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E4-92-282

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SUPERSYMMETRY OF GAUGE EQUATIONS AND GEOMETRIC NONADIABATIC PHASES

Submitted to "Physics Letters"

1992

Supersymmetric quantum mechanics allows us to find exact solutions for a broad class of problems including a lot of models obtained by the inverse scattering method and Darboux transformations [2, 3]. Exactly solvable models of the three-particle and many-dimensional problems in the adiabatic approach [4, 5] were developed on the basis of algebraic generalized Bargmann-Darboux transformations in our previous papers [6, 7].

The present work deals with the extension [7, 8] of supersymmetry quantum mechanics to a system of gauge equations, obtained in the adiabatic representation and also with additional geometric phases arising in a special case of supersymmetry and produced by singularities of gauge vector-potential A due to crossing of potential curves.

Nondiagonal elements of the induced connection operator A realise transitions between states of the parametric so-called instantaneous Hamiltonian H^f and generate nonadiabatic Aharonov-Anandan phases [1, 9]. In the realistic statement of the three-body problem [4, 5], the nondiagonal matrix elements of A have to be taken into account since without them it is impossible to right solve the problem. Moreover near the level crossing the adiabaticity is not valid. When the level crossing or quasicrossing between two or even three terms takes place, there arise singularities of A_{nm} and as a result, relevant additional geometric phases. This makes necessary to introduce geometrical nonadiabatic matrices in the presence of A singularities in addition to the Aharonov-Anandan phase. Berry phases [10] are obtained in the adiabatic limit when transitions between different states are ignored.

Induced gauge potentials appear naturally in the description of quantum-mechanical systems dependent upon slowly varying external parameters. This occurs in many real systems, there are fast and slow degrees of freedom and one should estimate the effect of the slow dynamic on the behaviour of the fast ones and vice versa. In this case the total Hamiltonian is decomposed into $H = H^s + H^f$, where $H^f(\mathbf{R})$ is the parametric family of the "fast" Hamiltonians, depending on slow variables. The searched wave function of H is expressed as a sum over the eigenfunctions $\Phi_n(\mathbf{R},\mathbf{r})$ of the instanteneous Hamiltonian H^f for each fixed value of the slow variable $\mathbf{R}$

$$\Psi(\mathbf{R},\mathbf{r}) = \sum \Phi_n(\mathbf{R};\mathbf{r}) F_n(\mathbf{R}); \qquad (1)$$

$$H^f(\mathbf{R})\Phi_n(\mathbf{R};\mathbf{r}) = E_n(\mathbf{R})\Phi_n(\mathbf{R};\mathbf{r}). \tag{2}$$

We use the orthonormalization $< n \mid m >= \delta_{nm}$ and completeness $\mid n >< n \mid = 1\delta(\mathbf{r}\text{-}\mathbf{r}')$ of the eigenstates $\mid \Phi_n(\mathbf{R}) >$ of the Hamiltonian H^f at fixed $\mathbf{R}$ and get the "slow" system of equations of the gauge tipe for the expansion coefficients F_n

$$-1/2[\nabla_{\mathbf{R}} I - iA(\mathbf{R})]^2 F(\mathbf{R}) + V(\mathbf{R})F(\mathbf{R}) = EF(\mathbf{R}) \;, \tag{3}$$

where $F = \{F_n\}$ is a column-vector of dimension M, I is the unit matrix and $\mathbf{A}, V$ are the vector and scalar components of the gauge field

$$A_{nm}(\mathbf{R}) =< \Phi_n \mid i\nabla_R \mid \Phi_m > \tag{4}$$

$$\begin{aligned} V_{nm}(\mathbf{R}) =< \Phi_n \mid H^f \mid \Phi_m > \delta_{nm} \;&+\; 1/2 \sum_{k\neq n,m} A_{nk}A_{km} \\ &+\; < \Phi_n \mid V^s \mid \Phi_m > \end{aligned} \tag{5}$$

Equation (3) possesses the unitary gauge symmetry $U(M)$ group. For a complete set Φ_n the second term in (5) vanishes and if A has not singularities one can find a gauge (pure gauge) where the induced vector potential A also vanishes. In the one-state approximation $F(R)$ becomes a scalar wave function.

