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OFF-SHELL EFFECTS IN THE DYNAMICAL MODEL OF PION PHOTOPRODUCTION ON NUCLEON

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1. Introduction

Recently, much attention has been paid to the pion photoproduction reaction in connection with the new accurate measurements for neutral pions near the threshold [1-2]. The reported value for the $E_{0+}(\gamma p\rightarrow p\pi^0)$ amplitude extracted from these measurements contradicts strongly the prediction of the low energy theorems (LET). The violation of the LET is very surprising especially in view of the recent analytical calculations by Naus *et al* [3]. Splitting the photoproduction amplitude into terms which contain an isolated pion or nucleon pole and imposing the gauge invariance on the operator level, the authors have reconsidered the low-energy region for the π^0-photoproduction. As a result, they have precisely obtained the LET value for the $E_{0+}(\gamma p\rightarrow p\pi^0)$ amplitude.

In this situation one faces a new task- to provide not only the successful description of the existing data but rather with the well established and physically founded approach to the elementary amplitude.

In this sense the coupled channels (or the dynamical) model for the photopion production reaction [4-6] seems to be very fruitful. On the one hand, this model does not violate the Watson theorem as the unitarity is guaranteed by the inclusion of the pion rescattering in terms of the separable pion-nucleon scattering T-matrix ($T_{\pi N}$-matrix). On the other hand, the Δ-isobar enters the model dynamically, i.e. through certain terms of the Hamiltonian. This provides one with the possibility to make the consistent and opaque extension to the nuclear matter case.

In the present paper, we are going to investigate the sensitivity of the dynamical model to the half-off-shell behaviour of the separable $T_{\pi N}$-matrix, which is the important input of the model. The parameters of this $T_{\pi N}$-matrix are fitted to the experimental data for the elastic pion-nucleon scattering. That is why various $T_{\pi N}$-matrices are practically equivalent in the on-shell region but can have different off-shell behaviour. The matter of our concern is whether this difference effects the results of the photopion production analysis in terms of the dynamical model.

2. General formalism

To the first order in $\alpha=1/137$, one can write the following expression for the photopion production multipole amplitude in the coupled channels model [4-6]:

$$\langle\vec{k}|T^{(\alpha)}_{\gamma N\to N\pi}(E_0)|\vec{q}_0\rangle = \langle\vec{k}|V^{(\alpha)}_{\gamma N\to N\pi}|\vec{q}_0\rangle + \int d\vec{q}\langle\vec{k}|V^{(\alpha)}_{\gamma N\to N\pi}|\vec{q}\rangle G(q,E_0)\langle\vec{q}|T^{(\alpha)}_{\pi N\to N\pi}(E_0)|\vec{q}_0\rangle , \qquad (1)$$

where $\vec{q}_0$ and $\vec{k}$ are the pion and photon momenta; α is the set of the channel quantum numbers; E_0 is the total energy of the pion (photon)-nucleon system; $G(q,E_0)$ is the propagator of the pion-nucleon system.

The photoproduction potential $V^{(\alpha)}_{\gamma N\to N\pi}$ in the considered model is supposed to be the sum of the Born diagrams of the $\gamma N\to N\pi$ process. In our calculations we use the Born terms of the BL-model [7] provided with the following time-ordered propagators in the pion pole and omega-exchange diagrams

$$\frac{1}{t-m_a^2} = \frac{1}{2E_a(E_\gamma - E_\pi(q) - E_a)} - \frac{1}{2E_a(E_N(q)-E_N(k)+E_a)} , \qquad (2)$$

where $a=\pi,\omega$; E_γ, E_π and E_N are the photon, pion and nucleon free energies in the c.m frame, respectively; $t=(k-q)^2$ is the invariant variable; $E_a=\sqrt{m_a^2+(\vec{q}-\vec{k})^2}$. In order to suppress the high- momentum contribution in the integral term of Eq.(1) one needs to include a cut-off formfactor in the expression for the Born term. In our calculations we shall use it in the dipole form

$$F(q) = \left(\frac{\Lambda^2+q^2}{\Lambda^2+q_0^2}\right)^2 , \qquad (3)$$

where $\Lambda=476.81$ MeV is the value obtained in Ref.5 from the analysis of the S-wave isotopic multipole photoproduction amplitudes. Note that this formfactor is included in the t-channel diagrams as well in order to preserve the gauge invariance of the model.

