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FERMION LOCALIZATION AND CAUSALITY

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Локализация фермиона и причинность

Рассматривается задача о скорости распространения фермиона от источника до детектора. Показана неприменимость простейшего подхода в виде задачи о расплывании пакета свободной частицы (Хегерфельдт и др.) ввиду невозможности локализации фермиона в конечном объеме. Предложен другой подход, использующий квантово-полевое описание источника и детектора фермиона. Установлено, что скорость распространения фермиона не превышает скорости света в пределах точности локализации источника и детектора.

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Fermion Localization and Causality

The velocity of a fermion (neutrino) propagation from its source to its detector is investigated. It is shown that a simple approach to the problem in terms of the quantum packet spreading (Hegerfeldt et al.) is not applicable because the fermion cannot be localized in a bounded space region. Another approach is elaborated which uses relativistic quantum field description of the fermion source and detector. It is demonstrated that the velocity of the fermion propagation does not exceed the light velocity C within the precision of the source and detector localization.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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Introduction

Relativistic quantum theory has a trouble, its simplest manifestation being the instantaneous spreading of the free particle packet. Let the particle be localized somehow in a bounded region V_S at a moment t = 0. Immediately after the moment the particle can be found in a region V_D, separated from V_S by an arbitrary distance R, see e.g. /1-8/. This means that it is possible to transmit a signal from V_S to V_D with the superluminary velocity. So one deals here with the problem of compatibility of the basic postulates of quantum theory and special relativity.

The acausal probability to find the particle at V_D at $t < R/c$ turns out to be very small if (R - ct) exceeds the particle Compton wave-length λ = h/mc, see /9,5/. Therefore, the acausality is particularly prominent in the cases of low mass (or massless) particles such as the photon or the neutrino. As to the photon, one meets problems with its localization, see e.g. /10,11/; so the photon packet spreading problem is ill defined. In spite of this the velocity of the photon propagation has been investigated since 1930 by using a more realistic approach than in /1-9/. For instance, an excited atom localized in V_S was considered as a photon source. Another (unexcited) atom localized in V_D served as a photon detector. For the relevant papers, see /12/. The list of references of this review can be supplemented by the papers /13-21/. The result of these investigations can be formulated as follows : if the problem is properly stated and accurately calculated, then the velocity of the signal transmission by means of the photon does not exceed the velocity c.

Let us stress that the problem arises only if particle observables are involved. The quantum fields and observables constructed in their terms (e.g., electric and magnetic fields, momentum and energy densities of the field, current density) behave causally in local theories, see, e.g., /22,12,23/. However, this does not secure the causal behaviour of the observables constructed from the positive energy parts of the fields, i.e. observables pertinent to the particle interpretation of quantum fields.

Here I investigate the propagation of a fermion, using neutrino as an example. Several possible statements of the problem are given in Sect. 1 in the order of increasing generality. It is shown in Sect. 2 why the simplest variants used in /1-5 / and /6/ cannot be exploited in the fermion case. The reason is that the free Dirac's fermion cannot be strictly localized in a bounded region. An approximate or "effective" localization of the fermion is possible but only within the precision not exceeding the fermion Compton wave-length λ .

Sect. 3 presents my approach. As in the photon case, it is grounded on the possibility of effective localizing the nonfree fermion within a region with dimension much less than λ . For instance, the electron or neutrino originating from β radioactive nucleus are emitted from the nucleus volume, its dimension being much smaller than λ_e or λ_ν . To describe the processes of neutrino creation in V_S and detection in V_D ; I use the Heisenberg picture of quantum field theory with the four-fermion weak interaction. The result of the calculation turns out to be causal : neutrino propagation velocity does not exceed c within the precision of the neutrino localization by its source and detector.

I. Forms of the Relativistic Causality Criterion

The simplest formulation is packet spreading. Let φ_S describe one free particle, localized at $t = 0$ in a volume V_S so that the probability of finding it outside V_S is zero. Then

$$|\langle \varphi_D | \varphi_S(t)\rangle|^2 = \langle \varphi_S(t) | \varphi_D\rangle\langle \varphi_D | \varphi_S(t)\rangle = \langle \varphi_S(t) | P_D | \varphi_S(t)\rangle$$

$$\varphi_S(t) = exp(-iHt)\ \varphi_S \qquad P_D \equiv |\varphi_D\rangle\langle\varphi_D| \qquad (I)$$

is the probability of finding the particle at the moment $t > 0$ in a state φ_D , localized in a region V_D at a distance R from V_S. H is the particle Hamiltonian, P_D is the projector on the one-particle state φ_D . According to special relativity, the probability (I) must vanish at $t < R/c$. Different variants of the formulation were used in /I-7/.

Let us generalize it.

a) One may replace P_D in (I) by another particle observable X_D which is in a sense localized in V_D . If the particle is described by a quantum field, then X_D may be the operator of the number of particles in V_D , e.g. see /24/ . Then, $\langle \varphi_S(t) | X_D | \varphi_S(t)\rangle$ would be the average number of particles in V_D at the moment t which also must vanish at $t < R/c$.

b) Real particle interacts with other particles, the interaction being described usually by a local quantum field theory.

Then, the probability of finding particle in $V_\mathcal{D}$ at $t > 0$ may not vanish even if there was no particle initially (let φ_o denote the initial no-particle state). For instance, the particle can appear in $V_\mathcal{D}$ as a virtial one (together with other particle). The probability must be considered as the "background" one, e.g. see /I2,23/. Let the state φ_s differ from φ_o only by the presence of one additional particle in V_s at $t = 0$. The difference

$$\Delta X_\mathcal{D}(t) = \langle \varphi_s(t) \,|\, X_\mathcal{D} \,|\, \varphi_s(t) \rangle - \langle \varphi_o(t) \,|\, X_\mathcal{D} \,|\, \varphi_o(t) \rangle \tag{2}$$

would be nonvanishing entirely due to the presence of the initial particle in V_s , other causes of the particle appearance in $V_\mathcal{D}$ being excluded in (2) . The difference must vanish if $t < R/c$ [x)].

