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VORTICAL "DISKS" OF NUCLEAR MATTER

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1 Introduction

In this paper we consider a possible state of incompressible nuclear matter, the plain nuclear vortical "disks". They are finite regions of the constant vorticity on the plane, are limited by the uniform-rotating boundary. These states can be considered as the generalization of the Elliptic Kirchhoff Vortex [1].

In this paper the nonlinear integro-differential equation for the "disk" shape evolution has been derived in the framework of the semiclassical nuclear hydrodynamics [2]. The method is suitable to analyze the "disks" of any type without further assumption about the ellipsoidal shape or approximations about the small deviations the "disk" shape from the circle [1].

The plan of our exposition is as follows. In §2 basic equations for the plain nuclear "disks" are derived. In §3 main facts concerning the elliptic Kirchhoff vortex are presented. A qualitative analysis of the main features of the "disk" states is given in §4.

2 Basic Equations

We shall describe the dynamical behavior of nuclear matter in terms of macroscopic fluid dynamical concepts. They appeal to our intuitive understanding of collective motion by involving as essential dynamical variables local macroscopic fields like single-particle density $\rho(\vec{x},t)$, current-density $\vec{j}(\vec{x},t)$, pressure tensor $P_{kn}(\vec{x},t)$, etc. We shall follow the semiclassical nuclear hydrodynamics [2]. This approach for the relaxation processes in heavy-ion collisions based on the current and density algebra and the soliton theory concepts.

The standard continuity and Euler equations are:

$$\frac{\partial\rho}{\partial t}+\sum_{k=1}^{3}\frac{\partial j_k}{\partial x_k}=0, \tag{1}$$

$$\frac{\partial j_k}{\partial t} = -\sum_{n=1}^{3} \frac{\partial}{\partial x_k}(j_k \rho^{-1} j_n - P_{kn}), \tag{2}$$

where the pressure tensor expressed in terms of the local density ρ and two-body interaction $U(\vec{\mathbf{x}} - \vec{\mathbf{y}})$. In this pare we shall not use the explicit form for the $P_{kn}(\vec{\mathbf{x}}, t)$. Aside from the currents and densities one is usually interested in the integrals of motion: the number of particles N , the total linear momentum $\vec{\mathbf{P}}$, the total angular momentum $\vec{\mathbf{L}}$, and the total energy E :

$$N = \int \rho d^3x, \qquad \vec{\mathbf{P}} = m \int \vec{\mathbf{j}}(\vec{\mathbf{x}}) d^3x, \qquad \vec{\mathbf{L}} = m \int \vec{\mathbf{x}} \times \vec{\mathbf{j}}(\vec{\mathbf{x}}) d^3x,$$

$$E = \frac{m}{2} \sum_{k=1}^{3} \int j_k \frac{1}{\rho} j_k d^3x + \frac{\hbar^2}{8m} \int \frac{|\nabla \rho|^2}{\rho} d^3x + \int \rho(\vec{\mathbf{x}}) U(\vec{\mathbf{x}} - \vec{\mathbf{y}}) \rho(\vec{\mathbf{y}}) d^3x d^3y.$$

For an incompressible ($\rho \equiv \rho_0$) nuclear vortical flow the Euler equation may be reduced to the following kinematical form:

$$\frac{\partial}{\partial t} rot\vec{\mathbf{v}} = rot[\vec{\mathbf{v}} \times rot\vec{\mathbf{v}}], \qquad \vec{\mathbf{j}} \equiv \rho_0 \vec{\mathbf{v}}, \tag{3}$$

where the velocity field of the nuclear matter $\vec{\mathbf{v}}(\vec{\mathbf{x}}, t)$ has been introduced in the usual way.

Further it is convenient to turn to the vorticity $\vec{\zeta}$, and the vector potential $\vec{\mathbf{A}}$:

$$\vec{\mathbf{v}} \equiv rot\vec{\mathbf{A}}, \qquad div\vec{\mathbf{A}} = 0, \tag{4}$$

$$\vec{\zeta} \equiv rot\vec{\mathbf{v}} = rotrot\vec{\mathbf{A}} = graddiv\vec{\mathbf{A}} - \Delta\vec{\mathbf{A}} = -\Delta\vec{\mathbf{A}}.$$

$$\frac{\partial}{\partial t}\vec{\zeta} + (\vec{\mathbf{v}}\nabla)\vec{\zeta} = 0.$$