In one-dimension case we can represent the Hamiltonian H of Eq.(3) and its supersymmetric partner in a factorized form

$$H^- = 1/2Q^-Q^+ \;,\quad H^+ = 1/2Q^+Q^- \tag{6}$$

with

$$Q^\pm = \pm D + \alpha(R) \;,\quad D = d_R - iA(R). \tag{7}$$

It follows that

$$\begin{aligned} H^\pm = 1/2\{[\;&-\; D^2 + \alpha^2(R)]I + \sigma_3(D\alpha(R) - \alpha(R)D)\}; \\ V_+(R) \;&=\; V_-(R) + D\alpha(R) - \alpha(R)D \end{aligned} \tag{8}$$

Here

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

In the two-component notation, (8) is rewritten as the supersymmetric Hamiltonian

$$H^s = 1/2\{Q_-, Q_+\} = \begin{pmatrix} H^+ & 0 \\ 0 & H^- \end{pmatrix}. \tag{9}$$

Here Q_- and Q_+ are determined as usual to be supersymmetric charges

$$Q_+ = \begin{pmatrix} 0 & Q^+ \\ 0 & 0 \end{pmatrix} \;,\quad Q_- = \begin{pmatrix} 0 & 0 \\ Q^- & 0 \end{pmatrix} \tag{10}$$

and they satisfy

$$Q_+^2 = Q_-^2 = [H^s, Q_+] = [H^s, Q_-] = 0. \tag{11}$$

If we ignore off - diagonal transition terms in the system of Eqs.(3), then we arrive at a system of noncoupled equations. It is the adiabatic approximation. In this case the supersymmetric Hamiltonian (9) is rewritten as

$$H^s = 1/2\{[-D^2 + \alpha^2(R)]I + \sigma_3 d_R\alpha(R)\},$$

where $\alpha(R)$ is related to $V(R)$ as usually by the Ricatti equation: $\alpha^2(R) - d\alpha(R) = V_-(R)$, $\alpha(R) = d_R W$ in the ground state wavefunction representation. The wave functions of H^+ and H^- at an arbitrary energy are related by the transformations

$$\begin{aligned} \chi_+(R,E) \;&=\; Q^+\chi_-(R,E) = [D + \alpha(R)]\chi_-(R,E) \\ &=\; \chi_0^{-1}(R)W_D\{\chi_0(R), \chi_-(R,E)\} \end{aligned} \tag{12}$$

generalizing the ordinary Darboux - Krum - Krein transformations. Here $\chi_0(R)$ is the wave function of the ground state of H^- represented in the form

$$\chi_0(R) = P\exp\{-i\int^R A(R')dR'\}exp\{-\int^R \alpha(R')dR'\} \tag{13}$$

and the generalized Wronskian W_D is defined by $W_D = \{\chi_0^\dagger D\chi_- - (D\chi_0)^\dagger\chi_-\}$, P denotes path ordering.

Next following [11, 8] one can represent three- or two-dimension extensions of (6)-(13) for the system of gauge equations (3) which describes a slow dynamics of nonrelativistic systems (in particular, a three-body system) and supersymmetry for it. Therefore we introduce the supercharges as

$$\mathbf{Q}^{+} = \frac{1}{\sqrt{2}}\tau_{+}\sigma\mathbf{Q}^{+} = \frac{1}{\sqrt{2}}\tau_{+}\sum_{\mu}\sigma_{\mu}Q_{\mu}^{+} ;$$
$$\mathbf{Q}^{-} = \frac{1}{\sqrt{2}}\tau_{-}\sigma\mathbf{Q}^{-} = \frac{1}{\sqrt{2}}\tau_{+}\sum_{\mu}\sigma_{\mu}Q_{\mu}^{-} , \qquad (14)$$

where

$$Q_{\mu}^{\pm} = \pm D_{\mu} + \partial_{\mu}W,\ D_{\mu} = \partial_{\mu} - iA_{\mu}, \qquad (15)$$

$\tau_{\pm} = 1/2(\sigma_1 \pm i\sigma_2)$, $\sigma_{\mu}(\mu = 1,2,3)$ are Pauli spin matrices. Then the supersymmetric Hamiltonian is given by

$$H^{S} = \frac{1}{2}\{\mathbf{Q}^{+},\mathbf{Q}^{-}\} = \frac{1}{2}\begin{pmatrix} (\sigma\mathbf{Q}^{+})(\sigma\mathbf{Q}^{-}) & 0 \\ 0 & (\sigma\mathbf{Q}^{-})(\sigma\mathbf{Q}^{+}) \end{pmatrix}. \qquad (16)$$