3. The Electric Dipole Amplitude Near the Threshold

The strong disagreement of the new experimental data [1] with the prediction of the LET occurs in the case of the neutral pion photoproduction near the threshold ($E_\gamma^{lab}=144.7$ MeV). To obtain the value which can be compared directly with the LET prediction

$$E_{0+}^{PCAC}(\gamma p\to p\pi^0) = \frac{eg}{8\pi m_N}\left(-\frac{m_\pi}{m_N} + \frac{\mu_p+2}{2}\left(\frac{m_\pi}{m_N}\right)^2\right) = -2.4\ 10^{-3}/\ m_{\pi^+}, \qquad (4)$$

where m_N and μ_p are the nucleon mass and the proton magnetic moment, one must extract the final state interaction (FSI) corrections from the $E_{0+}(\gamma p\to p\pi)$ experimental data. There are at least two approaches in the evaluation of FSI near the threshold. The first is based on the assumption that the real parts of the rescattering terms which involve the half-off-shell $T_{\pi N}$-matrices are negligibly small. In this approach the FSI corrections are fully determined by the on-shell π^+-photoproduction followed by the $n\pi^+\to p\pi^0$ scattering [1] :

$$\mathrm{FSI} = -ik_{th}\ a(p\pi^0\to n\pi^+)\ E^{Born}_{0+}(\gamma p\to n\pi^+) , \qquad (5)$$

where $k_{th}=i0.27/m_{\pi^+}$ is the $(n\pi^+)$ system momentum at the $(p\pi^0)$ threshold; $a(p\pi^0\to n\pi^+)$ is the isovector pion- nucleon scattering length. By using the value $E^{Born}(\gamma p\to n\pi^+)=27.7\ 10^{-3}/\ m_{\pi^+}$, one can obtain the following values from Ref.1: $\mathrm{FSI}\approx -1.0\ 10^{-3}/\ m_{\pi^+}$ and $E_{0+}(\gamma p\to p\pi) = -0.5\ 10^{-3}/\ m_{\pi^+}$.

The last number is in the strong disagreement with the LET prediction.

The more consistent and physically founded approach to the evaluation of the FSI corrections has been proposed in Ref.8 in terms of the dynamical model. It has been shown that the imaginary part of the π^+ rescattering diagram can appear only above the $(n\pi^+)$ threshold and that is why it does not contribute to the FSI at the $p\pi^0$ threshold. The rescattering corrections in the considered energy region are generated by the real parts of the rescattering diagrams, i.e. the principal value integrals

$$FSI_{N\pi}= P\int d\vec{q}<\vec{k}|V^{(\alpha)}_{\gamma N\to N\pi}|\vec{q}>G(q,E_0)<\vec{q}|K^{(\alpha)}_{\pi N\to N\pi}(E_0)|\vec{q}_0> , \qquad (6)$$

which involve the pion-nucleon K-matrix ($K_{\pi N}$-matrix). In terms of this approach with $\pi N= \pi^+ n$ and $p\pi^0$ Nozawa *et al* [8] have obtained the following values:

$FSI= 0.37\ 10^{-3}/\ m_{\pi^+}$ and $E_{0+}(\gamma p\to p\pi)= -1.92\ 10^{-3}/\ m_{\pi^+}$.

Now we are going to examine the sensitivity of the last approach to the off-shell behaviour of the $K_{\pi N}$-matrix. We recall the well-known relation between the off-shell K-matrix and the on-shell one in the separable model

$$K^{(\alpha)}(q,q_0)= K^{(\alpha)}(q_0,q_0)\ \frac{g^{(\alpha)}(q)}{g^{(\alpha)}(q_0)} , \qquad (7)$$

where $g^{(\alpha)}(q)$ is the formfactor. The S-wave pion-nucleon K-matrices are expressed in terms of the isotopic scattering length as follows

