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RESONANCE AMPLIFICATION OF THE NUCLEAR REACTION X(a,b)Y NEAR THE a+X CHANNEL THRESHOLD

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1. Introduction.

It is known that the S-wave cross section of the fusion reaction X(a,b)Y is described near the threshold $\varepsilon\to 0$ of the channel a+X by the formula /1/

$$\sigma^{in}(\varepsilon) = |\psi_\varepsilon(R=0)|^2 \cdot \frac{A}{V}, \quad A = \mathrm{const} \,, \tag{1}$$

where R is the distance between the fragments a and X , $\psi_\varepsilon(R)$ is the wave function of relative motion of a and X , and $V = \sqrt{2\cdot\varepsilon/M}$ is the relative velocity of their motion. The formula can be made more accurate by taking into consideration the coupling of the channels a+X and b+Y in the factor A (see, for example,/2/). For an arbitrary partial wave J it is convenient to write the reaction cross section through the Jost function $f_J(\varepsilon)=|f_J(\varepsilon)|\cdot e^{-i\delta_J}$ of the system a+X /2,3/:

$$\sigma_J^{in}(\varepsilon) = |f_J(\varepsilon)|^{-2} \cdot \frac{A}{V} \tag{1.a}$$

determined from the relation

$$k^J\cdot|f_J(\varepsilon)|^{-1}\cdot R^J \underset{R\to 0}{\longleftarrow} \psi_\varepsilon^J(R) \underset{R\to\infty}{\longrightarrow} \sin(kR-J\cdot\pi/2+\delta_J(\varepsilon))/kR \tag{2}$$

and coincident with $|\psi_\varepsilon^J(R=0)|\cdot e^{-i\delta_J}$ as $J=0$.
If the Coulomb interaction ($1/R$) occurs between the fragments a and X, formula (1) is reduced to the known Gamov formula

$$\sigma_0^{in}(\varepsilon) = C_0^2(\varepsilon)\cdot\frac{A}{V} \tag{3}$$

(where $C_0^2(\varepsilon) = |f_0(\varepsilon)|^{-1} = 2\cdot\pi/V/(e^{2\pi/V}-1)$ is the Gamov factor), which well describes a great amount of experimental data on fusion reaction cross sections of the type X(a,b)Y in the range 20-100 keV of the colliding energy ε and is used for the extrapolation of $\sigma_0^{in}(\varepsilon)$ in the limit $\varepsilon\to 0$ /1,4/.

Here we investigate deviation from the Gamov formula due to an interaction additional to the Coulomb one (it is not necessarily a short-range interaction) in the entrance channel and discuss manifestations of the effect.

2. Screening effects in fusion reactions of the type D(d,p)T near the threshold of the channel d+D

Recently, the problem how to improve the Gamov formula (3) has arisen in describing the screening effects in fusion reactions X(a,b)Y for slow collisions between nuclei (a) and atomic (or molecular) targets (X). Really, the latest experiments /5,6/ have shown the cross section of fusion reaction **D(d,p)T** to deviate noticeably from formula (3) at low colliding energies ($\varepsilon \leqslant 5$keV). It has been established in papers /6-9/ that it is necessary to take into account an electron screening in the channel **d+D.** At the energy $\simeq$ 1keV this effect makes the reaction cross section $\simeq$40% larger than the cross section for the reaction **d(d,p)T** on bare nuclei **d,** and this ratio grows exponentially if the energy decreases /8,9/. Let us investigate the electron shielding for **d+D** fusion in the limit $\varepsilon \to 0$. To calculate the searched-for Jost function $f_J(\varepsilon)$ (2) of relative motion of **d** and **D** , we use the adiabatic representation /10,11/ , successfully applied to the analysis of three-body systems such as **d+D** /9/.

