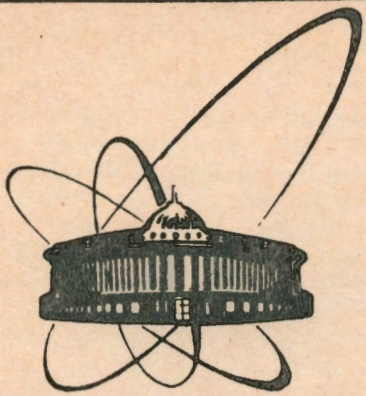

объединенный
институт
ядерных
исследований
дубна

E4-92-140

A.Adamczak [1], C.Chiccoli [2], V.I.Korobov,
V.S.Melezhik, P.Pasini [2], L.I.Ponomarev [3],
J.Wozniak [4]

MUON TRANSFER RATES IN HYDROGEN ISOTOPE MESIC ATOM COLLISIONS

Submitted to "Physics Letters B"

[1]Institute of Nuclear Physics, Cracow, Poland
[2]I.N.F.N., Sez. di Bologna and C.N.A.F., Bologna, Italy
[3]I.V.Kurchatov Institute of Atomic Energy, Moscow, Russia
[4]Institute of Physics and Nuclear Techniques, Cracow, Poland

1992

1. During the last two decades the physics of muon catalysis has been intensively studied by theory and experiment [1,2]. To improve this knowledge one has to describe the kinetics of muon catalyzed fusion. One important parameter that appears in the kinetics is the muon transfer rate $b\mu \to a\mu$ in mesic atomic collisions $b\mu + a$.

There are several calculations of these values using different approaches: two-level adiabatic approximation, variational methods, Faddev equations, etc. But the obtained results considerably differ from each other.

This paper presents the multi-level adiabatic calculations of the muon transfer rates for all the processes involving hydrogen isotope mesic atoms in the ground state:

$$p\mu + d \to d\mu + p, \tag{1a}$$

$$p\mu + t \to t\mu + p, \tag{1b}$$

$$d\mu + t \to t\mu + d. \tag{1c}$$

The calculations are performed in the adiabatic representation of the Coulomb three-body problem [3]. The method and its implementations are extensively described in paper [4]; it has been successfully applied to carry out the mesic atomic cross sections relevant for the muon catalyzed fusion kinetics [4-7].

2. The approach is based on the decomposition of the wave function $\Psi(\mathbf{r},\mathbf{R})$ describing the three-body system *mesic atom* + *nucleus* ($\mathbf{R}$ is the internuclear coordinate and $\mathbf{r}$ is the muon coordinate) in the adiabatic basis of states of discrete and continuous spectrum of the two-center problem of quantum mechanics (the problem of an electron motion in a Coulomb field of two fixed nuclei). The result is a system of equations for the amplitudes $\chi_i(R)$, that describe the relative motion of the nuclei:

$$\frac{d^2}{dR^2}\chi_i + \left(2M\varepsilon - \frac{J(J+1)}{R^2}\right)\chi_i - \sum_j U_{ij}\chi_j - \sum_j \int_k dk\, U_{ij}(k,R)\chi_j(k,R) = 0, \tag{2}$$

where the coefficients U_{ij} of the system are the effective potentials of the three-body problem in the adiabatic representation [3], ε is the colliding energy, J is the angular momentum and $M^{-1} = M_a^{-1} + M_b^{-1}$ is the reduced mass of the system, $M_a \geq M_b$ are masses of nuclei. After a discretization of the continuous spectrum

$$\sum_j \int_k dk \to \sum_j \sum_\alpha \Delta k_\alpha = \sum_{k=1}^{N},$$

we obtain a truncated system of ordinary differential equations of a finite number N.

In the present calculation we have taken into account the effective potentials $U_{ij}(R)$ connecting all the states of the first three shells of the discrete spectrum of the two-center problem (20 states) and $U_{ij}(K_\alpha, R)$ connecting the first shell of the discrete spectrum (2 states) with the states of the continuous spectrum of quantum numbers (lm): $m = 0, 1$;

$l \leq 5$. After a discretization $k \to k_\alpha = 0.1(0.1)3.0(0.5)10$ (44 points for every state (lm)) we get altogether 276 for $J = 0$, 546 states for $J = 1$ and 196 states for $J \geq 2$.

