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# ALPHA DECAY OF $^{210}$Po

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## 1. INTRODUCTION

Large interest has been attached to the study of the $\alpha$-decay fine structure with the development of the microscopic nuclear models. Good agreement between the experiment and the theory of the relative $\alpha$-decay intensities has been obtained using the simplified R-matrix theory/1/.We go into much trouble when calculating the absolute decay widths for composite emitted particles especially when the decay phenomenon is caused by strong nuclear forces. The microscopic calculations based on the frequently used theory of ref./1/,contain two hard obviating shortcomings. Firstly, a kind of a surface delta interaction that introduces a fiction channel radius parameter is used in the theories. Secondly, the microscopic model wave functions used for the parent (daughter) nucleus do not satisfy the boundary conditions which normally must be imposed upon a compound state.

It is our goal in this paper to give a more precise mathematical derivation of the expression of the decay width, expression than one can use the actual microscopic model wave functions. This theory leads to the necessity of introducing of: a) more sophisticated interactions responsible for the decay phenomenon, others than the phenomenological surface delta interactions used in the paper [1], and b) new interactions responsible for possible virtual transitions among different final channels after the preformation of the emitted particle and before its escape.

## 2. DECAY WIDTH

Following the time-dependent theory of the decay [3-5] we introduce, as in ref.[4,5], as an observable determining the decay law, the ratio $P(t)$ of the number of decayed nuclear systems at the time $t \geq 0$ to the undecayed ones at the time $t=0$

$$P(t) = \frac{\text{trace}\,\{\rho W(t)\}}{\text{trace}\,\{\rho W(0)\}} \,. \tag{1}$$

This definition is discussed very well in refs.[4,5], and we shall not dwell on it more. Here the density matrix

$$\rho = \sum_k |\lambda_k> \omega(k) < \lambda_k| \tag{2}$$

is defined in terms of incoherent, normalized to unity, states $|\lambda_k>$ describing the nondecayed nuclei in our instable nuclear system that is going to decay. The operator

$$W(t) = \int^R dx\, e^{iHt} |x><x| e^{-iHt} \tag{3}$$

projects onto the nuclear spatial domain (the statistical operator (2) describes $\alpha$ particles in the nondecayed nucleus, i.e., the spatial coordinates $|x| \leq$ nuclear radius R ). Assuming a pure state in the statistical ansamble (2) ($\omega(k) = \delta_{k0}$) the expression (1) becomes



$$P(t) = N \int^R dx\, |\psi(x,t)|^2 , \tag{4}$$

where

$$N^{-1} = \int^R dx\, |\psi(x,0)|^2 , \tag{5}$$

and

$$\psi(x,t) = <x| e^{-iHt} |\lambda_0> . \tag{6}$$

Defining now the asymptotic interaction "free" hermitean hamiltonian $H_0$ by $H = H_0 + H'$, in which three motions are studied adiabatically (the internal motions in the $\alpha$-particle, in the daughter nucleus and the relative motion described by a central potential) we can obtain a kind of a spectrum as shown in fig. 1.

Following the procedure of refs.[6-10] we introduce four projectors onto subspaces of $H_0$-states:

$$\begin{aligned} &P = |0><0| ; \quad Q = \sum_c \int d\epsilon\, |\phi_{c\epsilon}><\phi_{c\epsilon}| \\ &q = \sum_{c'} q_{c'} ; \quad P + Q + q + A = 1. \end{aligned} \tag{7}$$

The state $|0>$ is a bound state embedded in the continuum [6] (BSEC) having the eigenenergy $E_0$ (see fig. 1) in the nearest vicinity of the decay energy ($\epsilon$), Q projects onto the "active" open channels (the matrix elements of $H'$ between these channel states $|\phi_{c\epsilon}>$ and $|0>$ we consider of the first order, they describe the most intensive $\alpha$ transitions), q projects onto the "passive" open