In the approach , the system of three Coulomb particles d+D is described by a system of an infinite number of ordinary differential equations /10,11/

$$\left\{\begin{array}{l} \phi''_{1g}(R) + 2M(\varepsilon-E_{1g}(R)-\frac{Z_1Z_2}{R}-\frac{J(J+1)}{2MR})\phi_{1g}(R) - \sum\limits_{\substack{i=1\\ p=g,u}}^{\infty} U_{1g,ip}(R)\phi_{ip}(R)=0 \\ \\ \phi''_{1u}(R) + 2M(\varepsilon-E_{1u}(R)-\frac{Z_1Z_2}{R}-\frac{J(J+1)}{2MR})\phi_{1u}(R) - \sum\limits_{\substack{i=1\\ p=g,u}}^{\infty} U_{1u,ip}(R)\phi_{ip}(R)=0 \\ \dots\dots\dots\dots\dots \end{array}\right. \tag{4}$$

with the boundary conditions /9/

$$\left\{\begin{array}{l} \phi_{1g}(R) \underset{R\to 0}{=} |f_0|^{-1}\delta_{J0} \\ \phi_{1u}(R) = |f_0|^{-1}\cdot\delta_{J1} \\ \phi_{ip}(R) = 0\ ;\quad i=2,\cdots,\infty\ ;\quad p=g,u \end{array}\right. \tag{4.a}$$

$$\left\{\begin{array}{l} 1/\sqrt{2}\cdot(\phi_{1g}(R)+\phi_{1u}(R)) \underset{R\to\infty}{\longrightarrow} j_J(kR)-t_{aa}(\varepsilon)\cdot n_J(kR) \\ 1/\sqrt{2}\cdot(\phi_{1g}(R)-\phi_{1u}(R)) \longrightarrow -t_{ba}(\varepsilon)\cdot n_J(kR) \\ \qquad \phi_{ip}(R) \longrightarrow 0\ ;\quad i=2,\dots,\infty\ ;\ p=g,u \end{array}\right. \tag{4.b}$$

where $\phi_{ip}(R)$ are the wave functions of relative motion of the nuclei **d**, (ip) is the set of quantum numbers of the two-center problem (an electron in the field of two Coulomb centers Z_1 and Z_2 at fixed **R**) and **p=(g,u)** characterizes the parity of the two-center-problem wave functions relative to the rearrangement of nuclei /12/, $E_{ip}(R)$ are the eigenvalues of this problem, $k=\sqrt{2M(\varepsilon-E_1)}$ is the momentum of a channel , $E_1 = E_{1g}(\infty)=E_{1u}(\infty)=-\frac{1}{2}$, $U_{1p,jp'}$ are the effective potentials of the three-body problem /10/, J is the angular momentum of the system **d+D**, $t_{aa}(\varepsilon)$,$t_{ba}(\varepsilon)$ are the elements of the real symmetric reaction matrix, **M** is the reduced mass of the system **d+d.** For the "symmetric" system **d+D** (with the identical nuclei) there is no coupling of the states with different parity **p=(g,u)** ($U_{1g,ju}(R) = U_{1u,jg}(R) = 0$ /10/), which allows us to solve the problem (4) for the functions $\phi_{1g}(R)$ and $\phi_{ju}(R)$ separately.

Because the small parameter 1/2M of the problem equals $\simeq 2\cdot10^{-4}$ here , we can omit the terms $U_{1p,jp'}$ in the system of equations (4) . For this case only the first equation for $\phi_{1g}(R)$ with the potential

$$U_{1g}(R) = \frac{Z_1Z_2}{R} + E_{1g}(R) \tag{5}$$

remains in the system (4) and the boundary conditions (4.b) are reduced to $\phi_{1g}(R) \to j_J(kR)-t_{aa}(\varepsilon)\cdot n_J(R)$ /9/. It is known that this one-level approximation works well in the region of low energies /10,11/, for the system d+D it is $\leqslant$ 5-10keV /8,9/. This gives us a possibility of testing the screening effects in the range from extremely low energies to the experimentally attained energies $\simeq$ 3-10keV. In the paper/9/ the calculations in the approach were performed for the region 0.25–10keV , they are in good agreement with the semi-classical calculations based on the classical trajectory Monte Carlo method/8/. Here we have performed the calculations for low colliding energies $\varepsilon \to 0$. In Fig.1 the calculated function $|f_0(\varepsilon)|^{-1}$ of the system d+D is given ,where one can clearly see oscillations of the function.

