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INFLUENCE OF GIANT ANGULAR RESONANCES ON THE ELECTROMAGNETIC CHARACTERISTICS OF LOW-LYING STATES

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Introduction

Dipole magnetic transitions in deformed nuclei are not well enough studied in comparison with quadrupole electric ones. The magnetic properties of the ground (gr) and γ- rotational bands in even-even nuclei are influenced by the Coriolis admixtures of the states of $K^{\pi}=1^{+}$ bands [1]. The experimental observation of the low - lying collective states with $K^{\pi}=1^{+}$ [2], led to the appearance of new models [3-6] where the 1^{+} states admixed by the Coriolis coupling are considered as a "giant angular resonance" (GAR). The states of GAR are connected with the states of the ground band by dipole magnetic transitions. This fact gives rise to the alternative name of the resonance - the M1-mode. The models considering the coupling of the low-lying states with the states of GAR allow one to describe the M1 - transitions states from the β- and γ- bands to the ground band [7,8].

In ref.[5], the two -rotor model (TRM) with the Feshbach projection operator method [9] was developed for investigating the properties of the positive parity low-lying states in the transuranium nuclei. In the present paper this model is applied to study the properties of the rare earth nuclei. The effective Hamiltonian and expressions for the reduced probabilities of collective states are obtained. The calculations are performed for the ^{164,166,168}Er isotopes.

Model

Let us consider a model where a nucleus is considered as two axial rotators composed of protons and neutrons which can move with respect to each other but have fixed centre of masses. Internal states of each subsystem are characterized by the conserved quantum number associated with the angular momentum projection onto the symmetry axis of the subsystem (K_p and K_n). Thus we write

$$(\zeta_p \mathbf{I}_p)\,\psi = K_p\,\psi\,, \qquad (\zeta_n \mathbf{I}_n)\,\psi = K_n\,\psi\,, \qquad (1)$$

where ζ_p and ζ_n are the unit vectors directed along the symmetry axes of the proton and neutron components.

The relative motion of the subsystems is counteracted by the force, the corresponding potential of which in the harmonic approximation is

$$V(\theta) = \frac{1}{2} C\,\theta^2 \qquad (2)$$

$$\left(V\left(\frac{\pi}{2}-\theta\right) = \frac{1}{2} C\left(\frac{\pi}{2}-\theta\right)^2\right).$$

The angular variable θ is determined by the following expression

$$\cos(2\theta) = \zeta_p \cdot \zeta_n\,.$$

According to ref.[5], the nuclear Hamiltonian is

$$H = H_{0,0} + H_{0,1} + T'_{\gamma} + T'_{\beta}\,, \qquad (3)$$

where

$$H_{0,0} = \frac{1}{4}(A_p^0 + A_n^0)(\mathbf{I}^2 + \mathbf{S}^2) + V(\theta) + H_{int} \qquad (4)$$

$$H_{0,1} = \frac{1}{2}(A_p^0 - A_n^0)(\mathbf{I} * \mathbf{S}) \qquad (5)$$

$$T'_{\gamma} = T'_1 + T'_2 + T'_3 = b_1 I_+^2 + b_1^+ I_-^2 + b_2(I_+S_+ + S_+I_+) + b_2^+(I_-S_- + S_-I_-) + b_1 S_+^2 + S_-^2 b_1^+ \qquad (6)$$

$$T'_{\beta} = a_{\beta,1}(H_{0,0} - V(\theta) - H_{int}) + a_{\beta,2}\,H_{0,1}\,. \qquad (7)$$

Here A_i^0 are numerical parameters ($A_i^0=\frac{1}{2J_i}$, where J_i is the moment of inertia of the i^{th} subsystem), $I=I_p+I_n$ is the total angular moment of the whole system and $S=I_p-I_n$.

The eigenfunctions of $H_{0,0}$, which describe the system with the axial symmetry, have the following form [4,5]:

$$\psi(IMkK\kappa n)=\sqrt{\frac{2I+1}{16\pi^2(1+\delta_{k,0}\delta_{K,0})}}\;\varphi_{\kappa n}\left[D^I_{MK}\,\chi_k+(-1)^{I+K}\,D^I_{M-K}\,\chi_{-k}\right] \qquad (8)$$

here $K \geq 0$, $k=K_p+K_n$, $\kappa=|k-K|=0,1,2...$, $n=0,1,2...$

$$\varphi_{\kappa n}(\theta)=\left[\frac{2n!}{(n+\kappa)!}\right]^{1/2}\theta^{\kappa+1/2}e^{-\theta^2/2}\,L_n^{\kappa}(\theta^2)$$

where $L_n^{\kappa}(\theta^2)$- is the associated Laguerre polynomial. In the expression (8) χ_k- are the eigenfunction of the internal Hamiltonian H_{int}. The corresponding eigenvalues are denoted as $\varepsilon_{int}(k)$.

The eigenvalues of the operator $H_{0,0}$ are determined by

$$E(IkK\kappa n)=\frac{1}{2}\,\omega\,\theta_0^2\,I(I+1)+\omega(2n+\kappa+1)+\varepsilon_{int}(k). \qquad (9)$$

The motion of neutrons with respect to protons can be superimposed with exsitations of β- and γ - bands. As a result, in the framework of the considered model, an infinite number of states appears with quantum numbers k=0, K= κ =1,n (gr- and β - band) and k=2,K=1,3; κ=1,n (γ - band) above the gr-, β - and γ -vibrational bands.

The operator $\hat{a}_\beta$ in (7) does not change the quantum number k but influences the internal nuclear state. The operators $\hat{b}_1$ and $\hat{b}_2$ in (6) change the quantum number k and connect gr- and β - bands with the states in γ - band. Let us write down these operators in the following manner:

$$\hat{a}_\beta= a_\beta|\beta\rangle\langle gr|+ a^*_\beta|gr\rangle\langle\beta|$$
$$\hat{b}_\ell= b_\ell|\gamma\rangle\langle\alpha|+ b^*_\ell|\alpha\rangle\langle\gamma| \qquad (10)$$

where ℓ =1,2 and α = gr, β.