In addition to terms of $H^{+} = \frac{1}{2}Q^{+}Q^{-}$ and $H^{-} = \frac{1}{2}Q^{-}Q^{+}$ of the one - dimensional supersymmetric Hamiltonian (6) in Hamiltonians (16) there arise the matrix field strength tensor

$$F_{\mu\nu} = \partial_{\mu}A^{\nu} - \partial_{\nu}A^{\mu} - i[A^{\nu}, A^{\mu}] = \epsilon^{\nu\mu k}B^{k}, \qquad (17)$$

associated with magnetic field, and terms $\sigma(\nabla_{\mathbf{R}}W \times \mathbf{P})$ similar to spin-orbital couplings $\frac{2}{R}\partial_{R}W(\sigma\mathbf{L})$ for the central fields [11].The procedure (6) - (16) is an extension of ordinary procedure in the supersymmetric nonrelativistic quantum mechanics and it is based on the factorization of the total Hamiltonian of Eq.(3). It permits one to generate a wide class of exact solvable models for Eqs.(3) in the presence of effective vector potentials.

Let us thoroughly analyse approach, immediately generalizing the well-known problem of a spin - 1/2 charged particle moving in a plane under the influence of a perpendicular magnetic field [13, 12]. The supersymmetry in this case is based on the fact that in the two-dimensional space the Pauli Hamiltonian

$$H^{P} = 1/2[-i\nabla - e\mathbf{A}]^{2} - e/2B\sigma_{3} \qquad (18)$$

can be written as the square of the Dirac Hamiltonian ($\hbar = c = m = 1$)

$$H^{P} = 1/2(H^{D})^{2},$$
$$H^{D} = i\sigma_{-3}\nabla + e\sigma\mathbf{A}. \qquad (19)$$

Let us define the hermitian supercharges $Q_i \equiv Q_i(x,y)$

$$Q_1 = 1/2[\sigma_1(\pi_x - \partial_y W) + \sigma_2(\pi_y + \partial_x W)],$$
$$Q_2 = 1/2[\sigma_2(\pi_x - \partial_y W) - \sigma_1(\pi_y + \partial_x W)] \qquad (20)$$

with additional scalar functions ∂W_{μ} with respect to the classic problem (18) - (19) . In a multichannel case of the adiabatic approach $W(x,y)$ is a matrix and π_{μ} is related to the covariant derivative D by

$$\pi_{\mu} = -iD_{\mu} = [-i\partial_{\mu}I - A_{\mu}]$$

with matrices A_{μ} defined from (4). The supercharges Q_i satisfy the set of relations of the Witten supersymmetric quantum mechanics

$$[Q_i, Q_j] = \delta_{ij}H \text{ and } [H, Q_i] = 0, \qquad (21)$$
$$(i = 1, \ldots, N).$$

As usual,introduce the non-hermitian supercharges

$$\mathbf{Q}^{+} = \frac{1}{\sqrt{2}}[-Q_2 + iQ_1],$$
$$\mathbf{Q}^{-} = \frac{1}{\sqrt{2}}[-Q_2 - iQ_1]. \qquad (22)$$

Then they are represented as two-by-two matrices

$$\mathbf{Q}^{+} = \frac{1}{\sqrt{2}}\tau_{+}[(i\pi_x + \partial_x W) + (\pi_y - i\partial_y W)] = \frac{1}{\sqrt{2}}\tau_{+}[Q_x^{+} - iQ_y^{+}],$$
$$\mathbf{Q}^{-} = \frac{1}{\sqrt{2}}\tau_{-}[(-i\pi_x + \partial_x W) + (\pi_y + i\partial_y W)] = \frac{1}{\sqrt{2}}\tau_{-}[Q_x^{-} + iQ_y^{-}] \qquad (23)$$

corresponding to (14).Using them we can construct the supersymmetric Hamiltonian

$$H^{s} = 1/2\{\mathbf{Q}^{+},\mathbf{Q}^{-}\} = 1/2\begin{pmatrix} \mathbf{Q}^{+}\mathbf{Q}^{-} & 0 \\ 0 & \mathbf{Q}^{-}\mathbf{Q}^{+} \end{pmatrix}$$
$$= \frac{1}{2}\{\mathbf{Q}^{+},\mathbf{Q}^{-}\} + 1/2\sigma_3[\mathbf{Q}^{+},\mathbf{Q}^{-}] \qquad (24)$$

and the supercharges satisfy the relations of supersymmetry algebra (11).