We shall restrict ourselves to the simplest two-dimensional motion in the plane ($\vec{\mathbf{A}} = A\vec{\mathbf{e}}_z \quad \vec{\zeta} = \zeta\vec{\mathbf{e}}_z$):

$$\frac{\partial}{\partial t}\zeta + v_r \frac{\partial}{\partial r}\zeta + v_\phi \frac{1}{r}\frac{\partial}{\partial \phi}\zeta = 0, \qquad \zeta = -\Delta A, \tag{5}$$

$$\vec{\mathbf{v}}(r,\phi) = v_r \vec{\mathbf{e}}_r + v_\phi \vec{\mathbf{e}}_\phi, \qquad v_r(r,\phi) = \frac{1}{r}\frac{\partial A}{\partial \phi}, \qquad v_\phi(r,\phi) = -\frac{\partial A}{\partial r},$$

where (r, ϕ) are polar coordinates of a point.

The current function can be derived from the Poisson equation (5) with the help of the two-dimensional Green function for the Laplace operator:

$$A(r,\phi) = \frac{1}{2\pi} \int d\phi' dr' r' \ln(|\delta\vec{\mathbf{r}}|) \zeta(r', \phi'),$$

$$|\delta\vec{\mathbf{r}}| = \left(r^2 + r'^2 - 2rr' \cos(\phi - \phi')\right)^{1/2}.$$

If the region of the constant vorticity $\zeta(r', \phi', t) \equiv \zeta_0$, is limited by the contour:

$$\Gamma(r,\phi) \equiv r - R(\phi) = 0, \tag{6}$$

then the above formulae may be simplified to

$$A(r,\phi) = \frac{\zeta_0}{2\pi} \int_\Gamma d\phi' dr' r' \ln(|\delta\vec{\mathbf{r}}|). \tag{7}$$

The velocity projections v_r, v_ϕ can be determined from formulae (5) by differentiating $A(r, \phi)$ with respect to r and ϕ . So we have only one-dimensional integral is taken along the contour.

Up to now the consideration was pure kinematical. Therefore, it must be supplemented with the dynamical condition on the contour $(\vec{\mathbf{n}} \cdot \vec{\mathbf{v}}) = (\vec{\mathbf{n}} \cdot \vec{\mathbf{v}}_{contour})$ defined with the help of eq.(6). The normal vector $\vec{\mathbf{n}}$ is given by

$$\vec{\mathbf{n}} = \frac{\sigma \nabla \Gamma}{|\nabla \Gamma|} = \sigma\left(\vec{\mathbf{e}}_r - S(\phi)\vec{\mathbf{e}}_\phi\right)\left(1 + S(\phi)^2\right)^{-1/2}, \tag{8}$$

$$\Omega \frac{dR}{d\phi} + v_r - v_\phi(\frac{1}{R}\frac{dR}{d\phi}) = 0, \qquad S(\phi) \equiv \frac{1}{R}\frac{dR}{d\phi}, \tag{9}$$

where Ω is an angular velocity of the uniform-rotation of the contour, and σ defines the orientation of the contour.

With the help of the formulae (5-8) the equation (9) for the "disk" boundary may be cast into the form of the nonlinear integro-differential equation:

$$\frac{2\pi\Omega}{\zeta_0}\frac{dR}{d\phi} = \int_0^{2\pi} d\phi' R(\phi') \ln(|\delta\vec{\mathbf{R}}|)[(1 + S(\phi)S(\phi')) \sin(\phi' - \phi) +$$

$$+ (S(\phi) - S(\phi')) \cos(\phi' - \phi)]. \tag{10}$$

$$|\delta\vec{\mathbf{R}}| = \left(R(\phi)^2 + R(\phi')^2 - 2R(\phi)R(\phi') \cos(\phi - \phi')\right)^{1/2}.$$

It is to be noted that the equations of motion for the local density and the velocity field of the nuclear matter are nonlinear and the equations (9),(10) are derived without the usual procedure of linearization of the equations of motion in powers of $\vec{\mathbf{v}}$ and the deviation of the single particle density from the equilibrium one. Before to analyze our equations, let us remind the well known solutions.

3 The Elliptic Kirchhoff Vortex

At present there are well known elliptic Kirchhoff vortex [1] and the solution for small perturbations of the circle (we shall refer to the fundamental Lamb's monograph [1], where the references to the old original papers, hardly accessible now, can be found).