We are going to consider the states of the low - lying bands: gr- , β -(k=K=κ=n=0) and γ -(k=K=2,κ=n=0). vibrational bands. Following ref.[5] we introduce an operator P which is a projector onto the states under investigation (P - space):

$$P=\sum_{IM}\left\{|IMgr\rangle\langle IMgr|+|IM\beta\rangle\langle IM\beta|+|IM\gamma\rangle\langle IM\gamma|\right\}.$$

The states with the quantum numbers $(k=0,\ K=\kappa=1,n)_{gr}$, $(k=0,\ K=\kappa=1,n)_\beta$, $(k=2,\ K=\kappa=1,n)_\gamma$ and $(k=2,\ K=3,\ \kappa=1,n)_\gamma$ are included in the Q- space (GAR). For projected Hamiltonian one has:

$$H_{pp}= H^P_{0,0}+ PT_1'P. \qquad (11)$$

Here $H^P_{0,0}$, is diagonal with respect to basic wave functions $\psi^{(0)}$; T_1' describes the mixture of bands in the P- space (gr-, β- and γ- bands).

We write the following expressions for the Hamiltonians $H_{P,Q}$ and $H_{Q,Q}$:

$$H_{PQ} = P\,H_{0,1}Q + P\,T_\beta'\,Q + P\,T_2'\,Q = H^+_{QP} \qquad (12)$$
$$H_{QQ}= H^Q_{0,0}$$

The operators in (12) describe the mixture of states from the P- and Q- spaces.

The total wave function is determined as a sum:

$$\Psi = P\,\Psi + Q\,\Psi = \Phi + \chi$$

while the function Φ is represented in the model space as

$$\Phi^I_K = \sum_i C^{IK}_i\,\Phi^{IK}_i \qquad (13)$$

where C^{IK}_i is the amplitudes of the mixture of i states (in the P- space). The function Φ satisfies the equation:

$$\tilde{\mathcal{H}}\,\Phi = E\,\Phi.$$

We assume that energies of the low-lying states are small when compared with those of the operator H_{QQ}, and that the latter has the eigenvalues close to $H^{Q}_{0,0}=Q\ H_{0,0}Q$. This assumption makes it possible to attain the following expression for the m.e. of $\tilde{\mathcal{H}}$

$$\tilde{\mathcal{H}}_{ii'}=(H_{PP})_{ii'}+\sum_{j}(H_{PQ})_{ij}(E-E_j)^{-1}(H_{QP})_{ji'}\ , \qquad (14)$$

where j denotes the quantum numbers of the basic functions in the Q- space (additional to I and M). The energy E is assumed to be equal to the energy of the yrast-band states. Using expressions from ref.[5] for the m.e. of the Hamiltonian (3) and the Feshbach formalism of the projection operators (9) as well as taking into account that in our case the β - band is much higher than the γ one, we have the following expression for the effective Hamiltonian:

$$\tilde{\mathcal{H}}=\sum_{i,i'}\left[\omega_i\delta_{i,i'}+P_{i,i'}I(I+1)\right] \qquad (15)$$

where i,i'=gr,β and γ

$$P_{gr,gr}=A\left[1-A_0^2(1+|\langle gr|a_{\beta,2}|\beta\rangle|^2)\right]-\frac{4}{A}|\langle gr|b_2|\gamma\rangle|^2$$

$$P_{\beta,\beta}=A\left[1-A_0^2(1+\eta^2|\langle\beta|a_{\beta,2}|gr\rangle|^2)\right]-\frac{4}{A}|\langle\beta|b_2|\gamma\rangle|^2$$

$$P_{\gamma,\gamma}=A\left[1-A_0^2\sum{}'\right]-\frac{2}{A}\left[|\langle gr|b_2|\gamma\rangle|^2+|\langle\beta|b_2|\gamma\rangle|^2\right]\left[1-\frac{2}{I(I+1)}\right]$$

$$P_{\beta,gr}=A\left[\langle gr|a_{\beta,1}|\beta\rangle-A_0^2\langle gr|a_{\beta,2}|\beta\rangle(1+\eta)\right]$$

$$P_{gr,\gamma}=\sqrt{2}\left[\langle gr|b_1|\gamma\rangle-A_0\langle gr|a_{\beta,2}|\beta\rangle\langle\beta|b_2|\gamma\rangle\right]\left[1-\frac{2}{I(I+1)}\right]^{1/2}$$

$$P_{\beta,\gamma}=\sqrt{2}\left[\langle\beta|b_1|\gamma\rangle-A_0\eta\langle\beta|a_{\beta,2}|gr\rangle\langle gr|b_2|\gamma\rangle\right]\left[1-\frac{2}{I(I+1)}\right]^{1/2}$$

$A=\frac{1}{2}\ \omega\ \theta_0^2$- is the core inertial parameter

$A_0=(A_p^0-A_n^0)/(A_p^0+A_n^0)$, $\sum{}'=5+\sum_{n=1}^{\infty}\frac{4}{(2n+1)(n+1)}\approx 6.543$.

Let as introduce the following expression for the

quadrupole electric moment operator of a nucleus [5]:

$$M(E2;\mu)=\sum_{\nu}D^2_{\mu\nu}(\Omega)\ m'_{2\nu}+\left(\frac{5}{16\pi}\right)^{1/2}Q_0\ D^2_{\mu 0}(\Omega)$$

where

$$m'_{2\nu}=m_\nu\ (\ |\nu\rangle\langle gr|+|gr\rangle\langle\nu|\ ).$$

Here $m'_{2\nu}$- are determined in the nuclear c.m. frame; Q_0 is the internal nuclear quadrupole moment and $|\nu\rangle=|\beta\rangle, |\gamma\rangle$.