One may rewrite (2) as

$$\Delta X_\mathcal{D}(t) = \langle \varphi_s \,|\, X_\mathcal{D}(t) \,|\, \varphi_s \rangle - \langle \varphi_o \,|\, X_\mathcal{D}(t) \,|\, \varphi_o \rangle$$
$$X_\mathcal{D}(t) = e^{iHt} X_\mathcal{D}\, e^{-iHt}. \tag{3}$$

Here $X_\mathcal{D}(t)$ is the Heisenberg operator corresponding to $X_\mathcal{D}$. Its calculation may happen to be simpler than solving the Schroedinger equation in order to find $\varphi_s(t)$.

c) The localization of the initial particle in V_s is the result of a preparation process. Its idealized description can be included in theoretical formulation of the discussed problem. For instance, an excited atom can be considered as a source of localized photon state, the photon being the signal carrier. A theoretically simpler source is an external current localized in V_s /I2/. The external current is a prescribed function of time which does not change when emitting photons. In sect.3, I shall use an external source η of the fermion which carries the signal. Now

$$\Delta X_\mathcal{D} = \langle \varphi_o \,|\, X'_\mathcal{D}(t) \,|\, \varphi_o \rangle - \langle \varphi_o \,|\, X_\mathcal{D}(t) \,|\, \varphi_o \rangle \tag{4}$$

is the change of the observable $X_\mathcal{D}$ which is due to the switching on of the external source. Here $X'_\mathcal{D}(t)$ is the Heisenberg operator (coinciding with $X_\mathcal{D}$ at $t = 0$) when the source has been switched on, $X_\mathcal{D}(t)$ is the "background" Heisenberg operator when the source has not been switched on.

x) In order to compare $\langle \varphi_s(t) \,|\, X_\mathcal{D} \,|\, \varphi_s(t) \rangle$ with the background, one ought not use just the difference: one may consider its ratio instead. But it is the difference which is simpler to calculate. For instance, $\langle \varphi_s(t) \,|\, X_\mathcal{D} \,|\, \varphi_s(t) \rangle$ and $\langle \varphi_o(t) \,|\, X_\mathcal{D} \,|\, \varphi_o(t) \rangle$ may contain divergencies which are absent in their difference.

d) The appearance of the signal carrier in $V_\mathcal{D}$ can be detected by observing the change of the state of other particles in $V_\mathcal{D}$, the change being the consequence of their local interaction with the signal carrier. In particular neutral particles can be detected by means of charged ones. So one can replace $X_\mathcal{D}$ in (2)-(4) by other observables $Y_\mathcal{D}$ which are constructed in the same manner as $X_\mathcal{D}$ but from the operators of other particles. Of course, $Y_\mathcal{D}$ must be localized in $V_\mathcal{D}$ in a sense. The vanishing of

$$\Delta Y_\mathcal{D}(t) = \langle \varphi_o \,|\, Y'_\mathcal{D}(t) - Y_\mathcal{D}(t) \,|\, \varphi_o \rangle \tag{5}$$

at $t < R/c$ is the example of the general form of the causality criterion: no change in $V_\mathcal{D}$ (as compared to the background) can be till the moment R/c.

2. Fermion Localizability

2.I. The description of localization of the signal source and detector needs a notion of coordinates. The special relativity theory uses coordinates $\vec{x}$, t measured by rods and watches, i.e. the Minkowski coordinates, MC. The derivatives in the Klein-Gordon or Dirac equations are over just the MC. The coordinates are used also as parameters, numerating the field degrees of freedom $A(\vec{x},t)$, $\psi(\vec{x},t)$ etc. The locality of interaction of the fields is defined by using MC.

Other coordinates are also discussed in relativistic quantum theory, namely particle position operators and their eigenvalues, e.g., see the review /25/. Their properties (hermiticity, transformation properties, etc.) are usually discussed in the framework of the theory of one free particle (an example of the exception is the paper /26/). However, the main problem in this topic is the relevance of the position operators to the outcomes of real measuring devices. The relevance is obscure because the particle interacting with the measuring device is not free.

In the problem of signal transmission the choice of a coordinate is dictated by the following reason. One deals with the special relativity requirement "signal velocity must not exceed c ". The velocity must be defined in terms of MC and just the coordinates must be used.

2.2. The packet spreading problems, which are discussed in /I-5/, need the initial state φ_s which is strictly localized within a bounded region. However, positive-energy states of the free

Dirac fermion are described by the spinors $\psi_\mu(\vec{x})$ which cannot vanish outside a bounded region V, see Appendix A. In other words, there exists no free spinor ψ^s such that $\rho^s(\vec{x}) = \sum_\mu |\psi^s_\mu(\vec{x})|^2$ would vanish outside V.

The theorem "Violation of Causality" by Hegerfeldt /6/ allows nonlocalized initial states, but the state allowed by the theorem has the property of $\rho^s(\vec{x})$ decreasing as $|\vec{x}| \to \infty$ faster than $\exp(-K_2|\vec{x}|)$, where $K_2 > 2m$ (then one can realize the condition (I) from /6/ or (2I) from /7/). But this decreasing is forbidden for fermionic states, as is shown in Appendix A.

2.3. The papers /I-7/ define the probability of finding the spreading particle in the volume V using the expression $\langle \varphi_s(t), N_v\, \varphi_s(t)\rangle$ which in the case of the Dirac fermion assumes the form

$$\langle\psi^s(t), N_v \psi^s(t)\rangle = \int_V d^3x \sum_\mu |\psi^s_\mu(\vec{x},t)|^2; \quad N_v \equiv \int_V d^3x \sum_\mu |\vec{x},\mu\rangle\langle\vec{x},\mu|. \tag{6}$$

This is the limit of the sum of probabilities to find the particle in the states $\psi^i(x)$ localized in small volumes V_i which form the volume V: $V = \sum_i V_i$. The related measurement must reduce $\psi^s(t)$ in each measurement act to one of the states ψ^i, $\operatorname{supp}\psi^i = V_i$ /27,28/. There are reasons to suppose that no such measurements exist in the case of a fermion because there exist no physical means to localize strictly the fermion within an arbitrary small volume V_i. This is true, e.g, for the measurement which reduces to free fermion states. The paper /8/ argues the impossibility of such a fermion localization by means of a bounding potential.