Let us more closely examine two supersymmetric partners $H^+ = 1/2(\mathbf{Q}^+\mathbf{Q}^-)$ and $H^- = 1/2(\mathbf{Q}^-\mathbf{Q}^+)$ of H^s -Hamiltonian (24)

$$\begin{aligned} H^+ &= 1/2[Q_x^+Q_x^- + Q_y^+Q_y^- + i(Q_x^+Q_y^- - Q_y^+Q_x^-)], \\ H^- &= 1/2[Q_x^-Q_x^+ + Q_y^-Q_y^+ - i(Q_x^-Q_y^+ - Q_y^-Q_x^+)]. \end{aligned} \tag{25}$$

Using Eqs. (23) we present the expressions for Hamiltonians $H^\pm$ in explicit form

$$H^+ = 1/2[\pi_x^2 + \pi_y^2 + i(\pi_x\pi_y - \pi_y\pi_x) + (\partial_x W)^2 + (\partial_y W)^2$$
$$+(i\pi_x + \pi_y)(\partial_x W + i\partial_y W] + \partial_x W\pi_y - \partial_y W\pi_x],$$

$$H^- = 1/2[\pi_x^2 + \pi_y^2 - i(\pi_x\pi_y - \pi_y\pi_x) + (\partial_x W)^2 +$$
$$(\partial_y W)^2 + (-i\pi_x + \pi_y)(\partial_x W - i\partial_y W) + \partial_x W\pi_y - \partial_y W\pi_x]. \tag{26}$$

These relations are rewritten in more short and convenient form

$$H^+ = 1/2[\pi^+\pi^- + (\partial_x W)^2 + (\partial_y W)^2 +$$
$$\pi^+(\partial_x W - i\partial_y W) + \partial_x W\pi_y - \partial_y W\pi_x];$$

$$H^- = 1/2[\pi^-\pi^+ + (\partial_x W)^2 + (\partial_y W)^2 +$$
$$\pi^-(\partial_x W + i\partial_y W) + \partial_x W\pi_y - \partial_y W\pi_x]. \tag{27}$$

Here one takes into account ordinary determinations of non-hermitian supercharges

$$\pi^+ = (+i\pi_x + \pi_y)\ ,\ \pi^- = (-i\pi_x + \pi_y)$$

producing the matrix analog of the Pauli Hamiltonian (18)

$$H^P = 1/2\{\pi^+\ \ ,\pi^-\} = 1/2(\sigma\pi)^2$$

with the scalar potential matrix

$$V = F_{\mu\nu}\sigma_3. \tag{28}$$

As follows from Eqs (23) at $W(x, y) = 0$ the non-hermitian supercharges $\mathbf{Q}^+$ and $\mathbf{Q}^-$ turn into ones π^+ and π_- and supersymmetric Hamiltonian (24) H^s turns into H^p. It is equivalent to the principle of minimal switching on of interaction with the electromagnetic field as with the gauge one A. If the vector potentials $A_\mu = 0$ from the

relations (20) - (24) we easily obtain the Witten construction [15] of the supersymmetric quantum mechanic in two-dimensional space [11]. It becomes quite clear if generators $i\pi_x$ and $i\pi_y$ are replaced by the ordinary partial derivations in equations (26),(27). Thus the relations (20) - (25) represent the natural extension of the Witten construction for a system of gauge equations in two-dimensional space. A three-dimensional model of SUSY for gauge equations is obtained by the same way from Eqs. (14) - (16).

