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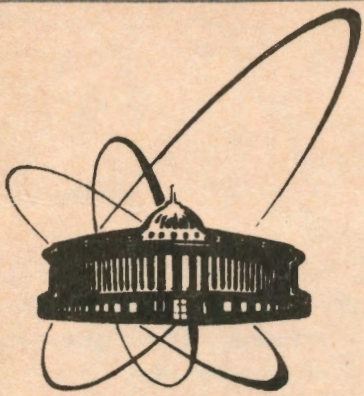

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GEOMETRICAL PHASES FOR QUASI-ENERGY STATES IN MULTI-LEVEL QUANTUM SYSTEMS

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Introduction

The discovery ov Berry's adiabatic phase and its generalizations and modifications [1–3] stimulated the search for relatively simple physical systems acquiring geometrical phases that might be detected experimentally (see e.g. reviews [4, 5]). Clear manifestations of the geometrical phases were revealed in optical polarization experiments [6–10], their classical theory [11–13] being developed long before the seminal Berry's works. Klyshko [14] revised the results of Refs. [6–10] in terms of quantum optics. Geometrical phases were demonstrated to be present in quadrupole [15] and magnetic [16, 17] resonance experiments.

In recent theoretical studies [18–21] Berry's phase was predicted for a two-level atom driven by a coherent quasi-resonant electric field of a light wave. Slow variations of the amplitude, frequency and phase of the field may be considered as an adiabatic evolution of the quasi-energy states (QES) [22]. Then the relevant geometrical phase manifests itself in the phase shift of the Rabi oscillations [18, 19], in the frequency shift of the Rabi triplet sidebands [21] or in the changes of the output photon statistics for a degenerate parametric amplifier [20].

In this paper we make use of the Berry's theory and its generalization [22] to periodically driven systems to derive the geometrical phases of the QES of a general-type multi-level quantum system driven by a coherent light wave having slowly varying parameters. Just as in the two-level atom [18, 19], the variation of the field phase is found to be necessary for the non-trivial geometrical phase to arise. Some errors of Ref. [18] are removed and the approach exhibited there is revised and extended to include not only an arbitrary number of energy le-

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vels, but QES degeneracy as well. The non-Abelian Wilczek-Zee factor is expressed in terms of the driving field phase and other QES parameters. The degeneracy of QES complicates the manifestation of geometrical phases in population oscillations, leading to the dependence of both the phase and the amplitude of the individual Rabi harmonics upon the phase of the driving field.

1. Berry phase for the quasi-energy-states of a multi-level system

An atom in an external light field E(t) envoles in time according to the Schroedinger equation

$$(i\hbar\partial/\partial t + H_o + V(t))\psi \equiv H\psi = 0, \qquad (1)$$

the potential V being taken in the dipole approximation

$$V(t) = -\mathbf{d}\cdot\mathbf{E}(t) = V^{(+)}e^{i\phi-i\omega t} + V^{(-)}e^{i\phi+i\omega t} \qquad (2)$$
$$V^{(+)} \equiv -(\mathbf{d}\cdot\hat{\mathbf{e}})E_o, \quad V^{(-)} \equiv -(\mathbf{d}\cdot\hat{\mathbf{e}}^*)E_o .$$

Here the amplitude E_o, frequency ω and phase ϕ of the light field, polarized along $\hat{\mathbf{e}}$, are considered to be adiabatically varying in time, i.e. $E_o=E_o(t)$, $\omega=\omega(t)$, $\phi=\phi(t)$, all rates being of the order of $\varepsilon\ll 1$. To the zeroth approximation in ε the potential is periodic: $V(t)=V(t+T)$, $T=2\pi/\omega$, and hence the standard approach [23-25] based on Floquet theorem yields the set of solutions of Eq.1, known as the quasi-energy states (QES):

$$\psi^\alpha = e^{-i\Omega^\alpha t}F^\alpha(t) , \qquad (3)$$

where F^α and Ω^α satisfy the eigenvalue equation

$$HF^\alpha = \hbar\Omega^\alpha F^\alpha \qquad (4)$$

with periodic boundary conditions $F^\alpha(t)=F^\alpha(t+T)$. The eigenvalue problem (4) is to be solved in the extended Hilbert space with the scalar product

$$\ll \varphi \mid \psi \gg = \frac{1}{T}\int_0^T \langle \varphi \mid \psi \rangle \, dt . \qquad (5)$$