There exists another expression for the probability of finding the particle in a volume V:

$$\Big|\int d^3x \sum_\mu \psi^{v*}_\mu(\vec{x})\,\psi^s_\mu(\vec{x},t)\Big|^2 = |\langle\psi^v|\psi^s(t)\rangle|^2. \tag{7}$$

It corresponds to the measurement which reduces $\psi^s(\vec{x},t)$ to the states ψ^v which is localized in V, the approximate localization being now allowed. The state ψ^v is determined by the concrete measurement process. If one represent ψ as $\sum_i \psi^i$, then (7) would depend on ψ^i phases while (6) does not depend on them. Expression (7) is allowed for fermions, and it was used in (I) and shall be used in sect. 3 below.

2.4. Let us comment the causality criterion used in /9,6,7/. It states that the probability of finding the particle at the moment t within a sphere V of the radius r must be less than the probability of finding the particle at t=0 within the sphere V_t which has

the same centre as V does but has the radius r = r + ct (or to find it within a larger region containing V_t /6/). Note, first, that, the definitions (6) of the probabilities (criticized above) were used in /9,6,7/. But it is more important to stress that this criterion differs from the requirement "signal velocity cannot exceed **c** "because the measurements related with the criterion do not determine any signal velocity.

2.5. There are empirical facts which give evidences for the existence of much better fermion localization than that permissible for the free fermion. For instance, one may conclude that neutrino emerging from the process $e^- + Be \to Li + \gamma + \nu$ was localized at some instant of time in the volume of the nucleus whose dimension is much smaller than λ_ν. It is possible also to detect neutrino in the region with dimension $\ll \lambda_\nu$, e.g., using the known reaction $\nu + Cl \to Ar + e^-$. Note that the localization of a nucleus as a whole also can be realized within the precision $\ll \lambda_\nu$ (e.g., in the crystal lattice). In these examples, the neutrino is not free; it is created or absorbed. The related localization measurement cannot be described entirely by classical means because the description of the creation or absorption needs quantum field theory. Using the theory I shall describe the first stage of the neutrino localization in which the neutrino transforms into heavier and easier detectable particles. As to the measurement of the latter, I use the usual postulates of the quantum measurement theory /27,28/.

3. Neutrino propagation velocity

The form d) of the causality criterion is used. Signal carrier is the neutrino. It is supposed to be detected using the process of the type $\nu + Cl \to Ar + e^-$. The related elementary reaction is $\nu + n \to p + e^-$, where n is one of the nucleus neutrons x). It is bounded, stable and is localized effectively in the nucleus volume V_n. The occurrence of the reaction is supposed to be detected by the observation of the resulting electron. All particles v,n,p,e are described by quantized fields and the interaction Hamiltonian is taken in the Fermi four-fermion form /3I,32/:

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x) The small probability of the reaction is irrelevant to the superluminary velocity trouble considered. If the probability of the superluminary velocity does not vanish within the measurement errors (however small it be), then there arises the logical paradox in the framework of the special relativity: the effect may abolish its cause /29,30/.

$$G/\sqrt{2}\,[\bar{\psi}_p Q \psi_n \; \bar{\psi}_e O \psi_\nu + \bar{\psi}_n Q \psi_p \; \bar{\psi}_\nu O \psi_e] \qquad (8)$$

(one may have in mind the V-A variant).

As an observable $Y_\mathcal{D}$ of the form d), I take the operator of the number of electrons in a state $\psi^\mathcal{D}$ which is approximately localized in and near a volume $V_\mathcal{D}$, $V_n \subset V_\mathcal{D}$, see Appendix B.

$$Y_\mathcal{D} = A_\mathcal{D}^+ A_\mathcal{D} \quad ; \quad A_\mathcal{D} = \int d^3x \, \Sigma_\mu \, \psi_\mu^{\mathcal{D}*}(\vec{x}) A_\mu (\vec{x}) \, , \qquad (9)$$

$$\Sigma_\mu \int d^3x \, |\psi_\mu^\mathcal{D}(\vec{x})|^2 = 1 \quad , \quad \{ A_\mathcal{D}, A_\mathcal{D}^+ \}_+ = 1 \, . \qquad (10)$$

$$A_\mu(x) = \int d^3p \, \Sigma_r \, a_{pr} \, u_\mu(\vec{p},r) \exp(i\vec{p}\vec{x}) = \int d^3x \, \Sigma_\nu \, \Pi_{\mu\nu}(\vec{x},\vec{y}) \, \psi_e(\vec{y}) . \qquad (11)$$

Here A_μ is the part of the electron-positron field ψ_e which annihilates electrons, Π is the operator projecting onto this part, see Appendix B.

The average of $A_\mathcal{D}^+ A_\mathcal{D}$ in a state φ is equal to the number of electrons in φ , irrespective of the presence of other particles in φ . As there can be only one electron in the state $\psi^\mathcal{D}$, the average is simultaneously the probability of finding the one- electron state $\psi^\mathcal{D}$ in φ (for more details see /33/, sect. 4).

The source of the neutrino (localized in V_S) is described by the prescribed Grassmann function $\eta(x)$. It is introduced in the Dirac equation for the neutrino field

$$(-i\gamma_\mu \partial_\mu + m)\psi_\nu = \eta + \ldots$$

by analogy with inserting an external current $\vec{J}$ into the equation for electromagnetic potential $\vec{A}$: $\Box \vec{A} = \vec{J} + \ldots$.

This is equivalent to the addition of the term $\bar{\eta}\psi_\nu + \bar{\psi}_\nu \eta$ to the interaction Hamiltonian (8). The function $\eta(\vec{x},t)$ is localized in the volume V_S and is switching on at the moment t = 0, being zero before.

Unlike the neutrino source reaction $e^- + Be \to Li + \gamma + \nu$ the external source η can create antineutrinos together with neutrinos (and also annihilate them). However, only the neutrinos are detected by the process $\nu + n \to p + e^-$,and therefore, the signal transfer from V_S to $V_\mathcal{D}$ is realized by the neutrino only.