At the application such approach for 1/2 -spin charged particles the so-called spin-flip effect takes place with the simultaneously changing coordinate dependence of wave functions, when the generators $\mathbf{Q}^+$ and $\mathbf{Q}^-$ (22) turn into superpartner states into each other

$$\chi_+ = \mathbf{Q}^+\chi_- \text{ and } \chi_- = \mathbf{Q}^-\chi_+.$$

If $\chi_{-\sigma_3}$ is an eigenstate of (24) then $\chi_{+\sigma_3}$ is given by

$$\chi_{+\sigma_3} = [(i\sigma_3(\pi_x - \partial_y W) + (\pi_y + \partial_x W)]\chi_{-\sigma_3}. \tag{29}$$

A zero eigen - state $\chi_0 = \chi_{-\sigma_3}$ of H must be annihilated by Q^+

$$Q^+\chi_0 = \sigma_3\{[\partial_x - i(A_x + \partial_y W)] + (-i\partial_y - A_y + \partial_x W)\}\,\chi_0 = 0. \tag{30}$$

One can easily see that in the case of the adiabatic representation (1)-(3) if the scalar potential satisfies the candition (28), we arrive at the well-known situation stated in [12] . The ground state of the Hamiltonian H (24) with $W = 0$ is degenerated the number of zero energy states being governed by the Atiah-Singer index theorem

$$\chi_{0_j} = (x + i\sigma_3 y)^j \exp\{-\sigma_3 \int\!\!\int F_{xy}(x, y)dx\,dy\}, \tag{31}$$
$$(j = 1, \dots, N-1)$$

Now it is to the point to cite a piece from paper [12] that "the index theorem relates the number of a zero modes of a particle moving in a external gauge field to the topological winding number (in our case the line integral of at infinity)". i.e.

$$\oint A_\perp = 2\pi(N + \epsilon)\ ,\quad 0 < \epsilon < 1.$$

Here N defines the degeneracy multiplicity of zero-energy eigen-states. Hence we obtained a simple example of the adiabatic quantum system giving rise to the phase factor, which apparantly is related with Wilczek and Zee one [14] . It can be expressed as a surface integral of a field strength tensor $F_{\mu\nu}$ of an induced gauge field A. The curvature tensor F is in general a complicated nonlocal functional of A. In fact ϵ also defines the geometric phase that can be called Aharonov - Casher phase. If $\epsilon = 0$ the flux $\Phi = \int\int F_{xy}(x,y)dx\,dy$ is quantized and one can said about the Hall effect.

What will happen with the degeneracy of the ground state χ_0 of the Hamiltonian (24) if it will be defined by the supercharges (23) and $W \neq 0$. In this case we have

$$\chi_0(x,y) = f(x,y)P\exp\{-\sigma_3 \int\int B_{xy}(x,y)dx\,dy\} \qquad (32)$$

where the matrix tensor B_{xy} is represented by

$$B_{xy}(x,y) = F_{xy}(x,y) - (\partial_x + i\partial_y)(\partial_x W(x,y) - i\partial_y W(x,y))$$

and the function f(x,y) satisfies equation

$$(\partial_x + i\sigma_3\partial_y)f(x,y) = 0.$$

Hence in the presence of $W(x,y)$ the function $f(x,y)$ is an entire function of $(x + i\sigma_3 y)$ as before (31) and as a result we have degeneracy of the ground state. It is now trivial to see from (32) and (31) that the presence of scalar potential can lead to decrease and even to cancell a positive integer N and as result to spontaneous breaking of supersymmetry. But the fact of the presence or the absent of the degeneracy due to the supersymmetry not eliminate the geometric Berry phase [10] and those related to the transfers between levels referred to as the nonadiabatic Aharonov - Anandan phase [1, 9] .

Of a special interest are the cases of potential curves crossing , which leads to singularities of gauge-vector potential A. In the presence of A singularities there arise the nontrivial geometrical phase in the wave functions χ_0 and hence $\chi(R,E)$ at an arbitrary energy (12)