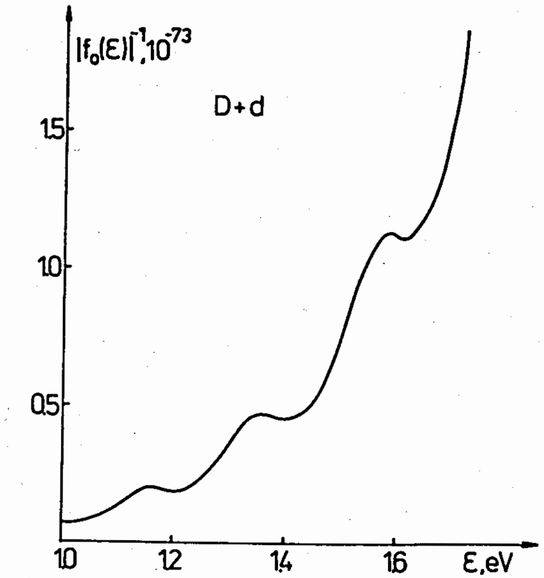

Fig. 1.

The function $|f_0(\varepsilon)|^{-1}$ of the system **d+D** for the state J=0 calculated in the one-level approximation of the adiabatic representation.

To understand the effect and to search for its possible stronger manifestations in other fusion reactions **X(a,b)Y** let us consider a simpler interaction than (5) in the entrance channel **a+X** of the reaction.

3. Resonance amplification of the reaction X(a,b)Y near the threshold of the channel a+X

First, let us consider the simplest model when there is no long-range Coulomb interaction in the entrance channel **a+X**, and an additional interaction is given as a rectangular potential well

$U(R)=\begin{cases} -V_0, & R_0 \leq 0 \\ 0, & R_0 > 0 \end{cases}$. In this case the Jost function of the system **a+X** is found from the equations*)

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*) The simple consideration is analogous to the problem of overbarrier reflection of a quantum particle for a potential barrier with spherical symmetry presented in book /13/.

$$\begin{cases} \frac{k}{q}\cdot|f_0(\varepsilon)|^{-1}\sin qR_0 = \sin(kR_0+\delta_0) \\ |f_0(\varepsilon)|^{-1}\cos qR_0 = \cos(kR_0+\delta_0) \end{cases} \tag{6}$$

where $k^2=2M\varepsilon$, $q^2=2M(\varepsilon+V\ )$, δ_0 is the S-wave phase shift for scattering on the potential **U(R)** . One obtains

$$|f_0(\varepsilon)|^{-2}=1+\frac{V_0}{\varepsilon_0}\sin^2(kR_0+\delta_0)=\frac{\varepsilon+V_0}{\varepsilon+V_0\cos^2 qR_0} \longrightarrow \begin{matrix} f_{max}^{-2}=1+\frac{V_0}{\varepsilon} \\ f_{min}^{-2}=1 \end{matrix} . \tag{7}$$

So, the S-wave cross section of the reaction **X(a,b)Y** oscillates according to (1.a) if the energy ε changes. The maxima of the oscillations are at the points

$$\varepsilon_n = \frac{\pi^2(1+2n)^2}{8MR_0^2} - V_0 \ . \tag{8}$$

where n= $\nu,\nu+1,\ldots$; ν is the number of bound states in the potential **U(R)**. The amplitude of the oscillations is defined by the depth V_0 of the well and decreases with growing **n**. The first maximum $f_{max}^{-2}=1+\frac{V_0}{\varepsilon}$ is more strongly marked. If it is close enough to the threshold $\varepsilon\to 0$, the reaction has the resonance behavior near the energy ε_ν. The oscillation period

$$\varepsilon_{n+1}-\varepsilon_n = \frac{\pi^2(n+1)}{MR_0^2} \tag{9}$$

is determined by the interaction range R_0 , the reduced mass **M** of the system **a+X** and increases if **n** grows.

In Fig.2 the dependence of the function $|f_0(\varepsilon)|^{-2}$ on the parameters V_0 and R_0 of interaction **U(R)** is demonstrated. The dotted curve has the resonance behavior as $\varepsilon\to 0$, since here the maximum of the first oscillation is at $\varepsilon_0=0.23$. A similar consideration is possible for the high waves J≠0 too. But the resonance amplification of the cross section as $\varepsilon\to 0$ is compensated for by the centrifugal barrier for the case **J≠0**.