For the reduced probabilities of transitions between the states in the P space one has:

$$B(E2;I_iK_i\rightarrow I_f0_{gr})=\left\{\left(\frac{5}{16\pi}\right)^{1/2}Q_0\left[C^{I_f gr}_{gr}\,C^{I_i gr}_{gr}\,C^{I_f 0}_{I_i 0;20}+\sum_{\nu}C^{I_f gr}_{\nu}\,C^{I_i K_i}_{\nu}\,C^{I_f K_\nu}_{I_i K_\nu;20}\right]+\sqrt{2}\left[C^{I_f gr}_{gr}\sum_{\nu}\frac{m_\nu\,C^{I_iK_i}_{\nu}}{\sqrt{1+\delta_{K_\nu,0}}}\,C^{I_f 0}_{I_i K_\nu;2-K_\nu}+C^{I_iK_i}_{gr}\sum_{\nu}\frac{m_\nu\,C^{I_f gr}_{\nu}}{\sqrt{1+\delta_{K_\nu,0}}}\,C^{I_f K_\nu}_{I_i 0;2K_\nu}\right]\right\}^2. \qquad (16)$$

Here m_ν are the parameters which can be determined using the experimental data.

Using the operator $M(M1)$ from [5], we have the following expression for the reduced m.e. of the M1- transition between the states from the P- space the following expression:

$$\langle\psi^{I_f}_{gr}||M_\theta(M1;\mu)||\psi^{I_i}_{K}\rangle=-\left(\frac{3}{16\pi}\right)^{1/2}(g_p-g_n)\left(\frac{e\hbar}{2mC}\right)\times$$

$$\times\sqrt{2I_i+1}\sum_{i,i'}C^{I_f gr}_{i}\,C^{I_i K}_{i'}\ a_{i,i'}\ , \qquad (17)$$

$$a_{gr,gr}=a_{\beta,\beta}=\frac{A_0}{\sqrt{2}}\left[\sqrt{I_i(I_i+1)}\ C^{I_f 0}_{I_i 1;1-1}-\sqrt{I_f(I_f+1)}\ C^{I_f 1}_{I_i 0;11}\right]$$

$$a_{\gamma,\gamma}=\frac{A_0}{2\sqrt{2}}\ \left[\sqrt{I_i(I_i+1)-2}\ C^{I_f 2}_{I_i 1;11}-\sqrt{I_f(I_f+1)-2}\ C^{I_f 1}_{I_i 2;1-1}-\right.$$

$$- 3 \sqrt{I_f(I_f+1)-6}\, C^{I_f 3}_{I_1 2;11} + 3 \sqrt{I_1(I_1+1)-6}\, C^{I_f 2}_{I_1 3;1-1}\Big]$$

$$a_{gr,\beta} = \frac{A_0}{\sqrt{2}} \langle gr|a_{\beta,2}|\beta\rangle \Big[\eta \sqrt{I_1(I_1+1)}\, C^{I_f 0}_{I_1 1;1-1} - \sqrt{I_f(I_f+1)}\, C^{I_f 1}_{I_1 0;11}\Big]$$

$$a_{\beta,gr} = \frac{A_0}{\sqrt{2}} \langle gr|a_{\beta,2}|\beta\rangle \Big[\sqrt{I_1(I_1+1)}\, C^{I_f 0}_{I_1 1;1-1} - \eta \sqrt{I_f(I_f+1)}\, C^{I_f 1}_{I_1 0;11}\Big]$$

$$a_{\alpha,\gamma} = \frac{\langle\alpha|b_2|\gamma\rangle}{A} \Big[\sqrt{I_1(I_1+1)-2}\, C^{I_f 0}_{I_1 1;1-1} + \sqrt{I_f(I_f+1)}\, C^{I_f 1}_{I_1 2;1-1}\Big]$$

$$a_{\gamma,\alpha} = - \frac{\langle\alpha|b_2|\gamma\rangle}{A} \Big[\sqrt{I_1(I_1+1)}\, C^{I_f 2}_{I_1 1;11} + \sqrt{I_f(I_f+1)-2}\, C^{I_f 1}_{I_1 0;11}\Big]$$

where $\alpha = gr, \beta$

Expression for the reduced probabilities of the M1 - transitions (is the case of the odd states from γ - band) can be written on the basis of (17) in the following manner:

$$B(M1; I_\gamma \to (I-1)_K) = \frac{3}{16\pi}(g_p - g_n)^2 \Big(\frac{e\hbar}{2mc}\Big)\Big(\frac{I+2}{2I+1}\Big)\Big\{\frac{\sqrt{2}}{A}\Big[\langle gr|b_2|\gamma\rangle C^{I-1K}_{gr} +$$

$$+ \langle\beta|b_2|\gamma\rangle C^{I-1K}_{\beta}\Big]\sqrt{I^2-1} + 4A_0 \sqrt{\frac{I-2}{I}}\, C^{I-1K}_{\gamma}\Big\}^2 (C^{I\gamma}_{\gamma})^2. \quad (18)$$

For the magnetic moment of the collective states one has

$$\mu = g_R(I) * I \quad (19)$$

where

$$g_R(I) = \frac{1}{2}(g_p + g_n) - \frac{1}{2}(g_p - g_n) \sum_{1,1'} C^{IK}_{1} C^{IK}_{1'} a_{1,1'} \quad (20)$$

$$a_{gr,gr} = a_{\beta,\beta} = A_0$$

$$a_{\gamma,\gamma} = A_0[1 - 8/I(I+1)]$$

$$a_{\beta,gr} = a_{gr,\beta} = \frac{A_0}{2} \langle\beta|a_{\beta,2}|gr\rangle(1+\eta)$$

$$a_{gr.\gamma} = a_{\gamma.gr} = \frac{\sqrt{2}}{A} \langle gr|b_2|\gamma\rangle[1 - 2/I(I+1)]^{1/2}$$

$$a_{\gamma,\beta} = a_{\beta,\gamma} = \frac{\sqrt{2}}{A} \langle\beta|b_2|\gamma\rangle[1 - 2/I(I+1)]^{1/2}.$$

In order to describe g_R- factor for the P - space states, one needs an additional parameter $\frac{1}{2}(g_p + g_n)$, which is determined from the experimental data.