3.1. Now the problem consists in the calculation of the change of the number of electrons, approximately localized in $V_\mathcal{D}$ which is

due to the switching on of the neutrino source in V_S

$$\Delta N_e(t) = \langle \varphi \,|\, A_\mathcal{D}^{\eta +}(t) A_\mathcal{D}^\eta(t) - A_\mathcal{D}^+(t) A_\mathcal{D}(t) \,|\, \varphi \rangle , \qquad (12)$$

where φ is the state "one neutron and no other particles"; $A_\mathcal{D}^\eta(t)$ is the Heisenberg operator (coinciding at t=0 with the Schroedinger one $A_\mathcal{D}$, see (9)) in the case when the total Hamiltonian H contains besides the free part H_0 the interaction

$$H_I = (8) + \bar{\eta}\psi_\nu + \bar{\psi}_\nu \eta \qquad (13)$$

and $A_\mathcal{D}(t)$ is the "background" Heisenberg operator of the case $\eta = 0$, $A_\mathcal{D}(t = 0) = A_\mathcal{D}$. The "backgroung" in (12) originates, e.g., from electrons which can appear in $V_\mathcal{D}$ at t> 0 due to the process $n \to p + e^- + \nu$, the process being the virtual one because n is a stable bounded neutron.

To calculate (12) one has to find the Heisenberg operators $\psi_{e\eta}(\vec{x},t)$and $\psi_e(\vec{x},t)$ because

$$A_\mathcal{D}^\eta(t) = \int d^3x \, \Sigma_\mu \, \psi_\mu^{\mathcal{D}*}(\vec{x}) \int d^3y \, \Sigma_\nu \, \Pi_{\mu\nu}(\vec{x},\vec{y}) \, \psi_{e\eta\nu}(\vec{y},t) \qquad (14)$$

see (9) and (11). Note that Π may be omitted in (14) because $\Pi\psi^\mathcal{D} = \psi^\mathcal{D}$, see App. A and B.

"To find the Heisenberg operator $\psi_e(\vec{x},t)$ "means expressing it in terms of all Schroedinger operators $\psi_e(\vec{x}), \psi_\nu(\vec{x}), \psi_p(\vec{x}), \psi_n(\vec{x})$ or, equivalently, in terms of the Schroedinger creation-destruction operators (because the former can be expanded in terms of the latter, see (20) below). This expression allows one to calculate (12) since we know how to calculate averages of the creation- destruction operators in the state φ, see below.

3.2.So we need equations for the Heisenberg operators and their solutions. The equations can be obtained by using $\dot{\psi} = -i[\psi, H]$:

$$(-i\gamma_\mu \partial_\mu + m_\nu)\psi_{\nu\eta} = \eta - G/\sqrt{2} \; O\psi_{e\eta} \, [\bar{\psi}_{n\eta} Q \psi_{p\eta}] \, , \qquad (15)$$

$$(-i\gamma_\mu \partial_\mu + m_e)\psi_{e\eta} = - G/\sqrt{2} \; [\bar{\psi}_{p\eta} Q \psi_{n\eta}] \, O\psi_{\nu\eta} \, , \qquad (16)$$

$$(-i\gamma_\mu \partial_\mu + m_n + U(\vec{x}))\psi_{n\eta} = - G/\sqrt{2} \; Q\psi_{p\eta} \, [\bar{\psi}_{\nu\eta} O \psi_{e\eta}] . \qquad (17)$$

The equation for $\psi_{p\eta}$ is similar to (17) ; $U(\vec{x})$ is the effective potential which binds nucleons in the nucleus. The equations for the "background" Heisenberg operators (ψ without the subscript η) differ only by omission of the term η in the equation for ψ_ν .

Let us rewrite the equations in the following integral form, e.g. see /34,35/

$$\psi_{\nu\eta}(\vec{x},t) = \psi_\nu^f(\vec{x},t) + \int_0^t dy_0 \int d^3y\, S_\nu(x-y)\,\eta(y) + g\int_0^t dy_0 \int d^3y\, S_\nu(x-y)\left[\bar{\psi}_{n\eta}(y)\, Q\, \psi_{p\eta}(y)\right] O\, \psi_{e\eta}(y), \tag{I5'}$$

$$\psi_{e\eta}(\vec{x},t) = \psi_e^f(\vec{x},t) + g\int_0^t dy_0 \int d^3y\, S_e(x-y)\left[\bar{\psi}_{p\eta}(y)\, Q\, \psi_{n\eta}(y)\right] O\, \psi_{\nu\eta}(y). \tag{I6'}$$

The equations for $\psi_{p\eta}$ and $\psi_{n\eta}$ are similar to (I6'), let us call them equation (I7') and (I8'), respectively. The following notation $g = G/\sqrt{2}$, $x = (\vec{x}, t)$, $y = (\vec{y}, t)$ was used; ψ^f are solutions of the corresponding Dirac equations in the case $\eta = 0$ and $G = 0$ ("free solutions"). They can be represented as

$$\psi^f(x,t) = \int d^3x' \left[-i\, S(\vec{x}, t\, ; \vec{x}', 0)\, \psi(\vec{x}')\right. \tag{I9}$$

see, e.g., (8.67) in /31/. It follows from (I9) that $\psi^f(\vec{x},t)$ coincides at t = 0 with the Schroedinger operator $\psi(x)$. Therefore, the Heisenberg operator $\psi(\vec{x},t)$ at t = 0 also coincides with $\psi(\vec{x})$: the integrals in the r.h.s. of (I5'), (I6') vanish at t = 0 . In the following I use another known representation of ψ^f as the expansion over (Schroedinger) creation-destruction operators, e.g.

$$\psi_e^f(\vec{x},t) = \int d^3p \sum_r \left[u(p,r)\, e^{i(\vec{p}\vec{x} - E_p t)} a_{p,r} + v(p,r)\, e^{i(\vec{p}\vec{x} + E_p t)} b_{p,r}^+\right]. \tag{20}$$

Here $E_p = \sqrt{\vec{p}^2 + m^2}$. Analogous expansions for ψ_n^f and ψ_p^f must contain spinors u and v which are the solutions of the time-independent Dirac equations with the nucleon potential. So the expansion must contain also the sum over bounded states.

One can represent the Heisenberg operators $\psi(\vec{x},t)$ in terms of $\psi^f(\vec{x},t)$ using the perturbation approach described in /36,37/. Simultaneously, the Heisenberg operators will be represented in terms of the Schroedinger ones $\psi(\vec{x})$ or a,b$^+$, see (I9), (20).