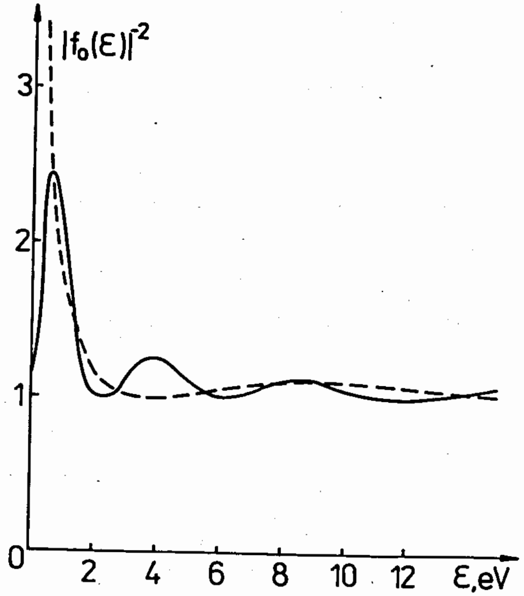

Fig.2.

The functions $|f_0(\varepsilon)|^{-2}$ for potential wells with $V_0=M=1$. The dotted curve corresponds to the well with the radius $R_0=1$, where there are no bound states, and the maxima of the first and second oscillations are at points $\varepsilon_0=0.23$ and $\varepsilon_1=10.1$, respectively. A sufficiently deep level appears in the well if its radius is increased to $R_0=2.5$, and the maxima of the first three oscillations are at points $\varepsilon_1=0.78$, $\varepsilon_2=3.93$ and $\varepsilon_3=8.66$. The solid curve corresponds to this case.

Let us apply this simple consideration to the above-analyzed reaction **D(d,p)T** . Using the model suggested in paper /7/ , we define the interaction in the channel **d+D** as

$$U(R)=\begin{cases} \frac{Z_1Z_2}{R} - V_0, & R\leq R_0 \\ 0, & R>R_0 \end{cases}, \tag{10}$$

where $V_0 = -E_{1s}(0) + E_{1s}(\infty) = -E_{1s}(He^{+}) + E_{1s}(D) \simeq 40\,\mathrm{eV}$, $Z_1=Z_2=+1$ and R_0

is of the order of the effective radius of the ground state of the D-atom*).

This rough model is very close to the one considered above ; nevertheless, such simplification yields sufficiently good results for the reaction **D(d,p)T** at low energies if compare with the numerical calculations /8,9/.
Using the above approach for the rectangular potential well one can get the Jost function for the potential (10) (see also/7/)

$$|f_0(\varepsilon)|^{-2}=\frac{C_0^2(q)\cdot(\varepsilon+V_0)}{\varepsilon+(V_0-R_0^{-1}+\frac{1}{4}\cdot(\varepsilon+V_0)^{-1}R_0^{-2})\cdot\cos^2\beta} \tag{11}$$

where $\beta = qR_0 - \frac{M}{q}\cdot\ln 2qR_0 + \arg\Gamma(1+i\frac{M}{q})$, $q=\sqrt{2M(\varepsilon+V_0)}$, M is the reduced mass of the system **d+D**, $C_0^2(q)=\frac{2\pi}{v_q(e^{2\pi/v_q}-1)}$, $v_q=\sqrt{2(\varepsilon+V_0)/M}$.

The value ε_n of the oscillating function $|f_0(\varepsilon)|^{-2}$maximum

$$f_{max}^{-2}=C_0^2(q_n)\cdot(1+\frac{V_0}{\varepsilon_n}) \tag{12}$$

can be found from the equation

$$q_nR_0 - \frac{M}{q_n}\cdot\ln 2q_nR_0 + \arg\Gamma(1+i\frac{M}{q_n}) = (\frac{1}{2}+n)\cdot\pi$$

having the solutions close to (8), where $n = \nu,\nu+1,\ldots$; ν is the number of bound states in the well (10).

