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OFF-SHELL FORM FACTORS IN THE SOFT PHOTON LIMIT FOR PION PHOTOPRODUCTION

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Recently, Naus *et al* [1] have reconsidered the derivation of the low energy theorems (LET) for pion photoproduction on the basis of the method originally developed by Gell-Mann and Goldberger [2]. They have splitted the photoproduction diagrams into those in which pion and photon vertices are separated by a single pion or nucleon-class A diagrams and the rest- class B diagrams (see Fig.1). This approach is more general than evaluation of the Born terms and the rest in several respects: i) some extra model dependent terms appear and ii) the dependence of the off-shell scalar variables takes place. Moreover, Naus *et al* have incorporated the gauge invariance on the operator level (strong gauge condition) and have come to the relation for the off-shell behaviour for the strong and electromagnetic vertices. The main result of the Naus calculations is in the recovering of the older results [3] for the pion photoproduction amplitude at the threshold.

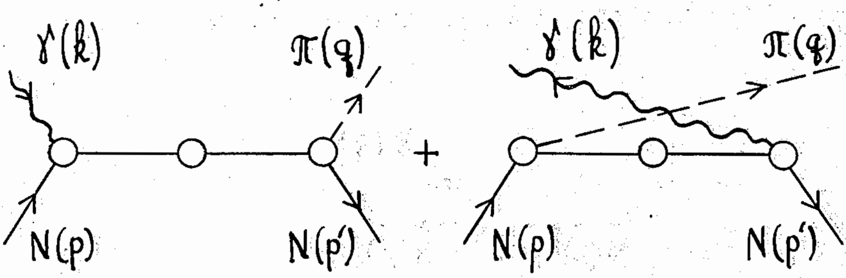

Fig.1 Class A diagrams for π^0 photoproduction reaction.

In this paper, we are going to show in a more straightforward and physically opaque manner, than in Ref.[1], how the off-shell behaviour and additional model dependent terms, caused by splitting the amplitude into class A and B diagrams, disappear in the soft photon limit. To get this goal we shall follow step by step the procedure to obtain the LET which is described in the paper by Adler and Dothan [4].

Let's start by writing down the amplitude for π^0 photoproduction using the quantities and notation as given in Ref.[1]

$$M_\mu(p',q;p,k)=M_\mu^{A_1}(p',q;p,k)+M_\mu^{A_2}(p',q;p,k), \tag{1}$$

where

$$M_\mu^{A_1}=\Lambda_5(p',p+k)\frac{1}{\hat{p}+\hat{k}-M}\left\{\gamma_\mu e_N+\frac{i\sigma_{\mu\nu}k^\nu}{2M}F_2^+ +\frac{\hat{p}+\hat{k}-M}{2M}\frac{i\sigma_{\mu\nu}k^\nu}{2M}(F_2^+-F_2^-)\right\} \tag{2}$$

and

$$M_\mu^{A_2}=\left\{\gamma_\mu e_N+\frac{i\sigma_{\mu\nu}k^\nu}{2M}F_2^+ +\frac{i\sigma_{\mu\nu}k^\nu}{2M}(F_2^+-F_2^-)\frac{\hat{p}'-\hat{k}-M}{2M}\right\}\frac{1}{\hat{p}'-\hat{k}-M}\Lambda_5(p'-k,p). \tag{3}$$

One can easily recognize in the above expressions the terms in which the photon couples to external charged particle lines of the corresponding nonradiative process (see Fig.2).

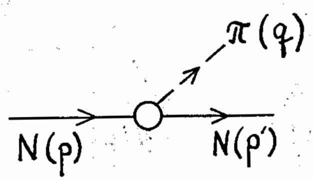

Fig. 2 Nonradiative diagram corresponding to pion photoproduction

According to the Adler-Dothan recipe [4], one must drop all terms in M_μ, which are explicitly independent or of first order in the photon four-momentum k to obtain the truncated amplitude M'_μ. It is clear that the terms proportional to $(F_2^+-F_2^-)$ are al least of an order of k^1 because the linear combinations of the Dirac operators in these terms cancel the nucleon propagator. And the first conclusion immediately comes: the extra model-dependent terms, proportional to $(F_2^+-F_2^-)$, are (according to the Adler-Dothan procedure) negligibly small in the soft photon limit.

