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QUASI-ENERGY OF ULTRACOLD NEUTRONS

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In a number of recent papers [2-4] the possibility is discussed of testing the consequences of the nonstationary Schrodinger equation in experiments with slow neutrons. It seems that most attractive in this connection are ultracold neutrons (UCN). They have a large wavelength, $\lambda = h/mv \geq 5\times10^{-6}$ cm. The characteristic quantum time is also sufficiently large in their case:

$$\tau \simeq \frac{\hbar}{\varepsilon} \simeq 6.10^{-9}\,\mathrm{sec} \tag{1}$$

(at the energy $\varepsilon \simeq 10^{-7}$ eV).

Therefore, the experiments seem feasible.

This paper discusses the possibility of performing an experiment which consists in transmitting a monochromatic beam of UCNs through a periodically acting fast quantum chopper [3-4]. The theory of the effect this chopper produces on an initially monochromatic neutron beam can be developed self-consistently. The energy spectrum of the transmitted neutrons appears to be a discrete one. The fact is rather a characteristic consequence of the nonstationary quantum mechanics possible to be checked on in the experiment.

In the simplest case it is a one-dimensional problem. The first step towards the solution of it was made by Moshinsky in 1952, who analysed the case of an instantaneous removal of an ideal absorber from the neutron beam [1]. The initial state is a semi-infinite plane wave:

$$\psi(x,0) = e^{ikx}\,\theta(-x), \tag{2}$$

filling the left half-space. The wave function a(k) in the momentum representation [5] determines the evolution of this state as:

$$\frac{a(k')}{2\pi} = \frac{1}{2}\,\delta(k'-k) + \frac{i}{2\pi}\,P\,\frac{1}{k'-k}$$

$$\psi(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} a(k')\,e^{i(k'x-\omega' t)}\,dk' \quad . \tag{3}$$

Here P means the principal value of integration. The Moshinsky's solution of eqs. (2)-(3) reveals strong dependence on the variable

$$\xi = vt - x. \tag{4}$$

The point $\xi=0$ corresponds to the classical beam front traveling with the velocity $v = \hbar k/m$. This front is gradually spreading in scale on propagation:

$$\Delta\xi \simeq \sqrt{\lambda\!\!\!^{-} x} \tag{5}$$

($x \simeq vt$; $\lambda\!\!\!^{-} = k^{-1}$). In the whole range of ξ values the picture reported in [1] reproduces the Fresnel light diffraction pattern from a sharp edge [6]. Note that the analogous result was obtained by Zommerfeld and Brilloin in 1914 for the case of an electromagnetic wave front traveling in a dispersive medium [7].

Now, turning our attention back to the chopper, we shall concentrate on the periodical mode of its operation. This regime of operation seems attractive from the theoretical and convenient from the experimental viewpoint. Let an absorbing shutter be put in the beam at the time $t=T$ and out at $t=2T$ with many repetitions. The total period is $2T$. The condition

$$T \gg \hbar/\varepsilon = \omega^{-1}. \tag{6}$$

ensures quick enough recovery of the initial plane wave all over the left half-space $x<0$, in analogy with eq. (2). Then in accordance with the superposition principle the wave function of the transmitted neutrons can be expressed through the Moshinsky's wave function (3).

After the (n+1)th shutter's operation, we have

$$\psi(x,t) = \int_{-\infty}^{\infty} e^{i(k'x-\omega' t)} \frac{1-e^{in[(\omega'-\omega)T+\pi]}}{1-e^{i[(\omega'-\omega)T+\pi]}} \frac{a(k')}{2\pi}\, dk'. \tag{7}$$

Then we may proceed to the limit $n \to \infty$. With each of the two terms in the numerator being independently integrated, some poles appear,

$$\omega_s = \omega + \frac{\pi(2s-1)}{T}; \quad k_s = \left(\frac{2m\omega_s}{\hbar}\right)^{1/2}, \tag{8}$$

which define the new energy values. Here s, an integer, is the number of a satellite line. The energy change occurs in accordance with the uncertainly principle (e.g. see [5])

$$\Delta E \Delta t \approx \hbar \tag{9}$$

and it is relatively small. The final result can be obtained in the form of a superposition

$$\psi = \frac{1}{2} e^{i(kx-\omega t)} + \frac{i}{\pi} \sum_{s=-\infty}^{\infty} \frac{e^{i(k_s x-\omega_s t)}}{2s-1}. \tag{10}$$

These equidistant satellites (8) together with the initial line ω, are related as the quasi-energy of the particle [8].