$$<\vec{q}_0|K_{\pi^+ n\to\pi^0 p}\ |\vec{q}>=\frac{\sqrt{2}}{3}\ (a_{3/2}\frac{g^{(S_{31})}\ (q)}{g^{(S_{31})}\ (q_0)} - a_{1/2}\frac{g^{(S_{11})}\ (q)}{g^{(S_{11})}\ (q_0)}),$$

$$<\vec{q}_0|K_{\pi^0 p\to\pi^0 p}\ |\vec{q}>=-\ \frac{1}{3}\ (a_{1/2}\frac{g^{(S_{11})}\ (q)}{g^{(S_{11})}\ (q_0)} + 2a_{3/2}\frac{g^{(S_{31})}\ (q)}{g^{(S_{31})}\ (q_0)}),$$

where $a_{1/2}= 0.173/m_{\pi^+}$ and $a_{3/2}= -0.101/m_{\pi^+}$ are the isospin 1/2 and 3/2 scattering lengths. We have used in our calculations the monopole S-wave formfactors

$$g^{(L_{2T,2J})}(q)= (q^2 + \beta^2_{2T,2J})^{-1} , \qquad (8)$$

where $\beta_{2T,2J}$ is the cut- off parameter.

One can see from eq. (8) that the off- shell values ($q\neq q_0$) of the pion- nucleon K-matrices depend on the choice of β.

We have investigated the contributions to the FSI which come from the ($p\pi^0$)- and ($n\pi^+$)-intermediate states separately. The different off-shell behaviour of the K-matrices has been provided by the choice of β to be equal to 2, 3 and 4 in the units of pion mass. These values correspond to the radius of the pion-nucleon interaction equal to (0.35- 0.7) Fm.

The calculations of the FSI in the case of the ($p\pi^0$)-intermediate state are summarized in Table 1. The results exhibit the strong sensitivity to the different off-shell behaviour of the $K_{\pi N}$-matrix. In our opinion the reason for this sensitivity is in the construction of the isoscalar pion-nucleon K-matrix. It is the sum of two comparable terms with opposite signs, multiplied by the function $g^{(\alpha)}(q)/g^{(\alpha)}(q_0)$. One can see that for all off-shell calculations the ($p\pi^0$)-intermediate state gives a small contribution to the FSI as compared with the Born term. It occurs because of the smallness of the isoscalar scattering length and the π^0-photoproduction amplitude.

Table 1. The contribution of the ($p\pi^0$)-intermediate state to the FSI [$10^{-3}/m_\pi$] at the different values of β in units of pion mass (see Eq.8)

β_{S11} β_{S31}	2	3	4
2	0.019	-0.093	-0.192
3	0.150	0.038	-0.062
4	0.266	0.154	0.055

On the contrary, the ($n\pi^+$)-intermediate state, which involves the large isovector scattering length and the π^+-photoproduction amplitude, gives the dominant contribution to the FSI, and its sensitivity to the off-shell behaviour of the $K_{\pi N}$-matrix is weak (see Table 2). However, our value of the FSI is larger than the corresponding value obtained in Ref.8. The reason for this discrepancy, as can be seen from Table 3, is in the choice of the pion propagator in the t-channel pion pole diagram. In the case of the time-ordered propagator (version I, see Eq.(2)) the estimations of the FSI give the biggest value. The choice $\Delta_\pi = -\frac{1}{2qk}$ (version II), which is adopted in Ref.8, leads to unexpected cancellation of the contributions from the seagull and pion pole diagrams.

Table 2. The contribution of the $(p\pi^+)$-intermediate state to the FSI [$10^{-3}/m_\pi$] at the different values of β in units of pion mass (see Eq.8)

β_{S11} β_{S31}	2	3	4
2	-1.75	-1.94	-2.05
3	-1.86	-2.05	-2.16
4	-1.92	-2.11	-2.22

Table 3. The contribution of the dominant diagrams in the FSI in $10^{-3}/m_\pi$ at the different choices of pion propagator.