Now following [26] we expand $F^\alpha(t)$ in a Fourier series using the complete set of eigenfunctions $\{\varphi_m\}$ of the bare Hamiltonian H_o:

$$H_o\varphi_m = E_m\varphi_m$$

The expansion involves a set of the Fourier coefficients A^α_{Nm}

$$F^\alpha(t) = \hbar\sum_m \sum_N A^\alpha_{mN}\varphi_m e^{iN\omega t} \qquad (6)$$

for which the following set of equations is to be satisfied

$$(\omega_m - \Omega^\alpha + N\omega)A^\alpha_{mN} + E\sum_k [V^{(+)}_{mk}e^{i\phi}A^\alpha_{k,N+1} + V^{(-)}_{mk}e^{-i\phi}A^\alpha_{k,N-1}] = O . \qquad (7)$$

Here $E=E_o d_o/\hbar$ is the field amplitude in frequency units, $V^{(+)}_{mk}=-\hbar(\mathbf{d}\cdot\hat{\mathbf{e}})_{mk}/d_o$, $V^{(-)}_{mk}=-\hbar(\mathbf{d}\cdot\hat{\mathbf{e}}^*)_{mk}/d_o$, d_o being the average transition dipole moment. Note that since we deal with high-frequency resonances, the integrals expressing A^α_{mN} in terms of the solution ψ may be extended up to the natural interval $\omega\in(-\infty,+\infty)$. The quasi-energies $\hbar\Omega^\alpha$ by definition are the solutions of the secular equation, corresponding to Eq.7, which is the explicit matrix form of the effective eigenvalue problem (4).

Consider now the slow time evolution equation corresponding to the Hamiltonian H:

$$i\varepsilon\hbar\partial\xi(t,\tau)/\partial t = H\xi(t,\tau) . \qquad (8)$$

According to the well-known adiabatic hypothesis [1] the instantaneous eigenvectors $F^\alpha(\mathbf{R}(\tau),t)$ of H depending upon τ via the parameters $\{E,\omega,\phi\}\equiv\mathbf{R}\in\mathbb{R}^3$ interpreted as cylindrical coordinates ρ,z,ϕ, differ only by a phase factor from the exact solution $\xi^\alpha(t,\tau)$ of Eq.8, provided that the initial condition is

$$\xi^\alpha(t,\tau=0) = F^\alpha(\mathbf{R}(0),t) .$$

To evaluate this factor Berry's approach [1] has been applied [22] to Eq.8, t being considered as an additional coordinate and τ as the evolution time. The corresponding adiabatic solution of Eq.1 may be easily recovered from $\xi^\alpha(t,\tau)$ by equating τ/ε and t in the final expressions:

$$\psi^\alpha(t) = \xi^\alpha(t,\varepsilon t) , \qquad (9)$$

which yields [22]:

$$\psi^\alpha(t) = F^\alpha(\mathbf{R}(t),t)\exp\{-i\int_0^t dt[\Omega^\alpha(\mathbf{R}) + t\dot{\omega}\partial\Omega^\alpha/\partial\omega] + i\gamma^\alpha(t)\} . \qquad (10)$$

The geometrical phase $\gamma^\alpha(t)$ is given by the expression

$$\gamma^\alpha(t) = \int_{\mathbf{R}(0)}^{\mathbf{R}(t)} \mathbf{A}^\alpha(\mathbf{R})\cdot d\mathbf{R} = -\mathrm{Im}\int_{\mathbf{R}(0)}^{\mathbf{R}(t)} d\mathbf{R}\cdot\langle F^\alpha(\mathbf{R}) \mid \nabla_{\mathbf{R}}F^\alpha(\mathbf{R})\rangle \qquad (11)$$

which is a sraightforward generalization of the original Berry's formula [1]. A more sophisticated approach is shown to

be necessary [22] to derive the correct expression for the dynamical phase in Eq.10. The term proportional to $t\dot{\omega}$ arises due to the change of the Fourier basis itself resulting from the adiabatic variation of ω.