Let us estimate the period and the amplitude of the oscillations as $\varepsilon\to 0$. Since 30 bound states ($\nu=30$) are located in the well describing the vibrations of the ion D_2^+ (here we approximated the well by the potential (10)),one can estimate the period of the oscillations as

$$\varepsilon_{n+1}-\varepsilon_n \underset{n\simeq\nu}{\simeq} \frac{\pi^2(\nu+1)}{MR_0^2} \simeq \frac{\pi^2 30}{2\cdot 10^3 10}\cdot 27\mathrm{eV} \simeq 0.4\mathrm{eV}$$

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*) For the present we cannot strictly fix the value of R_0 , but even this simple model is useful for a qualitative analysis of the reaction.

and the amplitude of the oscillating function $|f_0(\varepsilon_n)|^{-2}$ at $n=\nu$ can reach the value of the order of

$$f^{-2}_{max} \simeq C_0(\hat{q})\cdot(1+\frac{40eV}{\varepsilon_\nu}) .$$

The estimations are in a good agreement with the numerical result presented in the previous section (see Fig.1).

The oscillations manifest themselves for the system **d+D** only at low energies, that is in the region not reached experimentally. However, it is interesting to estimate the possibility of revealing oscillations caused by the electron shielding for other fusion reactions. Recently, the experimental data for reactions $D+^3He$ $(D+^3\overset{+}{He})$ at low energies $\approx$5keV have been obtained /14/,where the still unclear deviation of the function

$$A(\varepsilon) = \sigma^{in}_{exp}(\varepsilon)\cdot V\cdot C_0^{-2}(q)$$

from the constant value by the factor of the order of $\approx 5\cdot10^{-2}$ has been found /7/. Such growth of the function could be explained by the correction V_0/ε_0 ,estimated for the reaction to be /7/

$$\frac{V_0}{\varepsilon_n} \simeq \frac{10^2}{5\cdot10^3} \simeq \frac{1}{50} ,$$

and the period of the oscillations

$$\varepsilon_{n+1}-\varepsilon_n > \frac{\pi^2(\nu+1)}{MR_0^2}$$

exceeds here the period for the system **d+D** because of an essential increase in the number ν (the number of bound states in the well describing the system $D+^3He$) and a decrease in the quantity R_0 $(R_0\rightarrow R_0/Z_1Z_2)$. The final answer to the question could be obtained from a numerical calculation of the reaction analogous to that presented in Sect.2 (see also /9/).

4.Fusion reactions for light nuclei of the type $t(d,n)^4He$

It is surprising that clear formulae (8),(9) also describe some characteristics of ^5He nucleus. So, for the resonance series $\frac{3}{2}^+$ in ^5He , ε_ν=16.75MeV , $\varepsilon_{\nu+1}$=19.8MeV and $\varepsilon_{\nu+2}$=24-25MeV (the resonance energies referred to the ground state of ^5He) , determining the cross section of the reaction $t(d,n)^4He$ near the threshold **d+t** /15/,we have

$$\eta_\nu^{exp} = \left[\frac{\varepsilon_{\nu+2}-\varepsilon_{\nu+1}}{\varepsilon_{\nu+1}-\varepsilon_\nu}\right] = 1.4\text{-}1.7 . \qquad (13)$$

Since the first resonance, ε_ν=16.75MeV, above the threshold of the channel **d+t** is considered to be the first excited state of ^5He/15/ (the radial part of the wave function of the state is characterized by the 2S-state of a harmonic oscillator /16/), the number ν equals 1 and, according to the formula (9), the parameter η_ν is

$$\eta_\nu^{th} = \frac{\nu+2}{\nu+1} = \frac{3}{2} ,$$

which is in agreement with the experimental value (13).

Formula (9) also gives the realistic estimation for the channel radius of the ^5He -nucleus in the channel **d+t**:

$$R_0 = \pi\cdot\sqrt{\frac{\nu+1}{M(\varepsilon_{\nu+1}-\varepsilon_\nu)}} \simeq 14\ fm .$$

The simple formulae could be useful in analyzing of energy spectra of other light nuclei.