Let us expand

$$\psi(x) = \sum_{n=0}^{\infty} g^n \psi^{(n)}(x) \qquad , \qquad x \equiv (\vec{x}, t) \tag{2I}$$

insert the expansion in (I5') - (I8') and collect together terms having the same power of g . We obtain

$$\psi_{\nu\eta}^{(0)} = \psi_\nu^f + \int S_\nu \eta \quad ; \quad \psi_\nu^{(0)} = \psi_\nu^f , \tag{22}$$

$$\psi_{e\eta}^{(0)} = \psi_e^f = \psi_e^{(0)} \quad ; \quad \psi_{n\eta}^{(0)} = \psi_n^f = \psi_n^{(0)}, \tag{23}$$

$$\psi_{\nu\eta}^{(1)} = \int S_\nu\, O\, \psi_e^f \left[\psi_n^f Q\, \psi_p^f\right] = \psi_\nu^{(1)}, \tag{24}$$

$$\psi_{e\eta}^{(1)} = \int S_e \left[\bar{\psi}_p^f Q\, \psi_n^f\right] O\, \psi_{\nu\eta}^{(0)} . \tag{25}$$

Note that $\psi_e^{(1)}$ differs from $\psi_{e\eta}^{(1)}$, see (25), because $\psi_{\nu\eta}^{(0)}$ differs from $\psi_\nu^{(0)}$, see (22):

$$\psi_e^{(1)}(x) = \int d^4y\, S_e(x-y)\left[\bar{\psi}_p^f(y)\, Q\, \psi_n^f(y)\right] O\, \psi_\nu^f(y) . \tag{26}$$

3.3. Now one can calculate ΔN_e , see (I2), (I4), in the first nonvanishing approximation. Using $\psi_{e\eta} = \psi^f + g\psi_{e\eta}^{(1)} + g^2\psi_{e\eta}^{(2)}$ and analogous expansion for ψ_e , one obtains

$$\Delta N_e(t) = \int d^3x \int d^3x' \sum_{\alpha\beta} \psi_\alpha^{\mathcal{D}*}(\vec{x}') \cdot \langle \varphi | \psi_{e\eta\beta}^{+}(\vec{x},t)\, \psi_{e\eta\alpha}(\vec{x}',t) - \psi_{e\beta}^{+}(\vec{x},t)\, \psi_{e\alpha}(\vec{x}',t) \rangle\, \psi_\beta^{\mathcal{D}}(\vec{x}) = \tag{27}$$

$$= \int d^3x \int d^3x'\, \psi^{\mathcal{D}+} \langle \varphi | \psi_{e\eta}^{(0)+} \psi_{e\eta}^{(0)} - \psi_e^{(0)+} \psi_e^{(0)} + g\left(\psi_{e\eta}^{(0)+}\psi_{e\eta}^{(1)} - \psi_e^{(0)+}\psi_e^{(1)}\right) + c.c. + g^2\left(\psi_{e\eta}^{(0)+}\psi_{e\eta}^{(2)} - \psi_e^{(0)+}\psi_e^{(2)}\right) + c.c. + g^2\left(\psi_{e\eta}^{(1)+}\psi_{e\eta}^{(1)} - \psi_e^{(1)+}\psi_e^{(1)}\right) | \varphi \rangle\, \psi^{\mathcal{D}} .$$

The term of the order g^0 disappears in (27) because $\psi_{e\eta}^{(0)} = \psi_e^{(0)}$, see (23). In all the terms of (27) one may replace $\psi_{e\eta}^{(0)} = \psi_e^{(0)} = \psi_e^f$ by the positive-energy part $\Pi\psi_e^f$, see the note after (I4). The part gives zero when acting on φ : $\Pi\psi_e^f \varphi = 0$ and $\langle\varphi|(\Pi\psi_e^f)^+ = 0$ (recall that φ is no-electron state). Therefore, the terms $\sim g^1$ and also the next to the last term in (27) disappear. So using (25) and (26) and noting that $\langle\varphi|S|\varphi\rangle = 0$ if S contains an odd number of fermion operators, one obtains

$$\Delta N_e(t) = \langle \varphi | R^+ R | \varphi \rangle = \| R\varphi \|^2 , \tag{28}$$

$$R \equiv \int d^3x' \, \psi^{\mathcal{D}+}(x') \int d^4y \, S_e(x'-y) [\bar{\psi}_p^+(y) Q \psi_n^+(y)] \mathcal{O} \int d^4z \, S_\nu(y-z) \eta(z) . \quad (29)$$

The vector $R\Phi \sim \bar{\psi}_p^+ \psi_n^+ \Phi$ in (28) contains components of two types : I) one proton in all possible states, no other particles ($\bar{\psi}_p^+$ creates the proton, ψ_n^+ annihilates the initial neutron); 2) one proton plus one antineutron in all possible states plus the initial neutron (ψ_n^+ creates an antineutron). All components I) are orthogonal to all components 2) and

$$\Delta N_e(t) = \Delta N_1(t) + \Delta N_2(t) = \| R_1 \Phi \|^2 + \| R_2 \Phi \|^2 , \quad (30)$$

$$\| R_1 \Phi \|^2 = \Sigma_{pr} \left| \int d^3x \, \psi^{\mathcal{D}+}(x) \int_0^t dy_0 \int d^3y \; S_e(x-y) \, \bar{u}_{pr}(y) e^{iE_p y_0} \right. \quad (3I)$$
$$\left. Q \, u_n(y) e^{-iE_n y_0} \mathcal{O} \int_0^{y_0} dz_0 \int d^3z \; S_\nu(y-z) \, \eta(z) \right|^2 .$$

Here $u_{pr}(y)$ is the proton spinor of the energy E_p ; $u_n(y)$ is the initial neutron spinor of the energy $E_n < m_n$. The quantity $\Delta N_1(t) = \| R_1 \Phi \|^2$ is the inclusive probability to find the electron of the process $\nu + n \to p + e^-$ in the state $\psi^{\mathcal{D}}$: one does not observe the proton and therefore, the summing Σ_{pr} must be performed over all proton states; $\Delta N_2(t) = \| R_2 \cdot \Phi \|^2$ is the inclusive probability of finding the electron of the process $\nu + n \to n + p + \tilde{n} + e^-$ (or $\nu \to \tilde{n} + p + e^-$ because the initial neutron is not affected). This process takes place in the finite time interval (O, t) and neutrino and electron states are not plane waves. Therefore, the energy-momentum conservation does not forbid the process absolutely. One can avoid the discussion of the contribution ΔN_2 assuming that our detecting measurement allows one to separate the probability ΔN_1 to find the electron without the accompanying antineutron [x)].