The first-kind gauge transformation

$$F^{\alpha} \to F^{\alpha}\exp(i\mu^{\alpha}(\mathbf{R})) \qquad (12)$$

allowed by the eigenvalue equation (4), generates the second-kind gauge transformation

$$\mathbf{A}^{\alpha} \to \mathbf{A}^{\alpha}-\nabla_{\mathbf{R}}\mu^{\alpha}(\mathbf{R}) \qquad (13)$$

of the "connection form" or "vector potential" $\mathbf{A}^{\alpha}$, defined by Eq.(11). The arbitrary phase $\mu^{\alpha}(\mathbf{R})$ of the instantaneous eigenstate is to be a single-valued function of $\mathbf{R}$.

Making use of the Fourier decomposition (6) and the definition (11), one easily gets

$$\mathbf{A}^{\alpha}(\mathbf{R})=-\mathrm{Im}\sum_{nN} A^{\alpha *}_{nN}\nabla_{\mathbf{R}}A^{\alpha}_{nN}-(\pi/\omega)\hat{e}\sum_{nN}|A^{\alpha}_{nN}|^2 N \quad , \qquad (14)$$

$\hat{e}$ being the unit vector of the ω axis in the parameter space $\mathbb{R}^3$. For the cyclic evolution through a closed loop C in $\mathbb{R}^3$ the global phase is

$$\gamma^{\alpha}(C)=<\oint_{C} \mathbf{A}^{\alpha}\cdot d\mathbf{R} =\int_{S}(\mathrm{rot}\mathbf{A}^{\alpha}\cdot d\mathbf{S}) \qquad (15)$$

the gauge phase $\mu^{\alpha}(\mathbf{R})$ being unrelevant herein.

For loops lying in the (ω,E) plane the exterior derivative of $\mathbf{A}^{\alpha}$ in Eq.(15) vanishes identically. Thus, similar to the two-level case, the variation of the field phase ϕ remains to be a necessary condition for the non-trivial geometrical phase to arise.

Now let the phase ϕ be the only variable parameter. By means of a simple unitary transformation

$$A^{\alpha}_{nN} \to A^{\alpha}_{nN}e^{-iN\phi} \qquad (16)$$

ϕ may be removed from Eq.(7), thus making them to be independent of the adiabatic evolution time. From Eq.(14) it immediately follows that

$$\gamma^{\alpha}(t)=[\phi(t)-\phi(0)]\sum_{nN} N|A^{\alpha}_{nN}|^2-[\mu^{\alpha}(t)-\mu^{\alpha}(0)] \quad , \qquad (17)$$

where the coefficients A^{α}_{nN} are the solutions of Eq.7 with ϕ set to be zero. Arbitrary phases μ^{α} are included into γ^{α} to make our results comparable with those of Ref.18. For the cyclic evolution $\phi(t_C)-\phi(0)=2\pi$, $\mu^{\alpha}(t_C)-\mu^{\alpha}(0)=0$, and the final expression for the Berry's phase takes the form

$$\gamma^{\alpha}(t)=2\pi\sum_{nN} N|A^{\alpha}_{nN}|^2. \qquad (18)$$

This is a multi-level generalization of the formula derived in Ref.18. More details concerning the two-level system in the rotating-wave approximation will be discussed in Sec.2.

Since the phase and the frequency of the field are not independent, one can formally derive Eq.(18) from the dynamical part of the adiabatic phase (see also Berry [30]). For example, the adiabatic variation of the field phase may be attributed to a frequency shift $\Delta\omega(t)$:

$$\phi(t)-\phi(0)=-\int_0^t[\Delta\omega(t)+\Delta\dot{\omega}(t)t]dt. \qquad (19)$$

In the adiabatic limit $\Delta\omega$ must tend to zero , the evolution time t being infinitely long. The corrected set of instantaneous eigenvalue equations

$$[\omega_m-\Omega^{\alpha}+N(\omega+\Delta\omega)]A^{\alpha}_{mN} + E\sum_k [V^{(+)}_{mk}A^{\alpha}_{k,N+1}+ V^{(-)}_{mk}A^{\alpha}_{k,N-1}]=0 . \qquad (20)$$

may be solved using the perturbation theory. The first-order correction to the quasienergy Ω^{α} is

$$\Delta\Omega^{\alpha}=\Delta\omega\sum_{nN} N|A^{\alpha}_{nN}|^2. \qquad (21)$$

Since the perturbation is diagonal, the first-order correction to the coefficients A^{α}_{mN} is zero. From Eqs.(10), (19) and (21) it follows that the dynamical phase may be expressed as

$$-\int_0^t dt(\Omega^{\alpha}+\Delta\Omega^{\alpha}+t\Delta\dot{\omega}\partial\Delta\Omega^{\alpha}/\partial\Delta\omega)=-\Omega^{\alpha}t+[\phi(t)-\phi(0)]\sum_{nN} N|A^{\alpha}_{nN}|^2 \qquad (22)$$

in which the first term is a common dynamical phase and the second reproduces the Berry phase (18) in case of the cyclic evolution, when $[\phi(t)-\phi(0)]=2\pi$.