Rather a complicated wave structure of the transmitted neutron beam shows typical beats. At distances not far from the chopper

$$x \ll \frac{(vT)^3}{\lambda\!\!\!^{-2}}$$

their large-scale period is

$$L \simeq \frac{4mv^3T^2}{\pi\hbar} \gg vT, \tag{11}$$

as it follows from the last term in the expansion:

$$k_s x - \omega_s t \cong kx - \omega t - \frac{\pi\xi}{vT}(2s-1) - \frac{\pi^2}{2}\frac{\hbar x}{mv^3T^2}(2s-1)^2. \tag{12}$$

Expression (11) can be obtained by making use of a simple relationship:

$$(2s-1)^2 = 4s(s-1)+1, \quad \frac{4s(s-1)}{8} = \text{integer}. \tag{13}$$

It seems obvious that in the near chopper beam region there should appear some small-scale structure elements of linear dimensions $\approx vT$. These structure elements propagate with the velocity v and we analyse their evolution below. For the numbering of these structure elements of density it is convenient to introduce the following notations:

$$pvT < \xi < (p+1)vT, \qquad \zeta = \xi - pvT. \tag{14}$$

The same integer index p can also be a number also a thin transition zone (a kink) in the region $|\zeta| \ll vT$ between the corresponding elementary structures. (The variable ζ is counted from the position of the classical front related to the p-th operation of the chopper). Under assumption of the smallness of the distance from the chopper, we neglect the square term in the dispersion law (12) and, by performing summation after eq.(10), obtain:

$$x \ll \frac{(vT)^2}{\lambda\!\!\!^{-}}, \qquad \psi \cong \frac{1}{2}\left[1+(-1)^p\right] e^{i(kx-\omega t)} \tag{15}$$

in the interior of a given structure element. It is true, however, that close to the point $\zeta=0$ i.e. in the transition layer, the square term becomes important again. But, in this case, eq.(10) allows substitution of summation over index s for integration:

$$\psi = e^{i(kx-\omega t)}\left\{\frac{1}{2} + (-1)^p \frac{i}{2\pi}\int_{-\infty}^{\infty} e^{-i\frac{\hbar x}{2mv}æ^2} e^{-i\zeta æ}\frac{dæ}{æ}\right\}$$

$$= e^{i(kx-\omega t)}\left\{\frac{1}{2} + (-1)^p \frac{e^{-i\frac{\pi}{4}}}{\sqrt{2\pi}}\sqrt{\frac{mv}{\hbar x}}\int_0^{\zeta} e^{i\frac{mv}{2\hbar x}\zeta'^2} d\zeta'\right\}; \tag{16}$$

$$æ \cong k_s - k, \qquad |\zeta| \ll vT.$$

Expression (16) describes the shape of the fronts (kinks). Applicability areas for eqs.(16) and (15) overlap. Thus, in this region the fragment $(-1)^p = +1$ having the same

neutron density $|\Psi|^2 \simeq 1$ as the initial wave is confined within by Moshinsky's kinks [1].

With increasing distance x, the structure elements are merging due to the spreading of their fronts in accordance with estimate (5). However, the discrete character of the spectrum (8) leads to the situation, when, in some comparatively localized regions, the small-scale structuring recovers and sometimes in rather unexpected forms. For the illustration of this recovery we shall turn to the region:

$$x = \frac{mv^3T^2}{\pi\hbar} + x_1, \qquad |x_1| \ll \frac{(vT)^2}{\lambda\!\!\!^{-}}. \tag{17}$$

By using formulas (10), (12)-(14) and procedures analogous to the above mentioned we arrive at the following results:

$$\psi \cong \frac{1}{\sqrt{2}} \exp\left\{i\left[kx - \omega t - (-1)^p \frac{\pi}{4}\right]\right\}, \qquad |\Psi|^2 \simeq \frac{1}{2} \tag{18}$$

$$\psi = e^{i(kx-\omega t)}\left\{\frac{1}{2} + \frac{(-1)^p}{2\pi}\int_{-\infty}^{\infty} e^{-i\frac{\hbar x_1}{2mv}æ^2} e^{-i\zeta æ}\frac{dæ}{æ}\right\} \tag{19}$$

$$|\zeta| \ll vT$$

We see that here the structure elements of the phase type are present. The interior density of these elements is the same, but the sign of the additional phase $\varphi = \pi/4$ alternates. Thus, the wave function phases of two neighbouring elements differ by $\pi/2$.