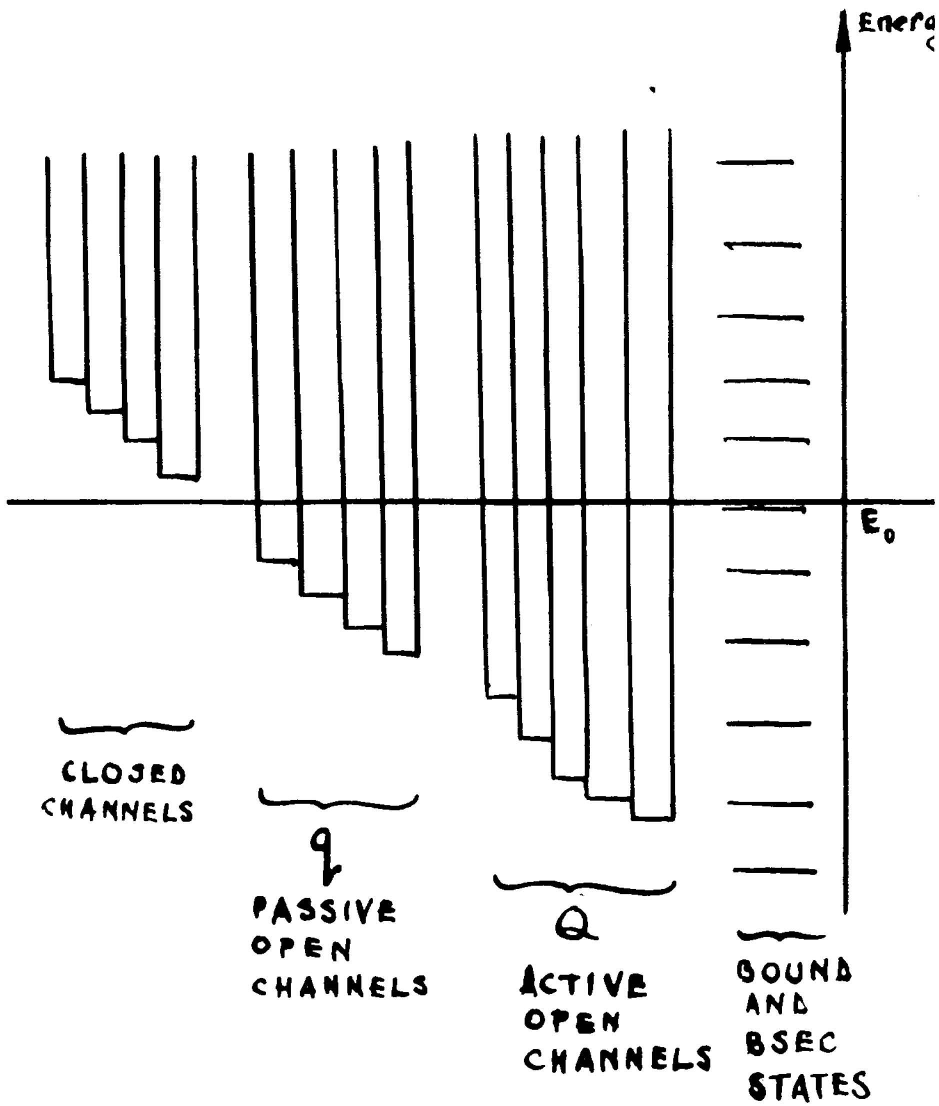


Fig. 1. The energy spectrum of the asymptotic interaction "free" $H_0$-Hamiltonian (schematically). The vertical boxes correspond to various open and closed channels and the full lines to the bound or BSEC (doorway) states.



channels (the corresponding $H'$-matrix elements are of the second order of magnitude), and A-projects onto the rest of (closed) channels. Expanding the $|\lambda_0>$ state in terms of $H_0$-states

$$|\lambda_0> = (P + Q + q + A)|\lambda_0> = \beta_0|0> + \Sigma\ \beta_n|n>. \quad (8)$$

we shall consider the case when the "sharp resonance" condition is fulfilled, i.e., $\beta_0 \approx 1$ and $\beta_n \approx 0$. Thus eq. (6) can be approximated by

$$\psi(x,t) \cong <x|e^{-iHt}|0>. \quad (9)$$

Taking into account now that the "sharp resonance" condition allows us to neglect the contribution to the integrals (4) and (5) of the vector $<x|(Q+q+A)e^{-iHt}|0>$, eq. (9) can be approximated by