To proceed with our consideration we note that $F_i^\pm$ in eqs.(1,2) are the functions of the (off-shell) scalar variables [1]: $F_i^\pm=F_i^\pm\left[(p+k)^2\right]$ in (2) and $F_i^\pm=F_i^\pm\left[(p'-k)^2\right]$ in (3). We expand $F_i^\pm$ with respect to k and get

$$F_i^\pm\left[(p+k)^2\right]=F_i^\pm(M^2)+(\hat{p}+\hat{k}-M)(\hat{p}+\hat{k}+M)\left.\frac{dF_i^\pm}{dp^2}\right|_{k\to0}+\ldots \tag{4}$$

and

$$F_i^\pm\left[(p'-k)^2\right]=F_i^\pm(M^2)+(\hat{p}'-\hat{k}-M)(\hat{p}'-\hat{k}+M)\left.\frac{dF_i^\pm}{dp'^2}\right|_{k\to0}+\ldots \tag{5}$$

For the same arguments as given above we must keep in these expressions only the first terms. Using the identity $F_2(M^2)=\kappa^S+\kappa^V\tau_3$ with $\kappa^S=(\kappa_p+\kappa_n)/2=-0.06$ and $\kappa^V=(\kappa_p-\kappa_n)/2=1.85$ one comes to the γNN vertex as used in the Born approximation.

Before considering a similar procedure for the off-shell πNN vertex Λ_5, we note, that, following the Adler-Dothan recipe one must add to the truncated amplitude M'_μ a ΔM_μ independent of k so as to make $k_\mu(M'_\mu+\Delta M_\mu)=O(k^2)$. This constraint leads to the following expression which is in the one-to-one correspondence with the condition for the class B terms of Naus *et al*

$$k_\mu\Delta M_\mu=e\left[\Lambda_5(p'-k,p)-\Lambda_5(p',p+k)\right] \tag{6}$$

The off-shell behaviour in the πNN vertices is removed in a similar manner as in the case of the γNN vertex. From the expression (A_1 diagram)

$$\Lambda_5(p',p+k)=\gamma_5 f_1\left[(p+k)^2,M^2,m_\pi^2\right]+\gamma_5 f_2\left[(p+k)^2,M^2,m_\pi^2\right]\frac{\hat{p}+\hat{k}-M}{M}$$

by means of the expansion of the form factors with respect to k, one easily comes to

$$\Lambda_5=\gamma_5\left[f_1(M^2,M^2,m_\pi^2)-2f_2(M^2,M^2,m_\pi^2)\right]+\frac{f_1(M,M^2,m_\pi^2)}{M}\gamma_5\hat{q}\ . \tag{7}$$

With the appropriate choice of the constants $f_i(M,M^2,m^2)$ it is the PS- or PV-coupling in the πNN vertex. Now we are in a position to make the second conclusion. The dependence of the off-shell scalar variables disappears in the derivation of the LET. This is a nice feature of the Adler-Dothan procedure because the final expressions are formulated in terms of the physically measurable (on-shell) quantities.

We encapsulate the above conclusions in the following statement: the splitting of the amplitude into classes A and B, and the introducing of the off-shell form factors *must* lead back to the Born terms. All additional terms, according to the Adler-Dothan recipe, are to be neglected in the derivation of the LET.

We don't continue the consideration of the topic by imposing the PCAC constraints, because this task can be found, for example, in the nice review by Kamal [5].

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Чумбалов А.А., Камалов С.С. E4-91-336

Внемассовые формфакторы в пределе мягких фотонов для фоторождения пионов на нуклонах

Низкоэнергетические теоремы для фоторождения пионов на нуклонах рассматриваются на основе метода Адлера-Дотана. Показано, что в пределе мягких фотонов все дополнительные модельно-зависимые слагаемые исчезают. Кроме того, показано, что в этом пределе нет необходимости вводить зависимость формфакторов от внемассовых скалярных переменных в различных вершинах.

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Off-Shell Form Factors in the Soft Photon Limit for Pion Photoproduction

The low energy theorems for the pion photoproduction are considered on the basis of the Adler-Dothan procedure. It is shown that the soft photon limit necessarily leads to the Born approximation for the π^0-photoproduction amplitude at the threshold. All additional terms, according to the Adler-Dothan recipe, are to be neglected in the derivation of the LET.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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