$$\delta = \frac{1}{2}\, i \oint_C A(\mathbf{R})\, d\mathbf{R}.$$

These phases are to be taken into account in addition to the standard ones. At the beginning consider some simple examples. If $A(R) = i(\nu R^{-1})$ and R is the radial variable independent of angles, then $\delta_- = (\pi\nu)/2$. It may be the case of the free motion in $\mathbf{R}_n$ described in spherical space parameterization $\mathbf{R}^1_+ \times \mathbf{S}^{\mathbf{N}-1}$.(Then $\nu = (N-1)/2)$). It is an artefact phase,because the nonvanishing centrifugal barrier $V^{\cdot}(R) = \nu(\nu-1)/R^2$ is related to the defect of embedding $\mathbf{S}^{\mathbf{N}-1}$ in $\mathbf{R}^{\mathbf{N}}$. According to Eq.(5), its supersymmetric partner is $V^+(R) = \nu(\nu+1)/R^2$, $A^+(R) = i(\nu+1)R^{-1}$ and $\delta_+ = \pi(\nu+1)/2$. In the three-body problem an analogous singularity potential $V(R)$ with $\nu = 5/2$ appears at the triple-collision point which coincides with the origin [5]. Generally, singularities of $A(R)$ are not obligatory situated at the origin $R = 0$ and the functional dependence of $A(R)$ may be more complicated. For example, if $A(R) = i\prod_j f(R)/(R - ib_j)$ with a smooth function $f(R)$ having an analytic continuation in the complex plane of R, then $\delta = \frac{\pi i}{2}\sum_j f(ib_j)$, at $f(R) = 2R$ or $f(R) = (R + ib_j)$ the geometric phase is $\delta = \pi\sum_j b_j$. Generally speaking, R is scaled and is dimensionless.

Let us return to our more complicate problem related to Eqs. (3), (4). The nondiagonal elements of $A_{n,m}(R) = < n|i\nabla_R|m >$ realise transitions between different eigenstates of the fast Hamiltonian H^f and the adiabatic assumption is not applied in particular near the level crossing. Let us rewrite the matrix elements of the induced vector-potential (4) in another form

$$A_{n,m}(R) = i\frac{< \Phi_n(R;r) \mid d_R H^f(R) \mid \Phi_m(R;r) >}{E_n(R) - E_m(R)}. \qquad (33)$$

It is a result of differentiation of Eq.(2) with respect to R and orthonormal relations of Φ .

It is quite clear, when the potential curves cross $E_n(R)$ (or quasi-cross) at some points $R = R_m$, the singularities appear in the matrix elements $A(R)$ which are responsible for geometrical phases. It is convenient to introduce the matrix of geometrical phase factors

$$S_{n,m} = \exp Im(\oint A_{nm}(R)dR). \qquad (34)$$

In our case we have

$$S_{n,m} = \exp \pi i \sum_m Res\frac{< \Phi_n(R;r) \mid d_R V(R,r) \mid \Phi_m(R;r) >}{E_n(R) - E_m(R)}. \qquad (35)$$

When an incomplete set, Φ , is taken into account, the state-vector is defined in an n-dimensional subspace of $(n+m) = M$- dimensional

Hilbert space, the second term in (5) does not vanish and nontrivial gauge fields are induced. Then, one has for the Berry phases

$$\delta_{ii} = \sum_{j\neq i}^{n} \oint A_{ij}A_{ji} = \sum_{j\neq i}^{n} \oint < \Phi_i \mid d_R \mid \Phi_j >< \Phi_j \mid d_R \mid \Phi_i > \quad (36)$$

analogously to [10]. Geometrical phases appear, as well related in off-diagonal elements of the effective matrix vector-potential $\tilde{A}_{ik}(R) = A_{ij}(R)A_{jk}(R)$ as well. In this case it is better to use the geometrical "S-matrix" (34)

$$S_{ik} = \exp \pi i \sum_{j\neq i,k}^{n} Res \frac{< \Phi_i(R;r) \mid d_R H^f(R;r) \mid \Phi_j(R;r) >}{E_i(R) - E_j(R)} \times$$
$$\times \frac{< \Phi_j(R;r) \mid d_R H^f(R;r) \mid \Phi_k(R;r) >}{E_j(R) - E_k(R)}. \quad (37)$$

In the last case there is an opportunity for three terms crossing at one point. Note, the nonabelian nonadiabatic phases manifest, even though only the radial dependence takes place. The reason is that in the presence of $A(R)$ singularities. The gauge transformation $U(R, R^\circ) = \int_{R^\circ}^{R} A(R')dR'$ does not eliminate them, and they occur in the scalar potential analogously to the nonvanishing curl of the vector potential in an N-dimensional space of slow variables.

In conclusion it should be remarked that topological effects and geometric phases arise in the presence of the supersymmetry for a system of gauge equations (3) describing slow dynamics of quantum - mechanical systems. The obtained relations of the supersymmetry (14)-(16) and (24),(25) with an additional scalar potential satisfy to the principal of minimal coupling of interaction with a gauge vector potential. Nonadiabatic extra geometric phase arise due to singularities of the connection operator A in points of level crossing.

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Received by Publishing Department

on June 30, 1992.