	Seagull	Pion	Sum
Var.I	-3.464	0.911	-2.553
Var.II	-3.464	2.224	-1.240
Ref.8	-3.07	3.54	0.47

4. The Electromagnetic Constants of the Δ- Isobar

Now we proceed with the off-shell effects in the P_{33}-channel. The Δ-isobar degrees of freedom are the substantial part of the π-photoproduction analysis in this channel. It is well known that the Born diagrams are not sufficient to describe the $M_{1+}(3/2)$ and $E_{1+}(3/2)$ multipoles. That is why the photoproduction amplitude has been constructed by adding the Δ-photoproduction term to the Born one. One of the most attractive features of the dynamical model is the inclusion of the Δ-isobar on the same footing with the photon, pion and nucleon. For the self-consistency it is necessary to have the Δ's with the same πNΔ-vertices both in the scattering and photoproduction amplitudes. In this case, the only undetermined quantity is the γNΔ-vertex for the pion photoproduction. Let's take it in the following form [4]

$$V_{\gamma N\Delta} = f^{(M1,E2)}_{\gamma N\Delta} \frac{e}{2m_N} , \qquad (9)$$

where $f^{(M1,E2)}_{\gamma N\Delta}$ are the electromagnetic constants of the Δ-isobar, which correspond to the magnetic dipole and electric quadrupole transitions. They are usually fitted to the data of the $M_{1+}(3/2)$ and $E_{1+}(3/2)$ photoproduction amplitudes. The question is whether this way of the $f^{(M1,E2)}_{\gamma N\Delta}$ determination is sensitive to the choice of the $T_{\pi N}$-matrix. We have followed the above described procedure, using three variants of the pion-nucleon T-matrix from Ref.9. The obtained overall good agreement is illustrated in Figs. 1,2. The two of the considered models (A and B) represent the situation when the scattering process is supposed to go by means of the background+resonance mechanism.

Table 4. The dependence of the Δ-isobar constants of the different variants of the separable interaction.

	Model A	Model B	Model C
$f^{(M1)}_{\gamma N\Delta}$	-2.75	-2.55	-2.80
$f^{(E2)}_{\gamma N\Delta}$	0.33	0.54	0.01

As can be seen from Table 4, the results of our calculations in the cases of these two models differ slightly. In our opinion, this happened due to the smallness of the background