5."In flight" fusion reactions in mesic atomic physics

The nuclear synthesis reaction , "in flight" fusion, in mesic atomic physics of the type

$$t\mu + d \rightarrow {}^4He + n + \mu \qquad (14.a)$$

$$p\mu + p \rightarrow d + e^+ + \mu \qquad (14.b)$$

formally differs from the reaction **D(d,p)T** considered above only in the length and energy scale:

$$a_e = 0.529\cdot10^{-8}cm \rightarrow a_\mu = a_e/m_\mu = 2.6\cdot10^{-11}cm$$

$$\varepsilon_0^{(e)} = 27\ eV \rightarrow \varepsilon_0^{(\mu)} = \varepsilon_0^{(e)}m_\mu = 5600\ eV .$$

Since there are maximum two bound states in the potential well (5) (they are bound states of the mesic molecules $dt\mu$ and $pp\mu$ /10/) and its depth is estimated as

$$V_0 = -E_{1s}(He^{+}\mu) + E_{1s}(p\mu, t\mu) = \frac{3}{2}\cdot\varepsilon_0^{(\mu)} \simeq 8 \text{ keV}$$

one can assume the following estimation for a typical period and an amplitude of the cross section oscillations:

$$\varepsilon_{\nu+1} - \varepsilon_\nu \simeq \frac{\pi^2(\nu+1)}{\mathcal{M}R_0^2} \simeq \frac{10\cdot 3}{10\cdot 10}\cdot 5\cdot 10^3 \simeq 10^2 - 10^3 \text{eV}$$

$$f_{max}^{-2} - f_{min}^{-2} \simeq C_0^2(q)\cdot\frac{V_0}{\varepsilon_\nu} \simeq C_0^2(q)\cdot\frac{8\cdot 10^3}{\varepsilon_\nu}.$$

The strongest manifestation of the effect is to be expected in the system $p\mu+p$ where the depth of the effective potential well is such that only one bound state can be located in the well (the ground state of the mesic molecule $pp\mu$) and even its insignificant deepening leads to a new bound state here /10,11/. In Fig.3 the

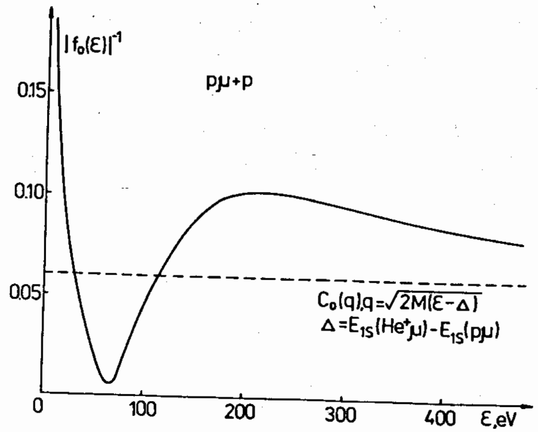

Fig.3.
The function $|f_0(\varepsilon)|^{-1}$ of the system $p\mu+p$ for the state J=0.

results of the calculation of the function $|f_0(\varepsilon)|^{-1}$ for the system $p\mu+p$ are presented. The calculation has been performed in the so-called "simple approach" of the adiabatic representation /17/: problem (4) for the first two states $\phi_{1g}(R)$ and $\phi_{1u}(R)$ of the adiabatic expansion with the reduced mass

$$\mathcal{M} = (M_p+m_\mu)\cdot M_p/(2M_p+m_\mu)/m_a ,$$

instead of $M = \frac{1}{2}\cdot M_p/m_a$, has been solved. The simplification yields accurate results as compared to multilevel approximations of the adiabatic approach for describing slow collisions in the system "mesic atom + nucleus" for the case of equal masses of nuclei /18/. For different masses, the "simple approach" yields less accurate results /19/. We have to note that the curve in Fig.3 really corresponds to the case of a resonance near the threshold demonstrated in Fig.1 (the dotted curve).

Let us estimate a possibility for the experimental study of the effect of oscillations in mesic atomic collisions. As is known (see, for example,/20,4/), the constant A_{dt} of the nuclear reaction $d+t \rightarrow {}^4He+n$ determined by formula (3) is at least two orders of magnitude as large as the constants of all other fusion reactions in hydrogen-isotope mixtures. As a result, the rate of reaction (14.a) is maximal as compared to other "in flight" fusion reactions. The rate of the reaction reduced to the liquid hydrogen density N_0 as $\varepsilon \to 0$ is estimated as /7/

$$\lambda_f(dt) = \lim_{\varepsilon\to 0} |f_0(\varepsilon)|^{-2}\cdot A_{dt}N_0 \simeq C_0^2(q)\cdot A_{dt}N_0 = \tag{15}$$
$$= 8.5\cdot 10^{-5} 1.3\cdot 10^{-14} 4.25\cdot 10^{22} \text{ s}^{-1} \simeq 0.5\cdot 10^5 \text{s}^{-1}$$