Now we make use of Eqs.(6), (11) to evaluate γ^{α}. The frequencies of the Fourier harmonics in Eq.(6) and the averaging period in Eq.(11) are affected by $\Delta\omega$, but the coefficients A^{α}_{mN} are $\Delta\omega$-independent up to the first order of approximation. Hence both the derivatives and the integral in Eq.(11) may be expressed explicitly:

$$\gamma^{\alpha}=-\mathrm{Im}\{i\pi\sum_{nN} N|A^{\alpha}_{nN}|^2+\sum_{n,M\neq N} A^{\alpha *}_{nN}A^{\alpha}_{nM}M/(M-N)\}\ln|(\omega+\Delta\omega(t))/(\omega+\Delta\omega(0))|.$$

In the adiabatic limit $\Delta\omega \to 0$ and γ^{α} tends to zero.

Although the additional phase is now derived in a "dynami-

cal" manner, it does not really depend upon the evolution rate and is determined only by the contour passed by the effective Hamiltonian in the parameter space.

2. Population oscillations in a multi-level system

Consider an atom prepared in an eigenstate φ_n of its own Hamiltonian H_o. At t=0 the periodical field is suddenly swiched on, $H_o \to H_o+V$, giving rise to the new basic set of states (10). The general solution of the Schroedinger equation (1) is a linear combination of these states

$$\psi(t)=\sum_\alpha C^\alpha \psi^\alpha(t) \qquad (23)$$

the coefficients C^α being time-independent in the adiabatic limit. To satisfy the initial condition $\psi(t=0)=\varphi_n$ the coefficients C_n^α must be chosen such that

$$\sum_\alpha C_n^\alpha \sum_N A_{Nm}^\alpha = \delta_{nm} , \qquad (24)$$

which follows immediately from Eqs.(10) and (6).

The probability for the atom to be found in its initial state $\varphi_n=\psi(0)$ at the time t, or the population of the n'th energy level, is expressed via the scalar product

$$W_n=|\langle\varphi_n|\psi(t)\rangle|^2 \qquad (25)$$

Let the field phase ϕ be slowly varying, the other parameters of the field being fixed. Eqs.(23), (10) and (6) make it possible to express W_n in terms of the solutions of the eigenvalue problem (7):

$$W_n=\Big|\sum_{\alpha,N} C_n^\alpha A_{nN}^\alpha \exp(-i\Omega^\alpha t+i\gamma^\alpha+iN\omega t)\Big|^2 , \qquad (26)$$

where the geometrical phase γ^α is given by Eq.(17) with the arbitrary phase μ^α set to be zero.

In non-linear spectroscopy it is common to treat atoms and molecules as having only a finite number of energy levels, each transition frequency being nearly resonant to a certain harmonic of the driving field [26-28]. In such systems the so-called rotating-wave approximation (RWA) is widely used [26,27]. Being applied to QES formalism, this approximation makes it possible to attribute a single Fourier harmonic $\exp(iN_m\omega t)$ to each state φ_m in the decomposition (6) [26,28]. Hence the reduced notation

$$A_{mN_m}^\alpha \equiv A_m^\alpha$$

may be introduced, and Eq.(26) takes the simplified form:

$$W_n=\sum_\alpha |A_n^\alpha|^4+2\sum_{\alpha<\beta}|A_n^\alpha|^2|A_n^\beta|^2\cos[\Omega^{\alpha\beta}t-\gamma^{\alpha\beta}(t)] ,$$

$$\Omega^{\alpha\beta}=\Omega^\alpha-\Omega^\beta; \qquad (27)$$

$$\gamma^{\alpha\beta}=\gamma^\alpha-\gamma^\beta=\sum_m N_m(|A_m^\alpha|^2-|A_m^\beta|^2)(\phi(t)-\phi(0)) .$$

Here we made use of Eq.(24) and of the orthonormality relations

$$\sum_\alpha A_n^{\alpha*} A_m^\alpha=\delta_{nm} , \qquad (28)$$

following from the Hermitian properties of the effective Hamiltonian.