The cross sections $\sigma_{21}(\varepsilon)$ under investigation have been calculated by the following formula*):

$$\sigma_{21}(\varepsilon) = \frac{4\pi}{k_2^2}\left(\frac{\hbar}{m_a e^2}\right)^2 \sum_J (2J+1)\frac{(t_{21}^J)^2}{(D_J-1)^2+(t_{11}^J+t_{22}^J)^2}, \tag{3}$$

where $D_J = t_{11}^J t_{22}^J - (t_{12}^J)^2$. The matrix elements $t_{ij}(\varepsilon)$ are obtained from the comparison of the numerical solution of the system (2) with the asymptotics of the scattering problem as $R \to \infty$ $(t_{12} = t_{21})$:

$$\begin{pmatrix} \chi_1^{(1)} \\ \chi_2^{(1)} \end{pmatrix} \underset{R\to\infty}{\sim} \begin{pmatrix} \sin(k_1 R - \pi J/2) + t_{11}\cos(k_1 R - \pi J/2) \\ (v_1/v_2)^{1/2} t_{21}\cos(k_2 R - \pi J/2) \end{pmatrix}, \tag{4a}$$

$$\begin{pmatrix} \chi_1^{(2)} \\ \chi_2^{(2)} \end{pmatrix} \underset{R\to\infty}{\sim} \begin{pmatrix} (v_2/v_1)^{1/2} t_{12}\cos(k_1 R - \pi J/2) \\ \sin(k_2 R - \pi J/2) + t_{22}\cos(k_2 R - \pi J/2) \end{pmatrix}, \tag{4b}$$

where:

$$k_1^2 = 2\mathcal{M}_a^{(N)}(\varepsilon + \Delta E_N), \qquad v_1 = k_1/\mathcal{M}_a^{(N)},$$
$$k_2^2 = 2\mathcal{M}_b^{(N)}\varepsilon, \qquad v_2 = k_2/\mathcal{M}_b^{(N)},$$

and

$$\mathcal{M}_a^{(N)} \to \mathcal{M}_a = M_b(M_a + m_\mu)/(M_a + M_b + m_\mu),$$
$$\mathcal{M}_b^{(N)} \to \mathcal{M}_b = M_a(M_b + m_\mu)/(M_a + M_b + m_\mu),$$

$\Delta E_N \to \Delta E$ as $N \to \infty$ [4,6], where ΔE is the isotope splitting of the ground state energies of the mesic atoms $a\mu$ and $b\mu$. The values of the channel wave numbers k_i are taken from the stationary solutions of Eq. (2) with finite N in the asymptotic region where potential curves can be regarded as constant. It is worth mentioning that the boundary conditions of the scattering problem (4) correspond to the asymptotics of the truncated system of equations (2) for every fixed N.

3. The results of our calculations of the isotopic exchange rates,

$$\lambda = \sigma_{21} v_2 N_0 \ \mathrm{s}^{-1}, \tag{5}$$

($N_0 = 4.25 \times 10^{22}\mathrm{cm}^{-3}$ is the liquid hydrogen density) of the reactions (1) are presented in Tables 1 – 3 for several collision energies in comparison with previous calculations. Most of them have been done in the framework of the two-level approximation: adiabatic PSS method, Belayev et al. (**BGZL**) [8], Matveenko, Ponomarev and Faifman (**MPF**) [9]; Faddeev equations, Brener, Zubarev (**BZ**) [10]; variational approach, Kobayashi et al. (**KIT**) [11]; improved two-level approximation, Bubak and Faifman (**BF**) [12], Cohen and Struensee (**CS**) [13,14]. The first multi-level adiabatic calculation of reaction (1.c) was done by Melezhik (**M**) [15], where, however, a more limited approximation of the effective potentials, U_{ij}, of the system (2) was used. The non-adiabatic coupled rearrangment channel approach has been applied to reaction (1.c) by Kamimura, (**K**) [16], where about one hundred short-range trial functions were used.