which is in agreement with the numerical results $\lambda_f(dt) = 1.2\cdot 10^5 \text{s}^{-1}$ /3/ and $\lambda_f(\varepsilon) = 1.15\cdot 10^5 \text{s}^{-1}$/21/. Fig.4 shows the function $|f_0(\varepsilon)|^{-1}$ for the system $t\mu+d$ calculated in the "simple approach". Here one can see the resonance (the first oscillation) at point $\varepsilon = \varepsilon = 76$eV ($\nu=2$), where $|f_0(\varepsilon)|^{-2}/C_0^2(q)$ increases to $\simeq 30$, and the rate of $t\mu+d$ "in flight" fusion (14.a) at the point is equal to

$$\lambda_f^{dt}(\varepsilon = 76\text{eV}) \simeq 2\cdot 10^6 \text{ s}^{-1} \tag{16}$$

and is nearly an order of magnitude as large as the rate of the muon decay $\lambda_0 = 0.45\cdot 10^6 \text{s}^{-1}$. However, it is quite difficult to distinguish the resonance from other mesic atomic processes accompanying the reaction (14.a) at these energies. For example, the rate of slowing down of $t\mu$ atoms at the energy range is estimated as /22/

$$\lambda_{el} \simeq \sigma^{el}\cdot v\cdot N_0 \simeq 10^{-19} 10^8 4.25\cdot 10^{22} \text{s}^{-1} \simeq 10^{11} \text{s}^{-1}.$$

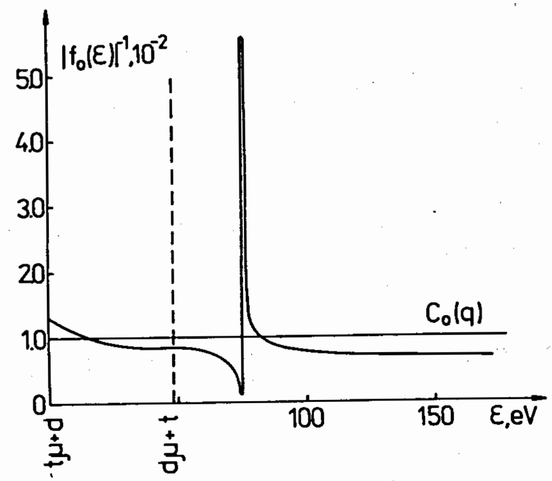

Fig.4.

The function $|f_0(\varepsilon)|^{-1}$ of the system $t\mu+d$ for the state J=0.

We have to note that the result (16) obtained in the "simple approach" for an "asymmetric" system $t\mu+d$ may be essentially different from that of multilevel calculations /19/. It is also necessary to take into consideration the molecular effects/23/ (that is, the influence of the change $t\mu+d$ by $t\mu+D_2$(DT) in the entrance channel of reaction (14.a)) near the thresholds of the channels $t\mu+d$ and $d\mu+t$ and the resonances for improving the accuracy of calculations of the rate of reaction (14.a). The estimation using an effective potential of electron screening in the entrance channel of reaction (14.a) /24,25/ shows that the correction for the electron screening is negligible.

The effect of increasing the quantity $|f_0(\varepsilon_\nu)|^{-2}/C_0^2(q_\nu)$ is the strongest for the system $p\mu+p$ (see Fig.3):

$$|f_0(\varepsilon)|^{-2}/\ C_0^2(q) \simeq 70 \qquad (17)$$

as $\varepsilon=\varepsilon_\nu \to 0$, and what's more , $\{C_0(q_{pp})/C_0(q_{dt})\}^2 \simeq 36$ (see Fig.3 and 4). So, the summary coefficient of amplification of the function $|f_0(\varepsilon=0)|^{-2}$ for the system $p\mu+p$ may be equal to the factor $2.5\cdot10^3$ as compared with the same function $|f_0(\varepsilon=0)|^{-2} \simeq C_0^2(q_d(\varepsilon=0))$ for

the system $t\mu+d$. But the constant A_{pp} of nuclear reaction $p+p \to d+e^++\nu$ is 25 orders as small as the constant A_{dt} of reaction $d+t \to {}^4He+n$, and the rate $\lambda_f^{pp}=|f_0(\varepsilon_\nu)|^{-2}\cdot A_{pp}\cdot N_0$ of nuclear reaction (14.b) even at the maximum is negligible as compared with the muon decay rate $\lambda_f^{pp} \ll \lambda_0$.