3.4. Let us discuss the obtained result for $\Delta N_1(t) = \| R_1 \Phi \|^2$. In the r.h.s of (3I) one has the product of the function $\theta(y) = \int_0^{y_0} dz_0 \int d^3z \, S_\nu(y-z) \eta(z)$ and the function $u_n(y) = u_n(\vec{y}) \exp(-i E_n y_0)$. The function $S_\nu(y-z)$ is known to vanish if the interval $(y - z)^2$ is space-like, e.g., see /39/. Therefore, $\theta(y)$ vanishes outside the future light cone of V_s . The latter is defined as the set of points $y = (\vec{y}, y_0)$ such

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x) Note that a similar situation occurs in the case of the photon exchange between the external current (the photon source) and the atom (the photon detector), see the footnote in sect 3 of the author paper /38/.

that $y_0 > 0$ and all intervals $(y - z)^2$ are time-like with respect to the points $z = (\vec{z}, 0)$, $\vec{z} \in V_s$ see fig. I.

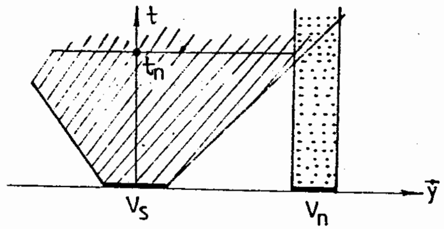

Fig. I.

The forward light cone of V_s is shaded; the support of $u_n(\vec{y}, t)$ is dotted.

The function $u_n(y)$ is localized in the four-dimensional region $\vec{y} \in V_n$, $y_0 > 0$. The product of $\theta(y)$ and $u_n(y)$ does not vanish only when supp $u_n(y)$ can intersect with supp $\theta(y)$, i.e. at $t > t_n = R/c$, R_n being the distance between V_s and V_n : $R_n = \min |\vec{z} - \vec{y}|$, $\vec{z} \in V_s$; $\vec{y} \in V_n$, see fig. I. So ΔN_1 vanishes at $t < R_n/c$. If $u_n(y)$ is not exactly zero outside V_n then $\Delta N_1(t)$ would not be exactly zero at $t < R_n/c$ but it is evident that this would not contradict the causality. One gets the causal result within the precision of the neutron localization. The precision is related in no way with the Compton wave-length of the signal carrier, the neutrino.

Recall that the electron of the process $\nu + n \to p + e^-$ is detected in the region $V_{\mathcal{D}}$ approximately. Note that the $V_{\mathcal{D}}$ dimension is of no importance for the obtained result (actually the $V_{\mathcal{D}}$ dimension is much greater that the V_n dimension).

3.5. The above calculation has been performed in the Heisenberg picture. In sect. 3 of /38/ I have calculated the inclusive quantity similar to ΔN_1 in the usual interaction picture. This was done for the case of external current $\to$ photon $\to$ detecting atom. The calculation turned out to be much more cumbersome than in the Heisenberg picture. It was demonstrated how in the final result there arised the strictly causal photon D function instead of the Feynman propagation function D_F which naturally appears in the interaction picture and does not vanish outside the light cone (see equation (33) in /38/).

Acknowledgement

I am thankful to M.A. Soloviev for the discussion of App. A.

Appendix A

An arbitrary state of the fermion is a superposition of positive-energy Dirac states

$$\psi_\mu(\vec{x}) = \int d^3p \sum_{r=1,2} C_r(\vec{p})\, u_\mu(\vec{p},r) \exp(i\vec{p}\vec{x}) \quad , \quad \mu = 1,2,3,4 \; : \qquad (A.1)$$

$$\sum_{\nu=1}^{4} (\bar{\alpha}_{\mu\nu}\bar{p} + \beta_{\mu\nu} m)\, u_\nu(\vec{p},r) = E_p\, u_\mu(\vec{p},r) \; , \qquad E_p = |\sqrt{p^2+m^2}| \, . \qquad (A.2)$$

Fourier transforms of $\psi_\mu(\vec{x})$ describe the same state in the momentum representation

$$\psi_\mu(\vec{x}) = (2\pi)^{-3} \int d^3p \; \phi_\mu(\vec{p}) \exp(i\vec{p}\vec{x}) \, , \qquad (A.3)$$

$$\phi_\mu(\vec{p}) = \sum_{r=1,2} C_r(\vec{p})\, u_\mu(\vec{p},r) \, . \qquad (A.4)$$

Four functions $\psi_\mu(\vec{x})$ or $\phi_\mu(\vec{p})$ are not independent because they are determined by two arbitrary functions $C_r(\vec{p})$ only. One can verify that $\phi_\mu(\vec{p})$ satisfies the equation

$$\phi_\mu = \sum_\nu \Pi_{\mu\nu}\phi_\nu \; ; \; \Pi_{\mu\nu} = (\bar{\alpha}_{\mu\nu}\bar{p} + \beta_{\mu\nu} m + \sqrt{p^2+m^2}\,\delta_{\mu\nu}) / 2\sqrt{p^2+m^2} \; , \qquad (A.5)$$

where Π is the known positive-energy projector. Equation (A.5) is equivalent to $(\bar{\alpha}\bar{p} + \beta m)\phi = E_p\phi$ and will be used instead of dealing with the explicit solutions $u_\mu(p,r)$ of equation (A.2).

Let us show that the four function $\psi_\mu(\vec{x})$ cannot have bounded supports so that $\rho(\vec{x}) = \sum_\mu |\psi_\mu(\vec{x})|^2$ cannot vanish outside a bounded volume V .

Suppose the contrary. Then $\phi_\mu(\vec{p})$, see (A.3), are entire (analytical everywhere) functions of three complex components p_1 , p_2 , p_3 of the momentum $\bar{p}$, see the theorem IX.I2 from /40/. Then ($\Pi\phi$)$_\mu$ also must be entire because $\phi = \Pi\phi$. But actually ($\Pi\phi$)$_\mu$ are not entire if ϕ_μ are, because Π contains the function $\sqrt{p^2+m^2}$ which is not single-valued (and consequently analytical) everywhere. So our supposition that $\psi_\mu(x)$ can have bounded supports is not consistent with the equation $\phi = \Pi\phi$. For another proof see /41/.