Eqs.(27) show that the population of the initial energy level exhibits a complex oscillation which is a superposition of harmonics. Each of these harmonics has the frequency $\Omega^{\alpha\beta}$ of a transition between two QES. Since in a two-level atom Ω^{21} is a generalized Rabi frequency, it is natural to call them Rabi harmonics. The Berry's phase contributes to the phase shift of each Rabi harmonic, proportional to the driving field phase increment $\delta\phi$. The proportionality coefficients between $\delta\phi$ and the Berry's phase depend upon the amplitude and the frequency of the field, the strengths and the frequencies of atomic transitions.

To compare our results with those of Ref.18, let us consider a two-level atom. In the RWA one may choose $N_1=0$, $N_2=-1$ [26] so that Eqs.(7) take the form

$$(\omega_1-\Omega^{1,2})A_1^{1,2}+EV_{12}^{(-)}A_2^{1,2}=0 ;$$
$$EV_{21}^{(+)}A_1^{1,2}+(\omega_2-\omega-\Omega^{1,2})A_2^{1,2}=0 . \qquad (29)$$

The solutions of (29) are [26] :

$$\Omega^{1,2}=\omega_1-\delta/2\pm\Omega_R/2 ; \qquad (30)$$

$$A_1^1=\cos\vartheta e^{i\mu_1};\ A_2^1=-\sin\vartheta e^{i\mu_1};\ A_1^2=\sin\vartheta e^{i\mu_2};\ A_2^2=\cos\vartheta e^{i\mu_2} , \qquad (31)$$

where $\delta=\omega-\omega_{21}$ is the frequency detuning ; $\delta\geq 0$ is assumed for unambiguity; $\Omega_R=(\delta^2+4V^2)^{1/2}$ is the generalized Rabi frequency; $V=|EV_{12}^{(\pm)}|$ is the Rabi frequency; $\tan 2\vartheta=2V/\delta$; $0\leq\vartheta\leq\pi/4$. Now from Eqs.(17),(31) the expression of γ^α immediately follows:

$$\gamma^1(t)=-\sin^2\vartheta(\phi(t)-\phi(0))-(\mu^1(t)-\mu^1(0));$$
$$\gamma^2(t)=-\cos^2\vartheta(\phi(t)-\phi(0))-(\mu^2(t)-\mu^2(0)). \quad (32)$$

The second of these expressions differs from the corresponding expression of Ref.18, in which $+\sin^2\vartheta$ stands for $-\cos^2\vartheta$. The error of Ref.18 is that ϕ is included into the arbitrary phase μ^2 of the second eigenvector. This is forbidden by the condition that μ^α must be a single-valued function of $\mathbf{R}$, since ϕ is a cyclic coordinate in $\mathbb{R}^3$.

This error, however, does not affect the final expression for the initial energy level population:

$$W_1(t)=\cos^4\vartheta+\sin^4\vartheta+2\cos^2\vartheta\sin^2\vartheta\cos[\Omega_R t-\cos 2\vartheta(\phi(t)-\phi(0))]. \quad (33)$$

Similar expression follows from our Eqs.(27),(30)-(32).

3. Wilczek-Zee factors for degenerate QES

The degeneracy of QES may be accidental as well as caused by the residual degeneracy of the original energy levels of the bare atom. In any of these cases an additional quantum number a appears, so that

$$HF^{\alpha a}=\hbar\Omega^\alpha F^{\alpha a}. \quad (34)$$

Now the vector potential (14) must be replaced by the gauge Wilczek-Zee potential [2,4]

$$A^\alpha_{ab}=-\mathrm{Im}\sum_{nN} A^{\alpha b*}_{nN}\nabla_R A^{\alpha a}_{nN}-(\pi/\omega)\hat{e}\sum_{nN} N A^{\alpha b*}_{nN}A^{\alpha a}_{nN}. \quad (35)$$