Nevertheless , the effect may influence the rate of weak processes /26/

$$p\mu \to n + \nu \qquad (18)$$

in hydrogen. Really, the strong growth of the function $|f_0(\varepsilon)|^{-2}$ for the system $p\mu + p$ as $\varepsilon \to 0$ increases the muon cloud density near the proton in the $p\mu$-atom at slow collisions $p\mu + p$ and, in principle, it may visibly change the effective rate of μ-capture (18) in dense mixtures due to an additional contribution of the channel $p\mu+p \to p+n+\nu$. It is interesting to study the possibility more carefully in view of planned experiments on measurement of the μ-capture rate in a mixture of hydrogen isotopes.

6. Conclusion

We have shown that the cross section of the nuclear reaction **X(a,b)Y** has an oscillating structure at low energies. If the maximum of the first oscillation is close to the threshold of the channel **a+X** ,the cross section has a resonance behaviour.To analyse the effect the clear formulas (7)-(9) have been derived.

Two interesting manifestations of the effect found in mesic atomic physics , the resonance amplification of the rate of $t\mu+d \to {}^4He+n+\mu$ "in flight" fusion and the growth of the muon cloud density near the nucleus in a $p\mu$-atom at slow collisions $p\mu+p$, demand , in our opinion , further careful consideration in view of planned experiments on muon catalyzed fusion and on measurement of the μ-capture rate.

The presented formulas would also be useful for a qualitative analysis of other threshold reactions of the type **X(a,b)Y** in quantum physics.

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Резонансное усиление ядерной реакции X(a,b)Y вблизи порога канала a+X

Исследуется отклонение сечения ядерной реакции X(a,b)Y от формулы Гамова за счет добавочного к кулоновскому взаимодействия во входном канале. Показано, что сечение реакции осциллирует при малых энергиях столкновения. Реакция носит резонансный характер, если максимум первой осцилляции находится вблизи порога канала a+X. Для качественного анализа эффекта получены простые соотношения, связывающие период и амплитуду осцилляций с параметрами взаимодействия. В частности, они предсказывают осцилляции сечений реакции синтеза X(a,b)Y при малых энергиях столкновения ядер (a) с атомами (или молекулами) мишеней (X), таких как D(d,p)T при столкновении ядер дейтерия (d) с атомами дейтерия (D), и описывают серию резонансов $3/2^+$ в ^{5}He, определяющих сечение реакции t(d,n)^{4}He вблизи порога d+t. Эта особенность сечений приводит к резонансному усилению скорости ядерной реакции "на лету" $t\mu+d \to {}^4He+n+\mu$ в проблеме мюонного катализа при $\epsilon \approx 76$ эВ и может повлиять на скорость μ-захвата в плотной смеси изотопов водорода.

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Resonance Amplification of the Nuclear Reaction X(a,b)Y Near the a+X Channel Threshold

Deviation of the cross section for the nuclear reaction X(a,b)Y from the Gamov formula due to an interaction additional to the Coulomb one in the entrance channel has been analyzed. It is shown that the reaction cross section has an oscillating structure at low energies. If the maximum of the first oscillation is close to the threshold of the channel a+X, it has a resonance behavior. To analyse the effect, simple relations between the period and the amplitude of the oscillations with parameters of the interaction have been derived. Specifically, they predict the cross section oscillations of fusion reactions of the type X(a,b)Y for slow collisions between nuclei (a) and atomic (or molecular) targets (X), as, for example, the reaction D(d,p)T between deuterons (d) and deuterium atoms (D), and describe the known resonance series $3/2^+$ in ^{5}He determining the cross section of the reaction t(d,n)^{4}He near the threshold of the channel d+t. The peculiarity of the cross sections leads to the resonance amplification of the rate for a muon catalyzed fusion reaction ("in flight" fusion) $t\mu+d \to {}^4He+n+\mu$ at the energy ≈ 76 eV and may influence the μ-capture rate in a dense mixture of hydrogen isotopes.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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