The simplest case of a circle of the constant vorticity was investigated by Kelvin [1]. Let us analyze small perturbations of the circle vortex : $\Gamma(r,\phi) \equiv r - R_0$, where R_0 is the radius of the circle. The direct calculation of the current function with the help of formula (8) gives for $r \leq R_0$:

$$A_0(r,\phi) = \frac{1}{4}\zeta_0(R_0^2 - r^2). \tag{11}$$

The small rotationless perturbation:

$$\delta A(r,\phi) = \alpha\frac{\zeta_0}{2}R_0^2\left(\frac{r}{R_0}\right)^l \cos(l\phi - \omega t), \tag{12}$$

where l is an integer, gives us the following (habitual for nuclear physics) contour equation (for small $\alpha \ll R_0$):

$$R(\phi) = R_0(1 + \alpha\cos(l\phi - \omega t)). \tag{13}$$

So the small perturbation given by trigonometrical functions (13), is a crimp moving along the circle vortex with the angular velocity

$$\Omega = \frac{\omega}{l} = \frac{(l-1)}{2l}\zeta_0. \tag{14}$$

For instance, at $l = 2$ the perturbed shape is an ellipse, rotating about its center with the angular velocity $\zeta_0/4$, that is half of the velocity of the fluid into the contour.
Perturbations of the higher symmetry $l \geq 3$ are rotating still slower.
A particular case of elliptic contour was solved exactly by Kirchhoff [1].

For the ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

one can derive the following connection between the frequency of the contour rotation and the vorticity

$$\Omega = \frac{ab}{(a+b)^2}\zeta_0 \tag{15}$$

Thus, from the above examples it follows that

[1] The author has paid his attention to elliptic vortex thanks to Prof. Smorodinsky Ya.A.

i) despite the internal part of the "disk" is rotating with a constant angular velocity, this motion differs from the motion of a rigid body, because the contour is rotating with a different velocity, more slowly;

ii) the contour velocity depends on the symmetry of a perturbation: the perturbations of higher symmetry are rotating more slowly;

iii) the fixed ratio Ω/ζ_0 and the symmetry of states define completely the shape of the contour (for instance, for the elliptic vortex its eccentricity).

The "disk" is unstable if its shape is not consistent with the ratio Ω/ζ_0 . Therefore, the parameter Ω/ζ_0 will be the bifurcation parameter.

4 Main Features of Nuclear Vortical "Disks"

For a quantitative analysis of the equation(10) it is necessary to build its discrete analogue. Such investigation is in progress. However, in this paper we restrict ourselves to the qualitative analysis, which can be done like in the previous section.

One may expect that the exact solutions of nonlinear equation (10) also may be described approximately by the symmetry relative to the turn by the angle $2\pi/l$. So if one makes Fourier analys of the contour, then the Fourier coeffitions of the corresponding $cos(l\phi)$ must be the largest.

One may also expect that the parameter Ω/ζ_0 will be the bifurcation parameter and will determine the stability of the "disk".

Equation (10) together with the definition of the velocity fields (5) will describe the motion of the contour as the propagation of a nonlinear dispersion wave on a plane. At the beginning the moving contour will be inevitably distorted. However, if this state is stable, then the interference between the nonlinearity and the dispersion will lead to the return of the initial contour shape. If one could prove the existence of these states, then the vortical "disks" of nuclear matter will be an analogue of the solitons on a plane.

It is necessary to note, that despite the thirty-years history of the soliton theory, up to now physical solutions in two- and three-dimensional spaces are practically absent. The generalization of the one-dimensional solution to the two- and three-dimensions is in principle highly difficalt.

If the contour motion is unstable, it is very interesting to investigate its evolution. The integrals of motion are the square of the "disk" which is a two-dimensional analogue of the particle number, and the circulation of the vortex defined by the help of ζ_0. So one mays expect the disintegration of the "disk" into the vortex filament and into the separate rotating "disks".

It is very interesting to analyze three-dimensional axially symmetric finite "disks". It is necessary to mention the recent paper [3], where the head-on collisions of two nearly symmetric heavy ions were simulated by the Boltzmann - Nordheim - Vlasov equation. During these calculations, nuclear "disks" formed. During a collision process a "disk" develops due to the side-squeezing of nuclear matter, whose thickness decreases and diameter increases monotonically with increasing bombarding energy. If the "disk" becomes thin, it breaks up into several fragments of a size commensurate with the thickness of the "disk". So the "disk" stability problem is connected with the multifragmentation.

Finally, in this paper we have derived the basic equations for nuclear plain vortex. The evolution of the shape of the "disk" is analogous to the propagation of a nonlinear dispersion wave on the plane. The qualitative analysis of the main features of the "disk" has been made. We have pointed out the possible relation of the "disk" stability problem to the multifragmentation process.

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