We consider our present results the most accurate that have been obtained so far. The performed analysis shows that the total errors of our multi-level adiabatic calculations due to the truncation of the system of equations (2) and to the numerical integration are less than 10%.

*) We consider the entrance channel of the reactions (1) as 2nd and the exit channel as 1st.

Table 1. A summary of the results for the isotope exchange rate $\lambda = \sigma_{21} v N_0 (10^{10}\mathrm{s}^{-1})$ of the reaction $p\mu + d \to d\mu + p$*).

ε_2 (eV)	BGZL 1960	MPF 1971	P 1978 BF 1987	KIT 1987	CS 1990	Present work
0.001	–	1.9	–	1.4	–	1.59
0.01	–	1.9	–	1.4	–	1.58
0.04	1.45	–	1.7	–	1.4	1.55
0.1	–	–	–	–	–	1.53
0.4	–	–	–	–	–	1.49
1.0	–	–	–	–	–	1.50

Table 2. Isotope exchange rate $\lambda(10^{10}\mathrm{s}^{-1})$ for the reaction $p\mu + t \to t\mu + p$.

ε_2 (eV)	BGZL 1960	MPF 1971	P 1978 BF 1987	BZ 1988	KIT 1987	CS 1990	Present work
0.001	–	0.92	–	0.56-0.50	0.58	–	0.717
0.01	–	0.91	–	0.56-0.49	0.58	–	0.713
0.04	0.64	–	0.75	–	–	0.55	0.707
0.1	–	–	–	0.55-0.49	–	–	0.702
0.4	–	–	–	–	–	–	0.709
1.0	–	–	–	0.54-0.47	–	–	0.761

Table 3. Isotope exchange rate $\lambda(10^{10}\mathrm{s}^{-1})$ for the reaction $d\mu + t \to t\mu + d$.

ε_2 (eV)	BGZL 1960	MPF 1971	P 1978 BF 1987	KIT 1987	K 1987	CS 1990	Present work
0.001	–	0.65	–	2.7	2.7	–	2.26
0.01	–	0.65	–	2.6	2.7	–	2.31
0.04	0.49	–	1.9	–	2.9	3.5	2.46
0.1	–	–	–	–	3.2	–	2.78
0.4	–	–	–	–	5.0	–	4.58
1.0	–	–	–	–	8.9	–	8.69

In Figures 1–3 the results of our calculations for partial rates $\lambda^J(\varepsilon)$ as for total rates

*) **BF** only calculated the cross sections and we have estimated λ from Eq.(5). **BGZL** did not calculate the energy dependence of λ.

$\lambda(\varepsilon) = \sum_J \lambda^J(\varepsilon)$ are presented as functions of the colliding energy in the center-of-mass system.

4. In experiments the muon transfer reactions (1) take place in the collisions of mesic atoms with hydrogen molecules. To compare the experimental data with the calculated rates it is necessary to estimate electron screening corrections. The influence of molecular binding on the considered rates is negligible since the energy released during the muon exchange is much greater than the energy of molecular binding. Screening corrections are significant for the mesic atom elastic scattering on hydrogen molecules at collision energies below 0.1eV. The method of calculation of these corrections has been presented in Refs.[17,18]*). The calculated cross sections for symmetric hydrogen molecules are given in Table 4.

Table 4. Cross sections of processes $p\mu + D_2 \to d\mu + HD, p\mu + T_2 \to t\mu + HT$ and $d\mu + T_2 \to t\mu + DT$ versus mesic atom energy in the laboratory cyctem. Cross sections are given in 10^{-20}cm^2.

ε(eV)	$p\mu + d$	$p\mu + D_2$	$p\mu + t$	$p\mu + T_2$	$d\mu + t$	$d\mu + T_2$
0.001	828.7	984.1	384.4	461.3	15.8	20.9
0.002	649.9	762.4	292.8	346.2	12.7	16.2
0.004	453.0	519.8	203.9	235.0	8.90	10.9
0.010	283.7	311.8	128.0	140.5	5.64	6.33
0.040	140.7	144.7	63.6	65.2	2.94	3.02
0.100	87.4	88.2	39.9	40.2	2.00	2.01
0.200	61.0	61.2	28.1	28.2	1.59	1.60
0.500	37.9	38.0	18.0	18.0	1.37	1.37
1.000	26.6	26.6	13.2	13.2	1.43	1.43
2.000	19.3	19.3	10.2	10.2	1.76	1.76
3.000	16.2	16.2	9.15	9.15	2.12	2.12
4.000	14.6	14.6	8.55	8.55	2.49	2.49
5.000	13.4	13.4	8.17	8.17	2.87	2.87

The respective screening corrections for asymmetric molecules (e.g. the processes $d\mu + DT, d\mu + HT$ etc.) are practically the same.

