathbf{a}_p^1 = N_p^1 \mathrm{rot\,rot}\, \mathbf{r}\, r\,(r^2 - R^2)\, P_1(\cos\theta). \qquad (3.5)$$

In spherical components eq.(3.5) is rewritten as

$$\begin{aligned}
(a_p^1)_r &= \frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta} = 2N_p^1(r^2 - R^2)\cos\theta, && (3.6)\\
(a_p^1)_\theta &= -\frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial r} = -2N_p^1(2r^2 - R^2)\sin\theta, && (3.7)\\
(a_p^1)_\phi &= 0. && (3.8)
\end{aligned}$$

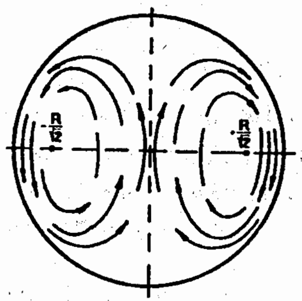

Fig.1. Meridional cross section of the poloidal dipole field of displacements for the spherical vortex of Hill associated with the dipole torus mode.

At this point it should be stressed that the obtained field of displacements precisely coincides with that for the spherical Hill vortex known in the theoretical hydrodynamics from the last century[33, 34]. The theory of the poloidal dipole flow is described in detail in ref.[35] which also contains the exhaustive mathematical treatment of solenoidal (poloidal and toroidal) vector fields in spherical geometry.

The Stokes stream function ψ for the vortex of Hill is given by

$$\psi(r,\theta) = N_p^1(r^2 - R^2)\,r^2\,\sin^2(\theta), \tag{3.9}$$

The meridional cross section of the poloidal dipole field for Hill vortex is pictured in Fig.1. The critical points are fixed by the conditions $a_r = 0$ and $a_\theta = 0$: $r_c = R/\sqrt{2}$ and $\theta_c = \pm\pi$. By virtue of the torus-like form of Hill vortex, this collective excitation of a nucleus is referred below to as the dipole torus mode.

Having established the field of an oscillating flow, we are able to evaluate the energy of a dipole torus mode. Inserting eq.(3.5) into eqs.(2.8) the inertia and stiffness become

$$B_1 = \frac{2}{3}(N_p^1)^2 M < r^4 >, \quad C_1 = \frac{10}{3}(N_p^1)^2 M < v^2 >< r^2 >. \tag{3.10}$$

The energy is given by

$$E(1^-_{\text{torus}}) = \hbar(C/B)^{1/2} = \hbar\left[5 < v^2 > \frac{< r^2 >}{< r^4 >}\right]^{1/2}. \tag{3.11}$$

For the elastic Fermi-globe of the radius $R = r_o A^{1/3}$ we get

$$B_1 = \frac{2}{7}(N_p^1)^2 M R^4, \quad C_1 = \frac{6}{5}(N_p^1)^2 M v_F^2 R^2, \quad E(1^-_{\text{torus}}) = \sqrt{\frac{21}{5}}\,\hbar\omega_F \approx 2\hbar\omega. \tag{3.12}$$

Using the parameters of the realistic Fermi-distribution for the density [2] in calculations of $< r^\lambda >$ we obtain

$$E(1^-_{\text{torus}}) = \approx 65 - 85\,A^{-1/3},\ MeV, \tag{3.13}$$

i.e. the mode under consideration turns out to be localized somewhat lower (approximately by 10-20%) than the isovector giant dipole resonance.

So, the nuclear fluid-dynamics predicts that the dipole torus mode may be interpreted as the $2\hbar\omega$ dipole isoscalar resonance, caused by the excitation in the nucleus volume of the spherical Hill vortex.

For the sake of comparison we note that in contrast with the transverse character of dipole torus excitations, the squeezing dipole mode ($3\hbar\omega$ resonance) is accompanied by the longitudinal oscillations with the field of displacements $\mathbf{a}_s = \nabla r^2 (r - 4/5R)\,P_1$. This

field is also corrected for the center of mass motion. However, it does not satisfy the equation of incompressibility (2.1), i.e. the squeezing dipole resonance is essentially the compression excitation [3, 37, 38].