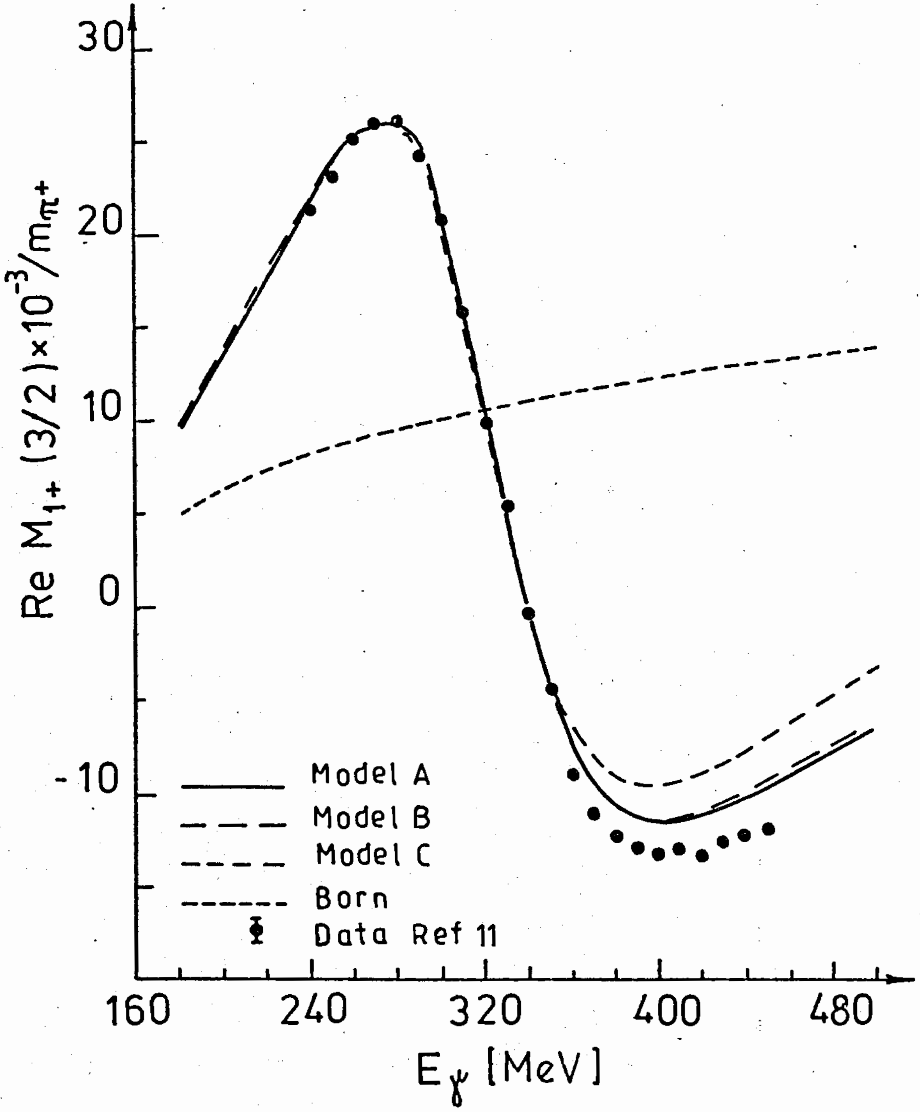

Fig.1 Fit to the real part of the $M_{1+}(3/2)$ multipole. Separable models from Ref.9.