Calculation for the $^{164,166,168}Er$ isotopes

Calculations were carried out in the case of the Er isotopes. We have used the following procedure for the parameter determination. The inertial parameter A was fixed by using the experimental value for the ground band (I=2) energy $E^{exp.}_{gr}(2)$. The values of A_0 and ω_γ were determined by fitting the experimental data for the energies of γ- band states with odd I. The metrix elements $\langle gr|b_1|\gamma\rangle$, $\langle\beta|b_1|\gamma\rangle$, which describe the direct mixture in gr, β and γ bands, influence both the spectrum and the branching ratios of γ-transitions. They are determined from the condition of the best reproduction of the branching ratios of E2-transitions from γ- band using the formulas A(3), A(4) (see Appendix). The parameters m_β and m_γ are defined by using the following expressions

$$B(E2; 2_\beta \to 0_{gr}) = |m_\beta C^{00}_{20;20}|^2$$
$$B(E2; 2_\gamma \to 0_{gr}) = 2|m_\gamma C^{00}_{22;2-2}|^2 \quad (21)$$

and having the experimental data B(E2) for ^{164}Er [10]. The free parameters: η, $\langle gr|a_{\beta,2}|\beta\rangle$, $\langle gr|b_2|\gamma\rangle$ and $\langle\beta|b_2|\gamma\rangle$, were

defined with in the χ^2 - method by fitting the experimental data. All the model parameters are summarized in Table 1.

With the quoted above values for m_K and Q_0= 742 fm^2 [10] we have calculated the reduced probabilities of E2 - transitions in the γ - vibrational band from (16).

The calculations of the spectra for the positive parity collective states in the cases of $^{164,166,168}Er$ are depicted in Figs. 1-3, respectively. At I≤ 12 the reproduction of expermental data is quite satisfactory. The discrepancy between the theory and experiment at I≥ 14 found in the case

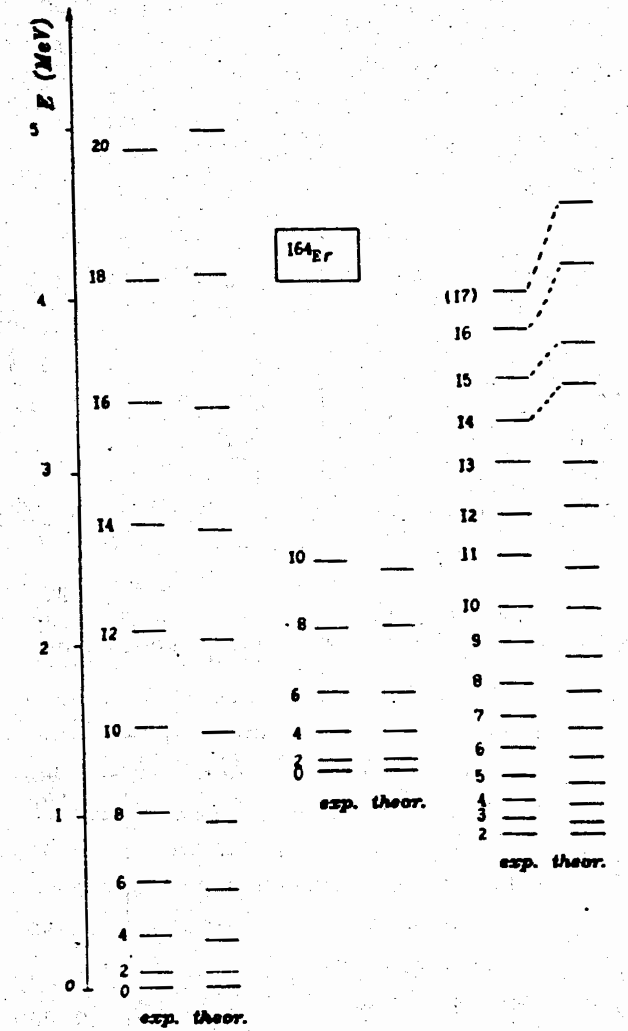

Fig.1. Comparison of the calculated and experimental spectra of positive-parity states for the ^{164}Er nucleus.

Table 3.

The ratio $B(E2;I_\gamma \rightarrow I'_{gr})/B(E2;I_\gamma \rightarrow I''_{gr})$ for the ^{164}Er nucleus

I_γ	I'_{gr}	I''_{gr}	experiment		theory	
			[13]	[6]	TRM	Alaga
2_γ	2_{gr}	0_{gr}	2.23(14)	1.97(30)	1.97	1.43
2_γ	4_{gr}	2_{gr}	0.11(5)	0.15(3)	0.09	0.05
3_γ	4_{gr}	2_{gr}	0.89(7)	0.82(20)	0.81	0.40
4_γ	4_{gr}	2_{gr}	13.3(19)	5.4(13)	7.1	2.94
5_γ	6_{gr}	4_{gr}	1.45(13)	1.3(3)	1.8	0.57
2_γ	4_{gr}	0_{gr}	0.25(10)	0.30(6)	0.18	0.08