4 Form factor and transition current density for dipole torus mode

To complete the analysis of the dipole torus excitation we present here calculations of the transverse electric form factor and transition current density making use of the plane wave Born approximation. The knowledge of these quantities is of interest since the form factor can be directly measured in reactions with inelastically scattered electrons [39, 40]. The disturbed (transition) current density in the fluid-dynamical representation is given by (see for comparison [13,14])

$$\delta\mathbf{J} = n_e\,\delta\mathbf{V} = n_e\,\mathbf{a}_p(\mathbf{r})\,\dot{\alpha}(t). \tag{4.1}$$

Here $n_e = Z/A\,n_o$ and n_o stands for the particle density; $\dot{\alpha}(t) = \alpha_o\,\omega\,cos\omega t$ and α_o is given by [2]

$$\alpha_o^2 = \left(\frac{\hbar^2}{2BC}\right)^{1/2}. \tag{4.2}$$

The transverse electric form factor is introduced on the basis of a tensor-function (see [39], eq. (1.25))

$$T_\lambda^E(k,t) = \frac{1}{k}\int [\nabla \times j_\lambda(kr)\mathbf{Y}_{\lambda\lambda;1}]\cdot\delta\mathbf{J}(\mathbf{r},t))\,d\tau. \tag{4.3}$$

This function depends harmonically on time due to an analogous time dependency of current $\delta\mathbf{J}$. The electric form factor $F_\lambda^E(k)$ may be defined by

$$|F_\lambda^E(k)|^2 = < |T_\lambda^E(k,t)|^2 >_t, \tag{4.4}$$

where the symbol $< ... >_t$ stands for the time averaging.

Taking into account eq.(4.1), the form factor can be represented as follows

$$|F_1^E(k)|^2 = \frac{1}{2}\alpha_0^2\omega^2\left[\int n_e\,(r^2 j_2(kr) - 5(r^2 - < r^2 >)j_0(kr))r^2\,dr\right]^2. \tag{4.5}$$

This equation shows that $F_1^E(k)$ depends only on two nuclear parameters: the ground state density and nucleus radius.

In the model of an elastic Fermi-globe with the uniform density, the integral (4.5) is taken analytically

$$F_1^E(k) = -\gamma \left[\frac{3}{k^5 R^5} (15 \sin(kR) - 6k^2R^2 \sin(kR) - 15kR\cos(kR) + k^3R^3 \cos(kR)) \right], \tag{4.6}$$

where $\gamma = N_1^p n_e \alpha_o \omega R^5$. In Fig.2, the plot of $|F_1^E(k)|^2$ for ^{90}Zr and ^{208}Pb is presented to illustrate the dependence of strength of the torus excitation on nucleus mass and charge. This figure displays the enhancement of the torus effect in heavy nuclei as compared to light ones; first maximum of the form factor is shifted towards the small momenta transfer when we proceed to nuclei with large Z and A.

The knowledge of form factor allows us to evaluate the contribution to the cross section of photoabsorption. As a result we obtain

$$\sigma_\gamma(1^-_{\rm torus}) = \frac{2\pi^2 e^2}{\hbar\omega c} |F_1^E(k=\omega)|^2 \approx 10^{-3} - 10^{-4} Z^2/A \text{ MeV fm}^2. \tag{4.7}$$

This value, probably, is too small to be observed in the photonuclear processes.

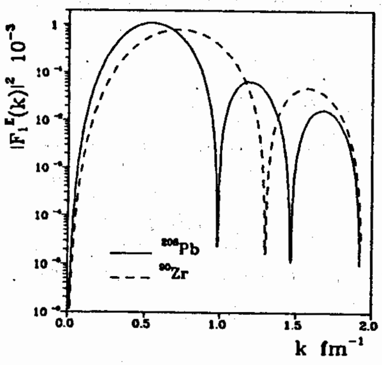

Fig.2. PWBA computed form factor $|F_1^E(k)|^2$ for $1^-, T = 0$ torus mode in ^{90}Zr and ^{208}Pb.

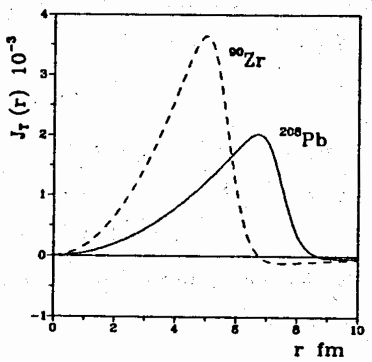

Fig.3. Transition current density for $1^-, T = 0$ torus excitation in ^{90}Zr and ^{208}Pb.