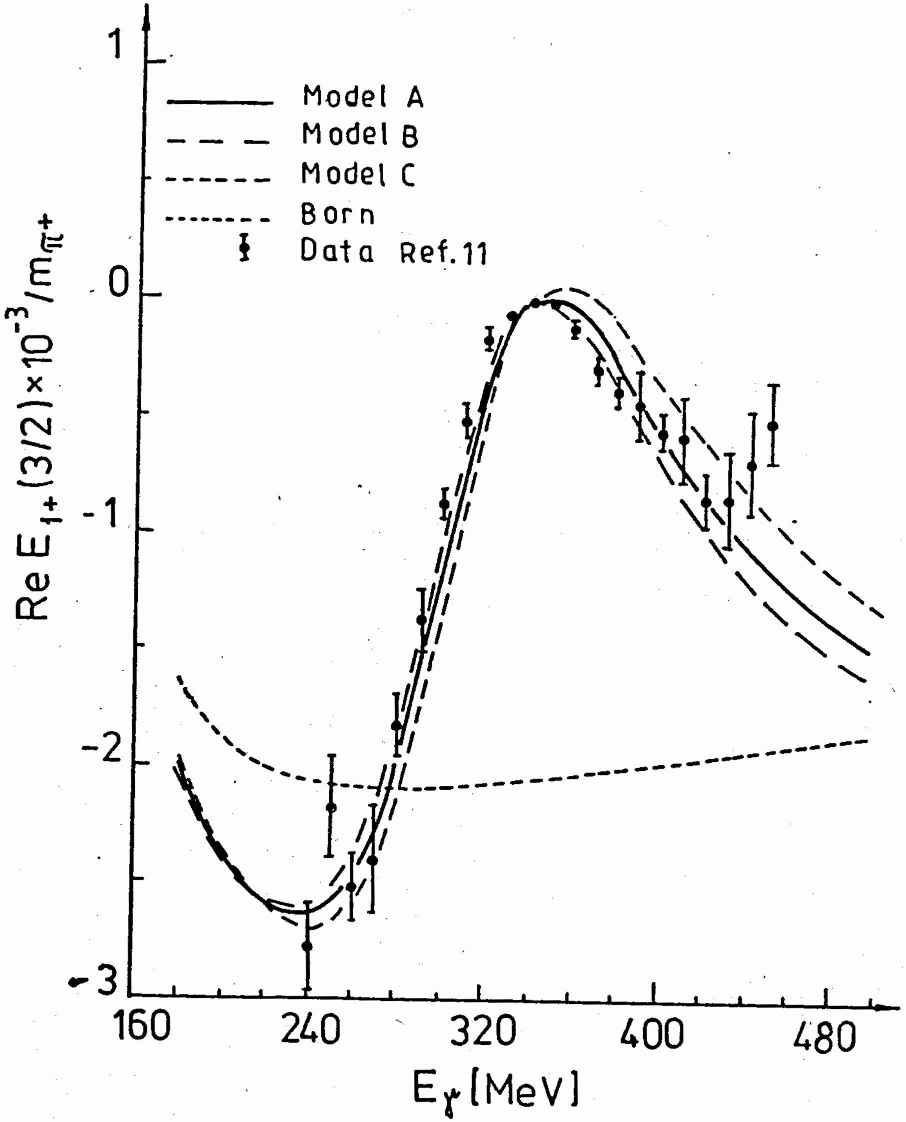

Fig.2 Fit to the real part of the $E_{1+}(3/2)$ multipole. Separable models from Ref.9.

interaction. As has been shown in Ref.10, all principal value integrals in this case are effectively absorbed in the renormalization procedure and no off-shell quantities appear in the final expression.

The situation is quite different when the pure resonance model C from Ref.9 is considered. The values of the electromagnetic constants, especially for the quadrupole transition, differ from those, obtained in the models A and B. To our mind, one can make the same with the nucleon-nucleon scattering case conclusion [9] about the doubtful validity of the pure resonance mechanism of pion-nucleon scattering in the P_{33}-channel.

5. Concluding Remarks

In the present paper we have investigated the sensitivity of the dynamical model to the off-shell behaviour of the pion-nucleon T-matrix. The main attention has been paid to the threshold region where the disagreement between the experimental data and the prediction of the LET occurs. In this energy region we have found that the rescattering corrections may be significant in the case of the time-ordered pion propagator in the t-channel pion-pole diagram. These corrections are almost determined by the $(n\pi^+)$-intermediate state and exhibit practically no sensitivity to the off-shell behaviour of the $T_{\pi N}$-matrix. However, this result can change drastically if the representation of the pion propagator as $-\frac{1}{2qk}$ is adopted. In this case, the two dominant diagrams, namely, the seagull and the pion pole become comparable and have the opposite signs. This point leads to the strong cancellation in the $(n\pi^+)$ channel.

Our estimations show that the $(p\pi^0)$ intermediate state gives a small contribution to the final result for the $E_{0+}(\gamma p\to p\pi)$ amplitude. This contribution is very sensitive to different off-shell $T_{\pi N}$-matrices.

We have investigated the influence of various choices of the $T_{\pi N}$-matrices in the P_{33}-channel on the electromagnetic constants of the Δ-isobar. Two versions of the separable $T_{\pi N}$-matrix have been considered. The results of our analysis in the case of models A and B of Ref.9 are rather similar. This indicates, to our mind, the validity of the procedure which has been proposed in Ref.10 for the construction of the π-photoproduction amplitude in the P_{33}-channel.

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Внеэнергетические эффекты в фоторождении
пионов на нуклонах

В рамках динамической модели фоторождения пионов на нуклонах исследуются эффекты взаимодействия в конечном состоянии для E_{0+}, M_{1+} и E_{1+} мультиполей. Обнаружена сильная корреляция между чувствительностью результатов анализа ко внеэнергетическому поведению пион-нуклонных Т-матриц и способом представления пионного пропагатора. В случае временно-упорядоченного пропагатора влияние взаимодействия в конечном состоянии определяется вкладом зарядово-обменного канала, для которого внеэнергетические эффекты не существенны. Показано, что величина электромагнитной константы Δ-изобары для E2-перехода зависит от выбора варианта сепарабельного пион-нуклонного взаимодействия.

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Off-Shell Effects in the Dynamical Model
of Pion Photoproduction on Nucleon

In terms of the dynamical model the rescattering effects for the E_{0+}, M_{1+} and E_{1+} multopoles are analyzed. Strong correlation between the sensitivity to the off-shell behaviour of the $T_{\pi N}$-matrix and the representation of the pion propagator in the photoproduction operator is shown to exist. For the time-ordered choice of a pion propagator the effect of a final state interaction is determined by the charge-exchange channel. In this case the off-shell effects are not important. It is also shown that the value of the coupling constant of the E2-transition in the $\gamma N\Delta$–vertex depends on a version of the separable model for the πN interaction in the P_{33}-channel.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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