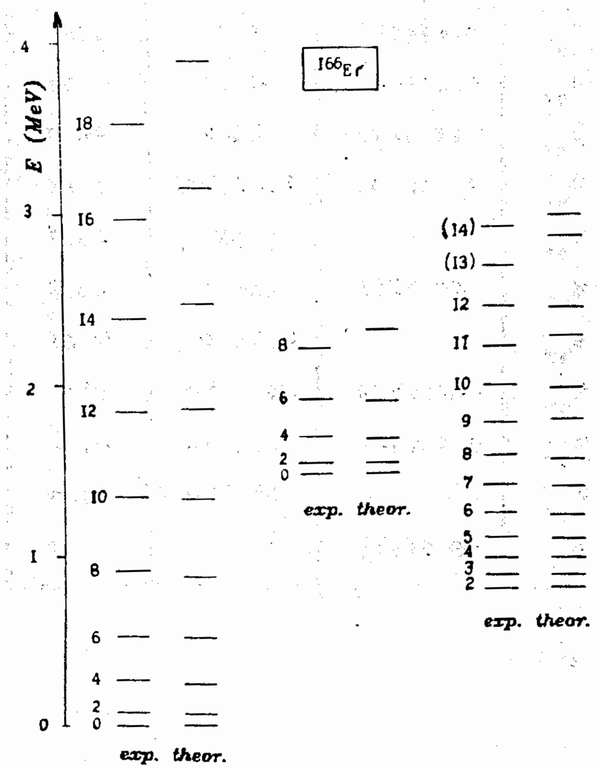

Fig.2. Comparison of the calculated and experimental spectra of positive-parity states for the ^{166}Er nucleus.

Table 4.

The ratio $B(E2;I_\gamma \to I'_\nu)/B(E2;I_\gamma \to I''_{gr})$ for the ^{166}Er nucleus

I_γ	I'_ν	I''_{gr}	experiment		theory	
			[15]	[16]	TRM	Alaga
2_γ	2_{gr}	0_{gr}	1.86(10)	1.91	1.92	1.43
2_γ	4_{gr}	2_{gr}	0.097(8)	0.063	0.091	0.05
3_γ	4_{gr}	2_{gr}	0.66(5)	0.73	0.78	0.40
3_γ	2_γ	2_{gr}	–	34.3	43.8	–
4_γ	4_{gr}	2_{gr}	5.67(45)	6.3	6.24	2.94
4_γ	6_{gr}	4_{gr}	0.26(6)	0.19	.21	0.09
4_γ	2_γ	2_{gr}	64.3(70)	53.5	66.8	–
5_γ	6_{gr}	4_{gr}	1.36(10)	1.52(25)	1.66	0.57
5_γ	4_γ	4_{gr}	–	25(20)	33.7	–
6_γ	6_{gr}	4_{gr}	10.9(8)	13(7)	13.6	3.7
6_γ	8_{gr}	6_{gr}	–	0.28(16)	0.35	0.11
6_γ	4_γ	4_{gr}	241(17)	220(150)	283	–
7_γ	8_{gr}	6_{gr}	2.12(18)	2.5(5)	2.96	0.67
7_γ	6_γ	6_{gr}	–	18.8(40)	26.9	–
8_γ	8_{gr}	6_{gr}	18.9(45)	–	31.8	4.17

Table 5.

The ratio $B(E2;I_\gamma \to I'_{gr})/B(E2;I_\gamma \to I''_{gr})$ for the ^{168}Er nucleus

I_γ	I'_{gr}	I''_{gr}	experiment		theory	
			[17]	[18]	TRM	Alaga
2_γ	2_{gr}	0_{gr}	2.27(45)	1.79(4)	1.73	1.43
2_γ	4_{gr}	2_{gr}	0.044(22)	0.075(4)	0.076	0.05
3_γ	4_{gr}	2_{gr}	0.65(30)	0.64(4)	0.62	0.4
4_γ	4_{gr}	2_{gr}	6.3(30)	5.27(55)	4.6	2.94
4_γ	6_{gr}	4_{gr}	0.08(4)	–	0.16	0.09
5_γ	6_{gr}	4_{gr}	1.0(4)	–	1.15	0.57
6_γ	6_{gr}	4_{gr}	10.7(42)	–	6.9	3.7
6_γ	8_{gr}	6_{gr}	0.19(8)	–	0.24	0.11
7_γ	8_{gr}	6_{gr}	1.64(80)	–	1.72	0.67

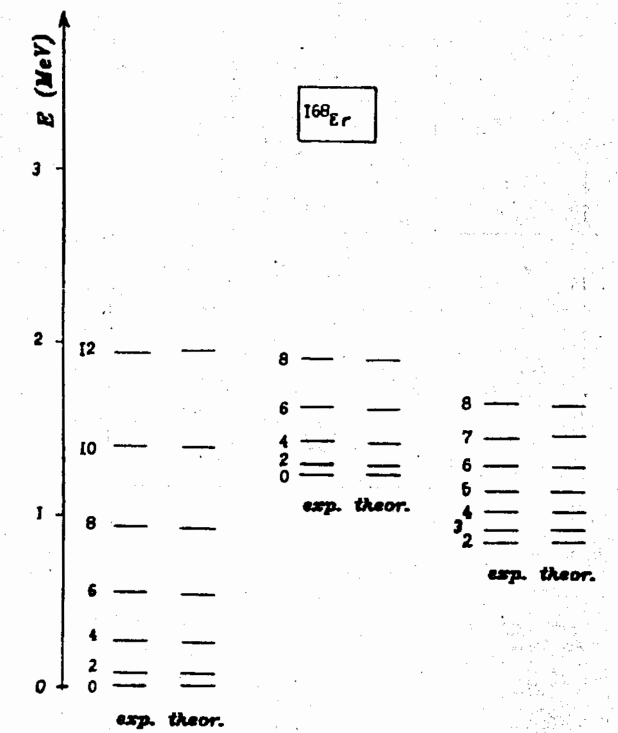

Fig.3. Comparison of the calculated and experimental spectra of positive-parity states for the ^{168}Er nucleus.