Having computed the form factor we can calculate the transition current density which is found [13,14] to be an important characteristics of collective oscillations as well. According to the general theory of electron scattering on nuclei, the transition current density is

defined as follows (see [40], eq.(9))

$$J_{\lambda,\lambda+1}(r) = -\frac{2}{\pi} \frac{\hat{J}_i}{\hat{J}_f} \frac{\sqrt{\lambda}}{\hat{\lambda}} \int F_\lambda^E(k) j_{\lambda+1}(kr) k^2 dk. \tag{4.8}$$

Here, the indexes i and f stand for initial and finale states and $\hat{J} = \sqrt{2J+1}$. For the torus mode $\lambda = 1, J_f = 1, J_i = 0$. So, we have

$$J_T = J_{1,2}(r) = -\frac{2}{\sqrt{3}\pi} \int_0^\infty F_1^E(k) j_2(kr) k^2 dk. \tag{4.9}$$

This integral may be computed only numerically. The transition current density is plotted in Fig.3.

It is worthwhile to note that according to ref.[36] (see also references therein), the poloidal torus-like current structures may be described by a new class of electromagnetic "toroid" multipole moments analogous to the electric and magnetic ones. The nuclear dipole toroid moment has been investigated in [16, 18].

Thus, the fluid-dynamical approach permits one to obtain rather a complete information to identify the dipole torus mode in the experiment.

5 Summary

In the present paper it is shown that the nuclear fluid-dynamics predicts a new macroscopic mechanism of resonance dipole excitation which is accompanied by transverse oscillations of nucleons. In the considered approach a spherical nucleus is modelled by an elastic-like globe of spin and isospin saturated incompressible Fermi-continuum. The restoring force of oscillations is interpreted in terms of quadrupole distortions of a spherical Fermi-distribution. The velocity field is derived by using the condition for the nucleus center of gravity to be at rest. This field corresponds to the spherical vortex of Hill which has a torus-like form of a flow pattern; the nucleus shape remains spherical. In view of this, the collective excitation under consideration is referred to as the dipole torus mode.

An analytic calculation of energy performed within the distorted Fermi-surface model predicts $E(1^-_{\rm torus}) \approx 65 - 85A^{-1/3}$MeV. This estimate allows one to interpret the dipole torus mode as $2\hbar\omega$ $1^-, T = 0$ current-dependent transverse resonance. The torus mode happens to be well discriminated by energy from the isoscalar low-energy $1\hbar\omega$ and high-energy $3\hbar\omega$ resonances and is expected to be found somewhat lower than the isovector GDR.

Concerning the experimental search for the dipole torus excitation one may note the following. An analogy with classical electrodynamics (see Sec.1) permits one to conclude that magnetic forces should play a dominant role in the considered mechanism. There by the torus mode should manifest itself in measurements with charged particles like e, p, α, $\pi^{\pm}$ etc. which produce the magnetic field. At nonrelativistic energies the magnetic effects happen to be appreciably suppressed compared to the electric ones. This is a probable reason why we have got the small magnitude of photoabsorption cross section which is found to be $\sigma_\gamma(1^-_{\text{torus}}) \approx 10^{-3} - 10^{-4} Z^2/A$ MeV fm^2.

We believe that the torus mode may be excited by inelastically scattered electrons at large angles. The conditions for such measurements should be the same as for the magnetic resonances (see for details [41]) because the torus excitation is caused by convectional transverse oscillations of nucleons. In particular, for ^{208}Pb, the dipole torus resonance is expected to be at energies 10.5-13.5 MeV, i.e. just in between the isovector dipole and isoscalar quadrupole resonances which in backward electron scattering are suppressed. Although this is region of $M3$ resonance predicted in [23], the excitation strength of the latter is somewhat weaker as compared to that for electric torus mode; the magnetic quadrupole resonance located at 6-8 MeV will be excited at far smaller momenta transferred to the nucleus. So, one may hope to detect the dipole torus response in (e,e') measurements under backward angles. With this in mind, we have computed the form factor and transition current density for such an experiment.

One should note that from the above presented considerations it follows that the torus excitation is not related to the spin and isospin degrees of freedom. This means that the measurements with isoscalar spinless probes is presumed to be more preferable. Therefore, the conditions of the KVI experiment reported in [42] are, in our opinion, quite suitable to suspect the excitation of the dipole torus mode in the $(\alpha, \alpha', \gamma)$ reaction.

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