The density kink at the interface of the phase elements is described with exp.(19). The variable:

$$\Delta\zeta \simeq \sqrt{\lambda\!\!\!^{-}|x|}$$

gives estimate on the kink's width scale. One can easily find corresponding asymptotic expressions at small and large values of this variable. Figure 1 gives a more complete idea of how the phase density kink looks like over the whole range of ζ values.

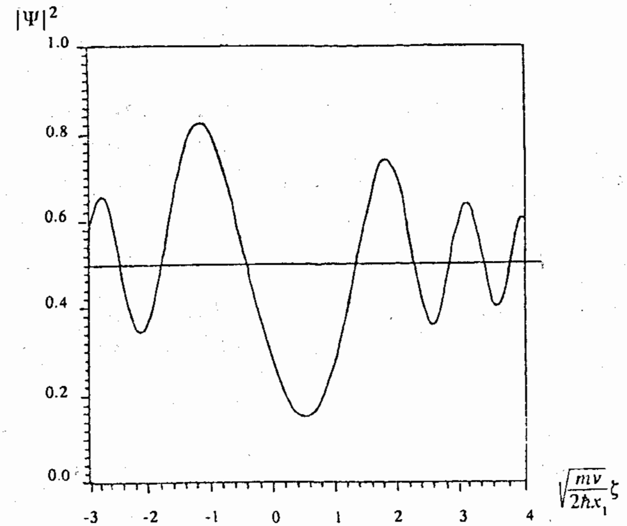

Figure 1.

Now we shall consider in brief the situation when the neutron beam density is measured with the detector positioned in some fixed point. It is obvious that in this situation the dependence of density on time is a strictly periodic function:

$$|\psi|^2 = \sum_{n=-\infty}^{\infty} C_n e^{-i\omega_n t}, \qquad \omega_n = \frac{n\pi}{T} \tag{20}$$

Coefficients C_n in the Fourier expansion of the neutron beam density (20) can be calculated explicitly if the detector is positioned in the area, where the small-scale structure elements are distinct enough. For example, for the nearest to detector region and with an appropriately chosen origin for t we obtain:

$$\begin{aligned} C_0 &= \frac{1}{2} \\ C_n &= -\frac{i}{\pi n} \operatorname{Cos} \frac{\pi^2}{2} \frac{\hbar x}{mv^3T^2} n^2 \qquad n = 2s - 1 \\ C_n &= -\frac{1}{\pi |n|} \operatorname{Sin} \frac{\pi^2}{2} \frac{\hbar x}{mv^3T^2} |n|^2 \qquad n = 2s \neq 0 \end{aligned} \tag{21}$$

$$x \ll \frac{(vT)^2}{ƛ}$$

Analogous expressions can also be obtained for the phase structuring area defined by exp.(17).

Neutron beam beats can be observed in experiments. Supplementary to them could be the experiments on neutron beam polarization in an external homogeneous magnetic field. It is important to emphasize here the feasibility of a direct experiment on the measurement of discrete energy spectra of UCNs after the periodic chopping of the beam.

In this experiment one must not necessarily shut the beam down fully and simultaneously over its whole cross-section. It appears sufficient just to periodically chop the beam at every point of the cross section. Thus we come to the problem of the motion of a periodical absorbing structure across the beam. Let us consider this problem in detail.

Imagine a plane wave, across of which at point $x = 0$ an infinite plane grating with a spatial period $2a$ is moving with a velocity V in the positive direction of the y axis. It is assumed as before that the time $T=a/V$ satisfies the condition (6). By solving the problem of diffraction in a moving reference frame connected with the grating, under the assumption that $ka \gg 1$, one obtains:

$$\psi_0(x,y,t) = e^{-i(k_w y+\omega_w t)} \left\{ \frac{1}{2} e^{i(kx-\omega t)} + \right. \tag{22}$$

$$\left. \frac{i}{\pi} \sum_{s=-\infty}^{\infty} \frac{e^{i\left[\left(\sqrt{k^2+2k_w æ_s - æ_s^2}\right) x + æ_s y - \omega t\right]}}{2s-1} \right\}$$

Here the expression under the square root is found from the energy conservation law and the following notations are accepted:

$$k_w = \frac{mV}{\hbar}, \qquad \omega_w = \frac{mV^2}{2\hbar}, \qquad æ_s = \pi(2s-1)/a. \tag{23}$$