Table 6.

The multipole mixture coefficients for the ^{164}Er nucleus

I_i	I_f	exp.[19]	TRM
2_γ	2_{gr}	–	-3.6
3_γ	2_{gr}	$0.13^{+0.28}_{-0.25}$ or $-7.7^{+5.1}_{-\infty}$	-3.1
4_γ	4_{gr}	$-1.15^{+0.35}_{-1.02}$ or > 7	-1.78
5_γ	4_{gr}	$-4.8^{+1.5}_{-5.8}$ or $0.0^{+0.07}_{-0.04}$	-1.63
6_γ	6_{gr}	$-1.19^{+1.6}_{-1.02}$ or >3.3	-1.16
7_γ	6_{gr}	$-6.5^{+2.2}_{-5.5}$	-1.0
8_γ	8_{gr}	$-1.5^{+0.75}_{-3.0}$ or $12.0^{+\infty}_{-6.8}$	-0.82
3_γ	2_γ	$/\delta/ > 3.7$ [6]	4.4

Table 7.

The multipole mixture coefficients for the ^{166}Er nucleus

I_i	I_f	experiment	TRM
1	2	3	4
2_γ	2_{gr}	$-38^{+24}_{-\infty}$ or -19^{+9}_{-38} [20]	-3.6
3_γ	2_{gr}	$-20^{+13}_{-\infty}$ [19] $-18^{+9}_{-\infty}$ [24]	-3.3
3_γ	4_{gr}	$-9^{+5}_{-31.9}$ [24]	-2.5
3_γ	2_γ	$/\delta/>2.6$ [25]	-2.6
4_γ	3_γ	$/\delta/=1.5(3)$ [25]	-2.65
4_γ	4_{gr}	$-3.3^{+1.2}_{-3.0}$ [20] -10^{+4}_{-27} [24]	-1.8
5_γ	4_{gr}	-37^{+3}_{-10} [20] $-84^{+57}_{-\infty}$ [19] $-20(4)$ [24]	-1.8
5_γ	4_γ	$/\delta/=1.61^{+0.53}_{-0.25}$ [15] $/\delta/=2.1$ [26]	-2.2

Continue table 7.

1	2	3	4
5_γ	6_{gr}	-25(3) [24] -5(3) [19]	-1.6
6_γ	6_{gr}	$-1.15^{+0.35}_{-0.80}$ or [19] $-6.3^{+\infty}_{-2.9}$	-1.1
6_γ	5_γ	$/\delta/=1.26$ [26] $/\delta/=1.61^{+0.40}_{-0.37}$ [15]	-3.0
7_γ	6_{gr}	$/\delta/=1.105$ [26] $-37^{+17}_{-\infty}$ [19] -22^{+5}_{-7} [24]	-1.14
7_γ	6_γ	$/\delta/=1.45^{+0.47}_{-0.32}$ [15]	-1.9
7_γ	8_{gr}	$-80<\delta<30$ [24] $-3.1^{+0.9}_{-1.5}$ [19]	1.1
8_γ	8_{gr}	$/\delta/>2$ [24] -0,75(20) or [19] $1.6^{+1.0}_{-0.55}$	-0.72
9_γ	8_{gr}	$-11^{+3}_{-\infty}$ [19]	1.1

Table 8.

The multipole mixture coefficients for the ^{168}Er nucleus

I_i	I_f	experiment	TRM
2_γ	2_{gr}	$/\delta/\geq6.4$ [21] $/\delta/\geq29$ [20] or >9.4 <-4.8 [22]	-4.4
3_γ	2_{gr}	$/\delta/=8.1$ [21] 16.5(23) [22]	-4.0
3_γ	4_{gr}	-4.9(3) [22]	-3.0
4_γ	4_{gr}	$-5.7^{+3.7}_{-5.7}$ [20] $25^{+\infty}_{-13}$ or [22] $50^{+\infty}_{-33}$	-2.5
5_γ	4_γ	$/\delta/=1.41$ [17] $/\delta/=1.38^{+2.05}_{-0.71}$ [23]	3.3
6_γ	5_γ	$/\delta/=1.05$ [17] $/\delta/=1.55^{+1.05}_{-0.76}$ [23]	2.3
7_γ	6_γ	$/\delta/=1.92$ [17] $/\delta/=1.52^{+1.14}_{-0.82}$ [23]	3.6
8_γ	7_γ	$/\delta/=0.245$ [17]	2.5