The result can be improved by using the theorem IX.I3 from /40/. I shall use its particular (onesided) variant:

"Let $f(\vec{x}) \in L^2(R^3)$ and let $f(\vec{x})\exp(b|\vec{x}|) \in L^2(R^3)$ at all $b < a$. Then $\hat{f}(\vec{p})$, Fourier transform of $f(\vec{x})$, is analytic in the tube region $\{ p : |Im\,\vec{p}| < a \}$ ".

Note that $\hat{f}(\vec{p})$ has one more property which I do not need. To apply the theorem to our case, suppose that $\int \rho(\vec{x}) d^3x < \infty$, then $\psi_\mu(\vec{x}) \in L^2$ for μ = 1,2,3,4. Suppose that $|\psi_\mu(\vec{x})| < C_\mu \exp(-a|\vec{x}|)$ outside a volume V . Then all premises of the theorem are satisfied for ψ_μ , in particular $\psi_\mu(\vec{x})\exp(b|\vec{x}|) \in L^2$ for all $b < a$. So $\phi_\mu(\vec{p})$ must be analytical in the tube

$$|Im\,\vec{p}| = [(Im\,p_1)^2 + (Im\,p_2)^2 + (Im\,p_3)^2]^{1/2} < a \, .$$

Let us show that the supposition $a > m$ contradicts the equation $\phi = \Pi\phi$. Indeed, if the functions ϕ_μ are analytical in the tube $|Im\,\vec{p}| < a$, then the functions $(\Pi\phi)_\mu$ cannot be analytical there because due to $\sqrt{p^2+m^2}$ the functions $\Pi_{\mu\nu}$ are not analytical in the region $|Im\,\vec{p}| \geq m$, in particular when $m \leq |Im\,\vec{p}| < a$ (see below for the proof of the statement). The obtained contradiction means that $|\psi_\mu|$ cannot decrease outside V faster than $C_\mu \exp(-a|\vec{x}|) = C_\mu \exp(-m(1+\varepsilon)|\vec{x}|)$, where $\varepsilon > 0$. The smaller is ε , the stronger is the restriction for ψ_μ . It is possible that the restriction can be further improved by other means, e.g., one would be able to show that $|\psi_\mu|$ cannot be smaller than $C_\mu |\vec{x}|^{-2} \exp(-m|\vec{x}|)$ outside V .

I still have to prove that $\Pi_{\mu\nu}$ is not analytical if $|Im\,\vec{p}| \geq m$, i.e. I should prove that $\sqrt{p^2+m^2}$ is not single-valued at $|Im\,\vec{p}| \geq m$.

The set of the branch points of $\sqrt{p^2+m^2}$ (the cut origins) can be found by using the equation $p_1^2 + p_2^2 + p_3^2 + m^2 = 0$. Let $p_k = \rho_k + i\sigma_k$ and rewrite the equation as

$$\sum_k \sigma_k^2 = m^2 + \sum_k \rho_k^2 \; ; \qquad \sum_k \rho_k \sigma_k = 0 \quad ; \qquad k = 1,2,3 \, . \qquad (A.6)$$

To describe the set, I use ρ_1 , ρ_2 and ρ_3 as running variables. Then, the first equation in (A.6) shows that $|\vec{\sigma}| = |Im\,\vec{p}| \geq m$ at all ρ_k . The second one means that $\vec{\sigma} \perp \vec{\rho}$ if $\vec{\rho}$ and $\vec{\sigma}$ are vectors in the same three-dimensional space. The function $\sqrt{p^2+m^2}$ has no branch points in the region $|Im\,\vec{p}| < m$ .The cuts from the branch points must be directed to the infinite point : this gives the needed branch of $\sqrt{p^2+m^2}$ which is equal to $|\sqrt{p^2+m^2}|$ both for positive and negative real values of p_1, p_2 , p_3 . So the branch points and cuts are outside the region $|Im\,\vec{p}| < m$. In particular, they are present at $m \leq |Im\,\vec{p}| \leq a$ if $a > m$.

So the following fact is proved: if $\int \rho(\vec{x})\, d^3x$ is finite, then $\rho(\vec{x})$ cannot decrease faster than $C \exp(-2(1+\varepsilon)|\vec{x}|/\lambda)$ outside a bounded region (in particular, at $|\vec{x}| \to \infty$). Here ε is positive and can be arbitrary small. This fact determines the possible approximate or effective fermion localization.

Appendix B

By definition the projector Π , see (A.5), is equal to the sum of projectors onto two independent positive-energy solutions of (A.2)

$$\Pi_{\mu\nu}(\bar p)=\sum_{r=1,2} u_\mu(\bar p,r)\, u_\nu^*(\bar p,r), \qquad \sum_\mu u_\mu^*(\bar p,r)\, u_\mu(\bar p,s)=\delta_{r,s} \tag{B.I}$$

In the momentum representation the projection with the help of $\Pi_{\mu\nu}(\bar p)$ is realized by summing over ν only, see (A.5), without integration over momentum. This means that Π is diagonal in the momentum representation

$$\Pi_{\mu\nu}(\bar p,\bar q)=\Pi_{\mu\nu}(\bar p)\,\delta(\bar p-\bar q)$$

$$(\Pi\phi)_\mu(\bar p)=\int d^3q\,\sum_\nu \Pi_{\mu\nu}(\bar p,\bar q)\,\phi_\nu(\bar q)=\sum_\nu \Pi_{\mu\nu}(\bar p)\,\phi_\nu(\bar p)\,. \tag{B.2}$$

Therefore, in the coordinate representation one has

$$\Pi_{\mu\nu}(\bar x,\bar y)=(2\pi)^{-3}\int d^3p\int d^3q\; e^{i\bar p\bar x}\,\Pi_{\mu\nu}(\bar p,\bar q)\,e^{-i\bar q\bar y}= \tag{B.3}$$

$$=(2\pi)^{-3}\int d^3p\; e^{i\bar p(\bar x-\bar y)}\,\Pi_{\mu\nu}(\bar p)=\Pi_{\mu\nu}(\bar x-\bar y)\,.$$

Here, one can use (A.5) or (B.I) for $\Pi_{\mu\nu}(\bar p)$.
Using (B.I) one can verify that $(\Pi\psi_e)_\mu(\bar x)$ (see (20) for ψ_e) is equal to $A_\mu(\bar x)$, see (II).