Linear transformation of the instantaneous eigenvectors

$$(A^{\alpha a}_{nN})'=\sum_b \Lambda_{ab}A^{\alpha b}_{nN} \quad (36)$$

generates a gauge transformation of the second kind

$$(A^\alpha_{ab})'=(\Lambda A^\alpha \Lambda^{-1})_{ab}+((\nabla_R\Lambda)\Lambda^{-1})_{ab}. \quad (37)$$

According to [2,4] the relation between the adiabatically evolving state and the corresponding instantaneous eigenvector now may be expressed as

$$\psi^{\alpha a}(t)=\sum_b U^\alpha_{ab}F^{\alpha b}(\mathbf{R}(t),t)\exp\{-i\int_0^t dt[\Omega^\alpha(\mathbf{R})+\dot{\omega}\partial\Omega^\alpha/\partial\omega]\}. \quad (38)$$

In this generalization of Eq.(10) the dynamical phase is not affected by the degeneracy since it depends only upon Ω^α and its derivative. Berry's phase factor $\exp(i\gamma^\alpha(t))$ is now

replaced by the non-Abelian Wilczek-Zee factor U^α_{ab}, which obeys the equation

$$\dot{U}^\alpha_{ab}=i\sum_c U^\alpha_{ac}A^\alpha_{cb}\cdot\dot{\mathbf{R}}. \quad (39)$$

If E and ω are constant, the field phase ϕ may be treated in the same way as in the non-degenerate case. The explicit differentiation with respect to ϕ in Eq.(35) brings Eq.(39) to the form

$$(d/d\phi)U^\alpha_{ab}=i\sum_c U^\alpha_{ac}B^\alpha_{cb}; \quad (40)$$

$$B^\alpha_{cb}=\sum_{nN} N A^{\alpha b*}_{nN}A^{\alpha c}_{nN}. \quad (41)$$

As in Sec.1, the coefficients $A^{\alpha a}_{nN}$ are the solutions of Eqs.(7) with ϕ set to be zero, corresponding to an eigenvalue Ω^α, which is, however, degenerate in the present consideration.

By means of the Euler substitution

$$U^\alpha_{ab}=y^\alpha_b\exp(i\kappa\delta\phi),\quad \delta\phi\equiv\phi(t)-\phi(0). \quad (42)$$

Eqs.(40) are reduced to the eigenvalue problem

$$\sum_c(B^{\alpha*}_{bc}-\kappa\delta_{bc})y^\alpha_c=0. \quad (43)$$

Denote the eigenvalues of the matrix $B^{\alpha*}$ and its eigenvector components by κ^α_d and y^α_{cd}, respectively. The general solution of Eqs.(40) is a superposition of the basic solutions:

$$U^\alpha_{ab}=\sum_d C^\alpha_d\, y^\alpha_{bd}\exp(i\kappa^\alpha_d\,\delta\phi). \quad (44)$$

Making use of the initial condition $U^\alpha_{ab}(0)=\delta_{ab}$ one finally gets

$$U^\alpha_{ab}=\sum_d y^{\alpha*}_{ad}\, y^\alpha_{bd}\exp(i\kappa^\alpha_d\,\delta\phi). \quad (45)$$

If, due to the favorable choice of the basis, the matrix B^α appears to be diagonal, its normalized eigenvectors being $y^\alpha_{ad}=\delta_{ad}$, then

$$U^\alpha_{ab}=\delta_{ab}\exp(i\kappa^\alpha_a\delta\phi);$$
$$\kappa^\alpha_a=\sum_{nN} N|A^{\alpha a}_{nN}|^2,$$

i.e. each a-component of the degenerate QES acquires its own Berry's phase. In general, B^α is non-diagonal and we are to deal with the non-Abelian phase factor (45).

The calculation of the level population is analogous to that of Sec.2. Comparement of Eqs.(10) and (38) shows that

Eq.(26) is to be replaced by

$$W_n=|\sum_{\alpha,N}\sum_{a,b} C_n^{\alpha a}U_{ab}^{\alpha}A_{nN}^{\alpha b}\exp(-i\Omega^{\alpha}t+iN\omega t)|^2 \quad , \qquad (46)$$

where the coefficients $C_n^{\alpha a}$ satisfy the equations

$$\sum_{\alpha,a} C_n^{\alpha a}\sum_{N} A_{mN}^{\alpha a}=\delta_{nm} \quad , \qquad (47)$$

which follow from the initial conditions

$$\psi(t=0)=\sum_{\alpha,a} C_n^{\alpha a}\psi^{\alpha a}(t=0)=\varphi_n \quad ; \quad U_{ab}^{\alpha}(t=0)=\delta_{ab}$$