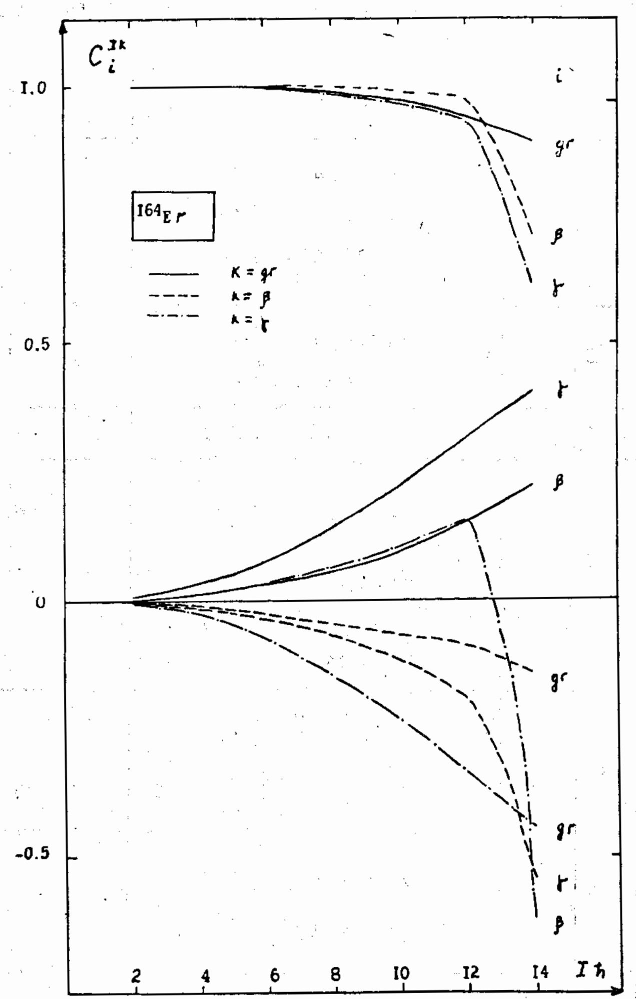

Fig.4. Structure of the wave - functions of gr-, β- and γ-bands for the ^{164}Er nucleus.

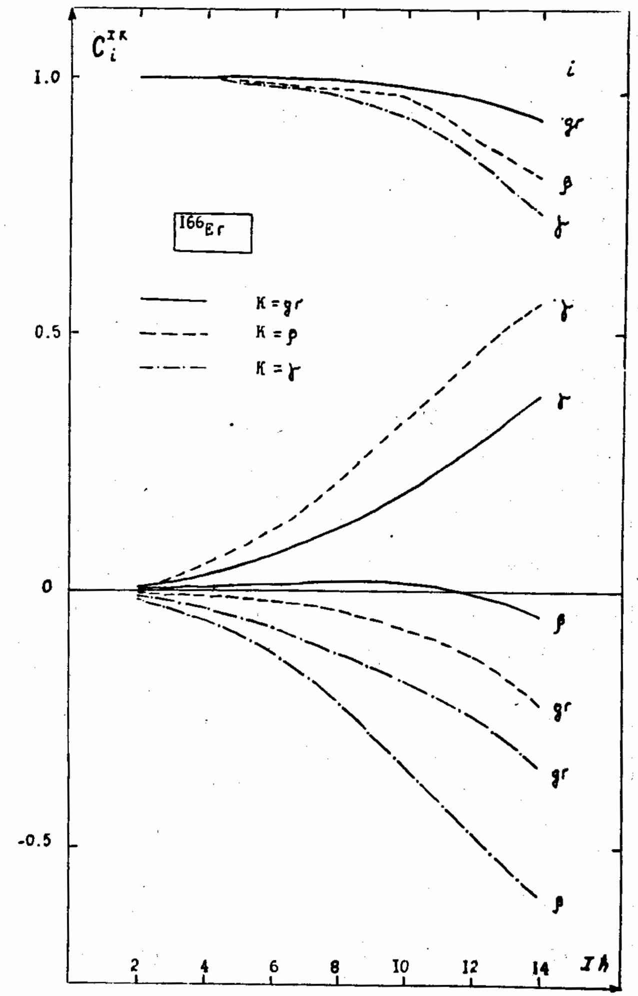

Fig.5. Structure of the wave - functions of gr-, β- and γ-bands for the ^{166}Er nucleus.

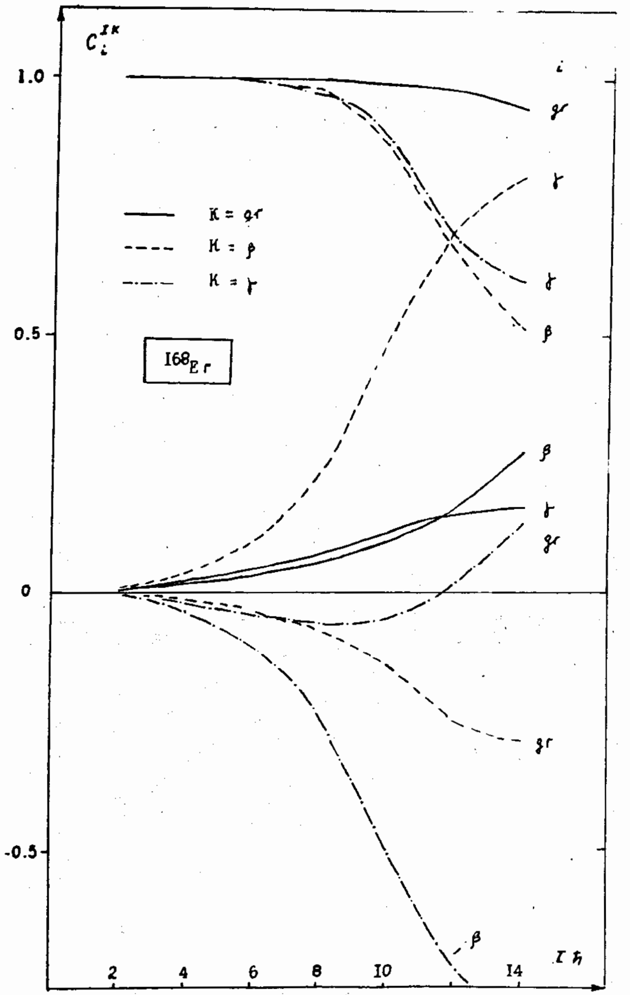

Fig.6. Structure of the wave - functions of gr-, β- and γ-bands for the ^{168}Er nucleus.

$B(M1;00_{gr}\rightarrow 1^{+}1) = 0.8\ \mu_N^2$ for ^{164}Er and $=1.75\ \mu_N^2$ for ^{166,168}Er at fixed value for $\omega_1 = 3$ MeV. Experiment [2] gives for ^{168}Er $B(M1) = 1.75\ \mu_N^2$ and $\omega_1 = 3.4$ MeV. Calculations in the IBM2 give $B(M1) = 1.5\ \mu_N^2$ [8].