The spinor $\psi^{\mathcal{D}}$ in (9) which is localized in and near $V_{\mathcal{D}}$ can be represented as $\psi^{\mathcal{D}}_\mu(\bar x)=\int d^3y\sum_\nu \Pi_{\mu\nu}(\bar x,\bar y)\,\xi_\nu(\bar y)$ where $\xi_\nu(\bar y)$ is superposition of positive-energy and negative-energy spinors such that $supp\,\xi_\nu=V_{\mathcal{D}}$, $\nu=$ I,2,3,4. It can be shown that

$$|\psi^{\mathcal{D}}_\mu(\bar x)|<const\,\exp(-m\rho) \tag{B.4}$$

at $\rho>>1/m$, where ρ is the distance between $\bar x$ and $V_{\mathcal{D}}$: $\rho=\min|\bar x-\bar y|$, $y\in V_{\mathcal{D}}$. Let us note that according to App. A $|\psi^{\mathcal{D}}_\mu(\bar x)|$ must satisfy simultaneously the inequality

$$|\psi^{\mathcal{D}}_\mu(\bar x)|>C_\mu\exp(-m(1+\varepsilon)\rho)\,,\qquad \varepsilon>0\,.$$

REFERENCES

I. G. Hegerfeldt. Phys. Rev. DIO (1974) 3320.
2. B. Skagerstam. Intern. Journ. Theor. Phys. I5 (1976) 2I3.
3. J. Perez, I. Wilde. Phys. Rev. DI6 (1977) 3I5.
4. G. Hegerfeldt, S. Ruijsenaars. Phys. Rev. D22 (1980) 377.
5. S. Ruijsenaars. Ann. of Phys. I37 (198I) 33.
6. G. Hegerfeldt. Phys. Rev. Lett. 54 (1985) 2395.
7. B. Rosenstein, M. Usher. Phys. Rev. D36 (1987) 238I.
8. P.N. Kaloyerou. Phys. Lett. AI29 (1988) 285.
9. M.I. Shirokov. Communication of the Joint Inst. for Nucl. Research E-I252 Dubna, I963.
IO. K.Kraus. Position Observables of the Photon, p. 293 in: The Uncertainty Principle and Foundation of Quantum Mechanics eds. W. Price, S. Chissick, (London, Wiley, I977).
II. H. Bacry. Localizability and Space in Quantum Physics (Berlin, Springer - Verlag , I988).
I2. M.I. Shirokov. Usp. Fiz. Nauk I24 (I978) 627. English translation Sov. Phys. Usp. 2I (I978) 345.
I3. P. Milonni, P. Knight. Phys. Rev. AIO (I974) IO96.
I4. M.I. Shirokov. Found. of Phys. II (I98I) 2I.
I5. V.P. Bykov, A.A. Zadernovsky. Zh. Eks. Teor. Fiz. (USSR), 8I (I98I) 37.
I6. B. De Haan. Physica A I32 (I985) 397.
I7. S.M. Golshan, D.H. Kobe. Phys. Rev., A34 (I986) 4449.
I8. M.H. Rubin. Phys. Rev., D35 (I987) 3836.
I9. A. Valentini. Phys. Lett., A I35 (I989) 63.
20. A. Valentini. Phys. Lett., AI53 (I99I) 32I.
2I. V.P. Bykov. Usp. Fiz. Nauk (USSR) I43 (I989), 657.
22. J.Knight, Journ. Math. Phys., 2 (I96I), 459.
23. Ph. Eberhard, R. Rose. Found. Phys. Lett., 2 (I989) I27.
24. A.S. Wightman, S.S. Schweber. Phys. Rev., 98 (I955), 8I2, sect. 2.
25. A.J. Kalnay. The Localization problem, pp. 93-IIO in Problems in the Found. of Phys. ed. M. Bunge (Berlin, Springer - Verlag, I97I).
26. A.A. Broyles. Phys. Rev., DI (I970), 979.
27. J. von Neumann. Mathematical Foundations of Quantum Mechanics Ch. 5 (Princeton University Press, I955).
28. M. Jammer. The Philosophy of Quantum Mechanics, ch II (New York, Wiley I974).

29. C. Møller. The Theory of Relativity.§2I, Oxford I952.
30. R.G. Newton. Causality and the Order of Cause and Effect in Time. pp. 49-57, in: Causality and Physical Theories ed W. Rolnick. Amer. Inst. Phys. Conf. Proc.New York, I974.
3I. S.S. Schweber. An Introduction to Relativistic Quantum Field Theory (New York, Row and Peterson, I96I).
32. E, Commins, P. Bucksbaum. Weak Interactions of Lepton and Quarks (Cambridge, Cambridge University Press I983).
33. M.I. Shirokov. Zh. Eks. Teor. Fiz. 8I (I98I) ,I2I0. Enlish translation: Sov. Phys. JETP 54 (I98I), 645.
34. G. Kallen. p. 243 in: Lectures in Theor. Phys. Brandeis Summer Inst. I96I v. I. (New York, Benjamin, I962).
35. M.I. Shirokov. Int. Journ. Mod. Phys. A, in press.
36. G. Källen.Quantum Electrcdynamics (New York, Springer - Verlaf, I972).
37. A.I. Akhiezer, V.B. Berestetskii. Quantum Electrodynamics, §23 (New York, Interscience, I965).
38. M.I. Shirokov. Communication of the Joint Inst. for Nuclear Research, E4-87-686, Dubna, I987.
39. J.D. Bjorken, S.D. Drell. Relativistic Quantum Fields, ch. I3, (New York, Mc-Graw Hill, I965).
40. M. Reed, B. Simon. Methods of Modern Mathematical Physics v. 2 Ch IX (New York, Academic, I975).
4I. B. Thaller, S. Thaller. Nuovo Cimento. A82 (I984), 222.

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