Here s, is the integer that defines the diffraction order 2s-1. Now we pass to the laboratory frame of reference by applying the Galilean transformation to the wave function. In accordance with ref.[5] we must replace the variable y in (22) for the y-Vt and then multiply the whole expression for the wave function by $e^{i(k_w y-\omega_y t)}$. Besides, we take into account that

$$æ_s V = (2s-1)\frac{\pi}{T} = \omega_s - \omega \; ; \quad æ_s = \frac{\omega_s - \omega}{V} \; ; \quad \omega = \frac{\hbar k^2}{2m},$$

and we arrive at the final formula

$$\psi(x,y,t) = \frac{1}{2} e^{i(kx-\omega t)}$$

$$+ \frac{i}{\pi} \sum_{s=-\infty}^{\infty} \frac{e^{i\left[\left(\sqrt{k^2+2k_w æ_s - æ_s^2}\right) x + æ_s y - \omega_s t\right]}}{2s-1} . \tag{24}$$

Provided the value for the inverse lattice vector $æ_s$ is

small, one has:

$$k_w \gg æ_s \; ; \qquad T \gg \frac{\hbar}{mv^2} . \tag{25}$$

$$\sqrt{k^2+2k_w æ - æ_s^2} \cong k + \frac{k_w æ_s}{k} = k + \frac{m}{\hbar k} V æ_s \cong k_s . \tag{26}$$

The term $æ_s y$ in (24) is purely diffractional. It disappears at large grating periods, i.e. at small $æ_s$. Now it is seen under what conditions exp.(24) transforms into the exp.(10). The transformation occurs, if the grating velocity V and the lattice constant *a* are simultaneously increased without changing the chopping time T=*a*/V. It can be shown that the experiment with a moving grating can indeed be very well described with formulas (10).

The idea seems attractive of the use of a phase grating in this experiment. Under the requirement that the thickness of a transparent plate, phase grating, varies with a step *a* and a period 2*a* by a value

$$\Delta d = \frac{\pi}{k(n-1)} , \tag{27}$$

where n is the refractive index of the transparent material of the plate, the phase of waves, transmitted through the neighbouring elements of the grating will differ by π, (a so-called "π-grating".

By repeating the calculation in an analogous manner we obtain:

$$\psi(x,y,t) = \frac{2}{i\pi} \sum_{s=-\infty}^{\infty} \frac{e^{i\left[\left(\sqrt{k^2+2k_w æ_s - æ_s^2}\right) x + æ_s y - \omega_s t\right]}}{2s-1} . \tag{28}$$

The difference from (24) consists in disappearance of the central line with the initial wave number k and in increasing intensity of satellite lines by a factor of four .

At $T \approx 2.5 \times 10^{-7}$s, which is an order of magnitude larger than in experiments with cold neutrons [4], in order to achieve the necessary energy resolution, $\Delta\varepsilon$, the following requirement should be satisfied:

$$\Delta\varepsilon \ll \hbar(\omega_s - \omega_s) \simeq \frac{2\pi\hbar}{T} \simeq 10^{-8}\text{eV}.$$

Such a resolution appears quite within the possibilities of the UCN gravitational spectrometry [9]. It seems attractive to apply to UCN the spin-echo method as well as the method of interference filters [10-12] having a resolution of the order of 10^{-9}eV.

In the latter case, the instrument could have comprised two such filters with a chopper positioned between them. One of the filters would serve as the monochromator and the other as the analyzer. One should have the opportunity to vary the neutron energy in front of the filter-analyser. This can be easily accomplished by one of the known methods: the Doppler shift or inhomogeneous magnetic field method (for which polarized neutrons must be available), or the method of UCN acceleration (deceleration) due to gravity. In the latter case the neutron beam should be sent vertically downwards (upwards). Then the energy analysis consists in the measurement of neutron transmission through the second filter in dependence on the height of the neutron rise in the gravitational field. It is clear that on neutron's traveling the distance of 1 cm its energy changes by 10^{-9}eV.

The main conclusions are the following. In accordance with the quasi-energy conservation law and in result of the periodical mode of neutron transmission a nonstationary state with a discrete energy spectrum is generated. In the configuration space the transmitted wave exhibits complex beats good for theoretical description. Nonstationary effects can be verified directly in the experiment by measuring energy changes which are of purely quantum character.

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