The screening corrections for the muon exchange reactions are not so high as for the elastic scattering [18]. Nevertheless, they are considerable at very low collision energies.

If one knows the collision energy distribution of the mesic atoms one can calculate the averaged rates. Assuming that the mesic atoms are thermalized in the hydrogen molecular target we have obtained the following averaged values for the molecular rates:

$$\lambda_{dT_2}(T = 30K) = 2.7 \cdot 10^8 \text{s}^{-1}, \quad \lambda_{dT_2}(T = 300K) = 2.5 \cdot 10^8 \text{s}^{-1},$$
$$\lambda_{pD_2}(T = 30K) = 1.7 \cdot 10^{10} \text{s}^{-1}, \quad \lambda_{pD_2}(T = 300K) = 1.6 \cdot 10^{10} \text{s}^{-1},$$
$$\lambda_{pT_2}(T = 30K) = 0.8 \cdot 10^{10} \text{s}^{-1}, \quad \lambda_{pT_2}(T = 300K) = 0.7 \cdot 10^{10} \text{s}^{-1}.$$

*) The influence of screening effects on the exchange rates for the mesic hydrogen scattering on hydrogen atoms has been estimated in Ref.[14]

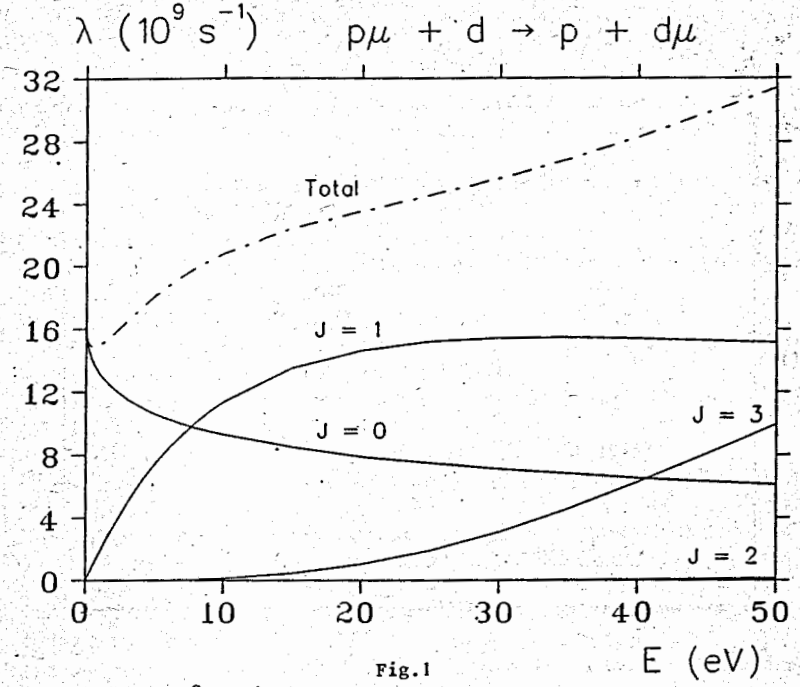

Fig.1

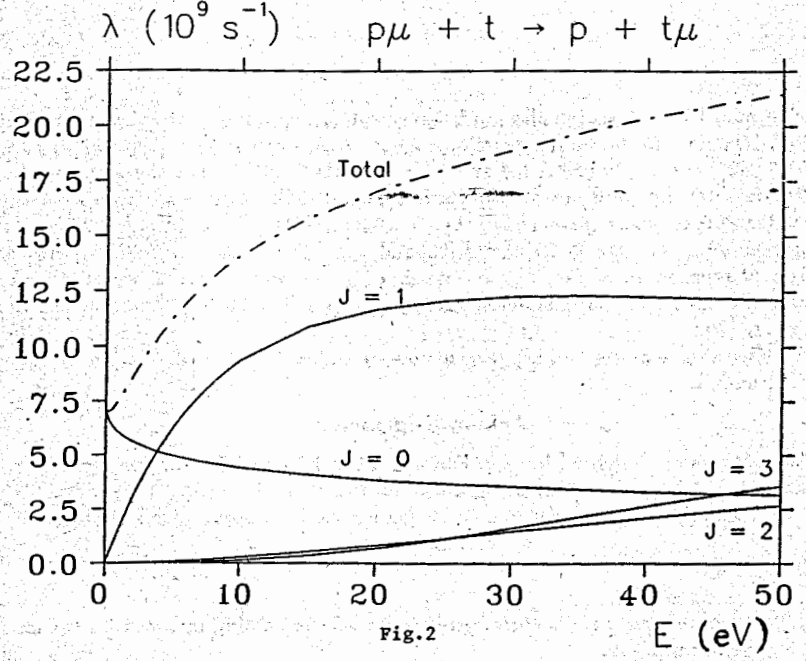

Fig.2

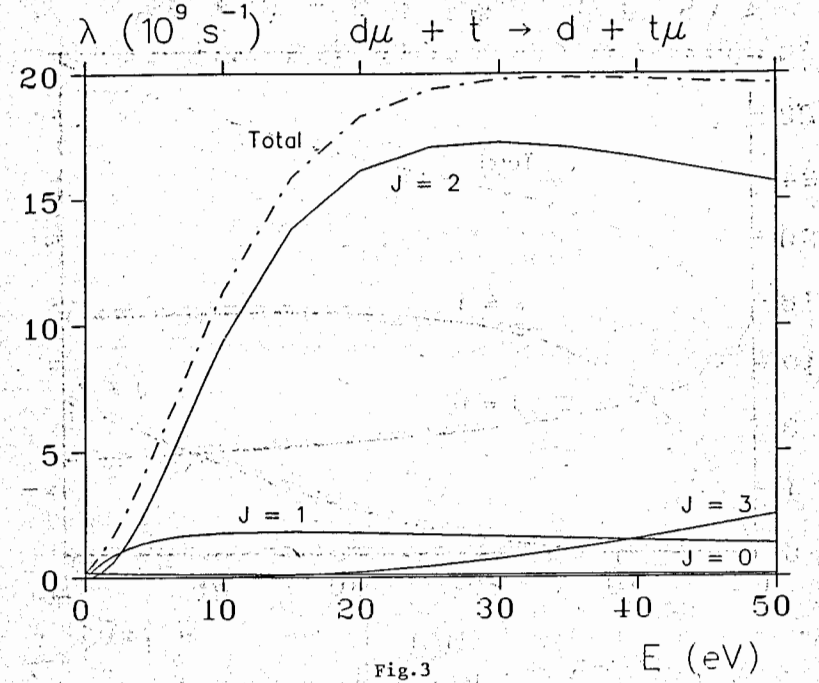

Fig.3

We would like to mention the good agreement of our results with the experimental data obtained for the $d\mu \to t\mu$ reaction: $\lambda_{dt} = (2.8 \pm 0.5)10^8 s^{-1}$ [19], $\lambda_{dt} = (2.8 \pm 0.4)10^8 s^{-1}$ [20] and $\lambda_{dt} = (2.8 \pm 0.4$ and $3.5 \pm 0.5)10^8 s^{-1}$ [21]*). For the other reactions the experimental data are not yet conclusive, for example in the case $p\mu \to d\mu$ three independent experiments [22-24] gave $\lambda_{pd} = (0.95 + 0.34, -0.25)10^{10} s^{-1}$ [22] in liquid hydrogen and $\lambda_{pd} = (1.43 \pm 0.13)10^{10} s^{-1}$ [23] and $\lambda_{pd} = (0.84 \pm 0.13)10^{10} s^{-1}$ [24] in gaseus hydrogen at room temperature. Let us note that for the molecular rates we use the same normalization as for the nuclear ones, namely $N_0 = 4.25 \times 10^{22} cm^{-3}$ is the density of hydrogen nuclei.