Now we restrict ourselves by considering a finite number of resonant energy levels in the RWA (see comments to the transition from Eq.(26) to Eq.(27) in Sec.2). Being transformed to a sum of real harmonics, Eq.(46) yields

$$W_n(t)=W_{no}(\phi(t))+2\sum_{\alpha<\beta} W_n^{\alpha\beta}(\phi(t))\cos[\Omega^{\alpha\beta}t+\Phi_n^{\alpha\beta}(\phi(t))] \quad , \qquad (48)$$

where $$W_{no}(t)=\sum_{\alpha a}(D_n^{\alpha a})^2+2\sum_{\alpha}\sum_{a<b} D_n^{\alpha a}D_n^{\alpha b}\cos[(\kappa_a^{\alpha}-\kappa_b^{\alpha})\delta\phi(t)] \quad ; \qquad (49)$$

$$W_n^{\alpha\beta}(\phi(t))=\left\{\sum_{a,b,c,d} D_n^{\alpha a}D_n^{\beta b}D_n^{\alpha c}D_n^{\beta d}\cos[(\kappa_a^{\alpha}-\kappa_b^{\beta}-\kappa_c^{\alpha}+\kappa_d^{\beta})\delta\phi(t)]\right\}^{1/2} ; \qquad (50)$$

$$\Phi_n^{\alpha\beta}(\phi(t))=\operatorname{atan}\frac{\sum_{a,b} D_n^{\alpha a}D_n^{\alpha b}\sin[(\kappa_a^{\alpha}-\kappa_b^{\beta})\delta\phi(t)]}{\sum_{a,b} D_n^{\alpha a}D_n^{\alpha b}\cos[(\kappa_a^{\alpha}-\kappa_b^{\beta})\delta\phi(t)]} \quad ; \qquad (51)$$

$$D_n^{\alpha a}=\sum_{b,c} A_n^{\alpha b*}y_{ba}^{\alpha *}y_{ca}^{\alpha}A_n^{\alpha c}=D_n^{\alpha a*} \quad . \qquad (52)$$

The main conclusion which may be drawn from Eqs.(48)-(52) is that in the degenerate case not only the phases, but also the amplitudes of the individual Rabi harmonics appear to be dependent upon the field phase $\delta\phi(t)$. This is an observable manifestation of the non-Abelian Wilczek-Zee phase factors. Such amplitude manifestations of the geometrical phases were previously mentioned as typical for degenerate systems by Segert [31] in connection with the problem of the optical pumping of atoms in an external magnetic field.

Our next studies are to deal with the geometrical phases in damped driven multi-level systems. For a two-level damped driven atom successful approaches to this problem were realized in Refs.19,32. More complicated paths in the parameter space, involving simultaneous variations of the frequency, phase and amplitude of the field, are also to be considered.

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Геометрические фазы для квазиэнергетических состояний многоуровневых квантовых систем

Теория Берри и ее обобщения применяются для описания адиабатической эволюции квазиэнергетических состояний (КЭС) многоуровневой квантовой системы общего вида, взаимодействующей с интенсивным полем излучения когерентной световой волны с медленно меняющимися параметрами. Для вырожденных КЭС неабелевый фактор Вильчека-Зи выражен в терминах фазы интенсивного возбуждающего поля и других параметров КЭС. Вырождение КЭС обеспечивает проявление геометрических фаз в осцилляциях заселенностей, приводящее к зависимости как фаз, так и амплитуд индивидуальных гармоник Раби от фазы возбуждающего поля.

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Geometrical Phases for Quasi-Energy States in Multi-Level Quantum Systems

The Berry's theory and its generalization are applied to describe an adiabatic evolution of the quasi-energy states (QES) of a general-type multi-level quantum system driven by a coherent light wave having slow varying parameters. For degenerate QES the non-Abelian Wilczek-Zee factor is expressed in terms of the driving field phase and other QES parameters. The degeneracy of QES provides the manifestation of geometrical phases in population oscillations, leading to the dependence of both the phases and amplitude of the individual Rabi harmonics upon the phase of the driving field.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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