Let us note that the signs of δ in Tables correspond to Steffan-Becker convention [10].

We are now in a position to make the following conclusions:

1) The deviation of $R_{I\gamma}$ from Alaga rule are is due to the gr-, β -, and γ - bands mixture.
2) The presence of the GAR components in the wave functions of β - and γ- vibrational bands leads to the M1-transitions from these states on the ground band. The rule of these components increases with the spin, and this explains the decrease of the coefficient of multipole mixture δ in the case of γ - band with increasing angular momentum I.
3) The best description of $\delta^{exp.}$ in the case of the low - lying levels is obtained with $B(M1;00_{gr}\rightarrow 1^{+}1) = 0.8\ \mu_{Я}^2$ for ^{164}Er and $\approx 1.75\ \mu_{Я}^2$ for ^{166,168}Er at fixed value for $\omega_1 = 3$ MeV.

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APPENDIX A

The state mixture leads to the strong violation of the rules of the adiabatic theory for the transition branching. These effects are easily interpreted by using the wave functions of the first perturbation order in the parameter of the bands mixture (in the P- space)

$$|grIM\rangle=|gr(K=0)IM\rangle - \varepsilon_{gr,\beta}\, I(I+1)\, |\beta(K=0)IM\rangle - \sqrt{2}\, \varepsilon_{gr,\gamma}\, \tilde{I}\, |\gamma(K=2)IM\rangle$$

$$|\beta IM\rangle=|\beta(K=0)IM\rangle + \varepsilon_{gr,\beta}\, I(I+1)\, |gr(K=0)IM\rangle - \sqrt{2}\, \varepsilon_{\beta,\gamma}\, \tilde{I}\, |\gamma(K=2)IM\rangle \qquad (A1)$$

$$|\gamma IM\rangle=|\gamma(K=2)IM\rangle + \sqrt{2}\, \tilde{I}\, [\varepsilon_{gr,\gamma}|gr(K=0)IM\rangle + \varepsilon_{\beta,\gamma}|\beta(K=0)IM\rangle]$$

were

$$\tilde{I} = I(I+1)\left[1 - \frac{2}{I(I+1)}\right]^{1/2}, \qquad \varepsilon_{gr,\beta} = \frac{A\langle gr|a_{\beta,1}|\beta\rangle}{\omega_\beta},$$

$$\varepsilon_{\alpha,\gamma}= \frac{\langle\alpha|b_1|\gamma\rangle}{\omega_\gamma - \omega_\alpha}, \qquad \alpha = gr,\ \beta.$$

Using the expression for the m.e. of the quadrupole moment, which were obtained above, one comes to the following formula:

$$\langle grI_f\|M(E2)\|\gamma I_i\rangle = gr(K=0)I_f\|M(E2)\|\gamma(K=2)I_i\rangle +$$

$$\sqrt{2}\, \tilde{I}_i\left[\varepsilon_{gr,\gamma} + \varepsilon_{\beta,\gamma}\left(\frac{16\pi}{5}\right)^{1/2}\left(\frac{m_\beta}{Q_0}\right)\right]\langle gr(K=0)I_f\|M(E2)\|gr(K=0)I_i\rangle$$

$$-\sqrt{2}\, \tilde{I}_f\varepsilon_{gr,\gamma}\, \langle\gamma(K=2)I_f\|M(E2)\|\gamma(K=2)I_i\rangle . \qquad (A2)$$

For the ratio of the reduced probabilities of E2-transitions from the γ- vibrational band we have

$$R_{I\gamma} = \frac{B(E2; I_{1\gamma} \longrightarrow I_{fgr})}{B(E2; I_{1\gamma} \longrightarrow I'_{fgr})} =$$

$$= \left| \frac{C^{I_f 0}_{I_1 2; 2-2} + \frac{Z_\gamma(0)}{\sqrt{24}} \tilde{I}_1 \, C^{I_f 0}_{I_1 0; 20} - \frac{Z_\gamma(2)}{\sqrt{24}} \tilde{I}_f \, C^{I_f 2}_{I_1 2; 20}}{C^{I'_f 0}_{I_1 2; 2-2} + \frac{Z_\gamma(0)}{\sqrt{24}} \tilde{I}_1 \, C^{I'_f 0}_{I_1 0; 20} - \frac{Z_\gamma(2)}{\sqrt{24}} \tilde{I}'_f \, C^{I'_f 2}_{I_1 2; 20}} \right|^2 . \qquad (A3)$$

Transition branching from γ- bands are completely determined by two spin independent parameters

$$Z_\gamma(0) = \left(1 - \frac{\langle\beta|b_1|\gamma\rangle}{\langle gr|b_1|\gamma\rangle} \frac{\omega_\gamma}{\omega_\beta - \omega_\gamma} \sqrt{\frac{16\pi}{5}} \frac{m_\beta}{Q_0} \right) Z_\gamma(2)$$

$$Z_\gamma(2) = \frac{\langle gr|b_1|\gamma\rangle}{\omega_\gamma} \sqrt{24} \sqrt{\frac{5}{16\pi}} \frac{Q_0}{m_\gamma} . \qquad (A4)$$

The spin dependence of reduced transitions probabilities, thus calculated, coincides with that for the mixture of two bands having different quadrupole moments [15].

The factor

$$1 + \left(\frac{\mathcal{E}_{\beta,\gamma}}{\mathcal{E}_{gr,\gamma}} \right) \left(\frac{m_\beta}{Q_0} \right) \left(\frac{16\pi}{5} \right)^{1/2}$$

normalizes the internal quadrupole moment of the γ- band.

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