We hope that a new set of experiments performed at PSI [25] will clarify the experimental situation.

Acknowledgements.

The authors are indebted to M.P. Faifman, M. Jeitler and P. Kammel for numerous discussions. V.I.K. and V.S.M. are grateful to Professor A. Vitale, Director of INFN (Sezione di Bologna) and to Professor A. Bertin for the kind hospitality in Bologna.

*) The two different results correspond to the different fitting methods for the analysis of the experimental data.

References

[1] L.I. Ponomarev, Contemporary Physics, **31** (1991) 219 and references therein.

[2] W.H. Breunlich *et al.*, Annu. Rev. Nucl. Part. Sci. **39** (1989) 311, and references therein.

[3] S.I. Vinitsky and L.I. Ponomarev, Elem. Chast. Atomn. Yadr. **13** (1982) 1336.

[4] L. Bracci *et al.*, Muon Catalyzed Fusion, **5** (1991) 21.

[5] L. Bracci *et al.*, Muon Catalyzed Fusion, **4** (1989) 247.

[6] V.S. Melezhik, L.I. Ponomarev and M.P. Faifman, Zh. Eksp. Teor. Fiz. **85** (1983) 434 [Sov. Phys. JETP **58** (1983) 254].

[7] V.S. Melezhik, J. Comp. Phys. **65** (1986) 1.

[8] V.B. Belayev, S.S. Gershtain, B.N. Zakhariev and S.P. Lomnev, Zh. Exper. Teor. Fiz. **37** (1959) 1652 [Sov. Phys. JETP, **10** (1960) 1171].

[9] A.V. Matveenko, L.I. Ponomarev and M.P. Faifman, Zh. Exper. Teor. Fiz. **68** (1975) 437 [Sov. Phys. JETP, **41** (1975) 212].

[10] S.E. Brener and A.L. Zubarev, Muon Catalyzed Fusion **2** (1988) 157.

[11] K. Kobayashi, T. Ishihara and N. Toshima, Muon Catalyzed Fusion, **2** (1988) 191.

[12] M. Bubak and M.P. Faifman, Preprint JINR E4-87-464, Dubna, 1987; L.I. Ponomarev, Proceedings of 6th Int. Conf. on Atomic Physics, Zinatne, Riga, Plenum Press, (New York), (1978), 182; L.I. Ponomarev *et al.*, Yad. Fiz. **29** (1979) 133 [Sov. J. Nucl. Phys. **29** (1979) 67].

[13] M.C. Struensee *et al.*, Phys. Rev. A, **34** (1986) 3605; M.S. Struensee *et al.*, ibid., **37** (1988) 340.

[14] J.S. Cohen and M. Struensee, Phys. Rev. A, **43** (1991) 3460; J.S. Cohen, ibid., **43** (1991) 4668.

[15] V.S. Melezhik, Muon Catalyzed Fusion, **2** (1988) 117.

[16] M. Kamimura, Muon Catalysed Fusion, **3** (1988) 335.

[17] A. Adamczak, V.S. Melezhik and L.I. Menshikov, Z. Phys. D, **4** (1986) 153.

[18] A. Adamczak, Muon Catalyzed Fusion, **4** (1989) 31.

[19] V.M. Bystritsky *et al.*, Zh. Eksp. Teor. Fiz., **80** (1981) 1700. [Sov. Phys. JETP **53** (1981) 877].

[20] S.E. Jones *et al.*, Phys. Rev. Lett. **51** (1983) 1757.

[21] W.H. Breunlich *et al.*, Phys. Rev. Lett. **58** (1987) 329.

[22] V.P. Dzhelepov *et al.*, Zh. Eksp. Teor. Fiz., **42** (1962) 439 [Sov. Phys. JETP **15** (1962) 306].

[23] E.J. Bleser *et al.*, Phys. Rev. **132** (1963) 2679.

[24] A. Bertin *et al.*, Lett. Nuovo Cimento, **4** (1972) 449.

[25] V.E. Markushin, E.I. Afanasieva and C. Petitjean, Preprint PSI-PR-91-21, Villigen PSI, 1991.

Received by Publishing Department
on March 27, 1992.