$$\psi(x,t) \cong <x|Pe^{-iHt}|0> = <x|0>a(t) \quad (10)$$

with

$$a(t) = <0|e^{-iHt}|0>.$$

$|0>$ is a sufficiently nice vector in the sense that $d<0|E_\lambda|0>$ falls off exponentially where $dE_\lambda$ is the spectral measure associated to H, then

$$a(t) = (2\pi i)^{-1} \int_C e^{-izt} <0|G(z)|0>, \quad (11)$$

when C is a contour as in fig. 2 running from $ip+\infty$ to $ip-\infty$ and the rezolvent $G(z) = (z-H)^{-1}$ defined in the complex $z$ plane so that has no singularities lying above C. We can simultaneously rotate C and

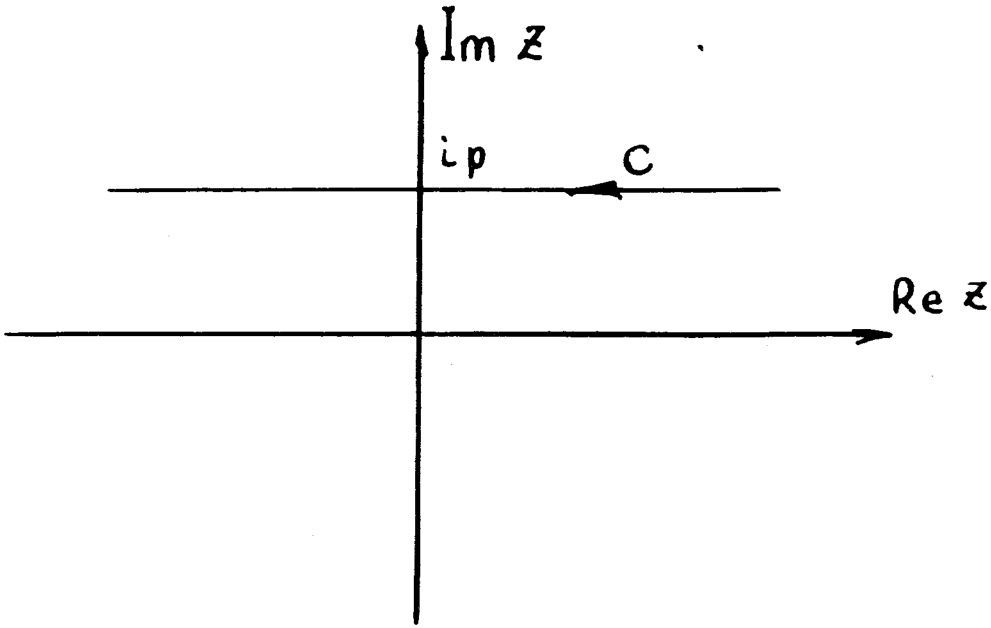


Fig. 2. The contour to perform the integral (11).

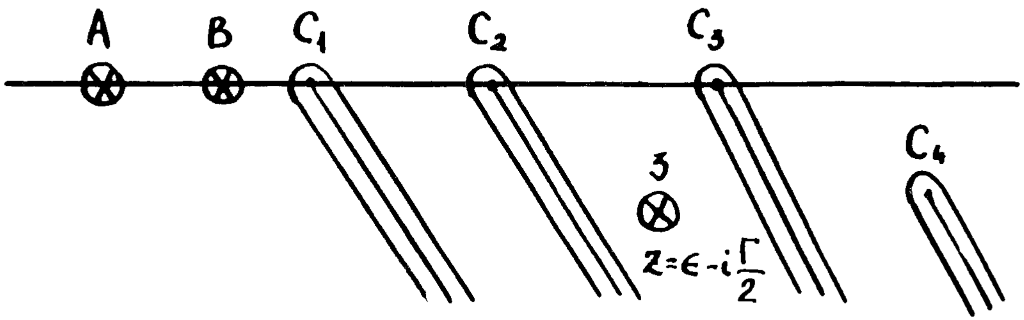


Fig. 3. The remaining contours after the rotation $U(\theta)$. By $A$ and $B$ are denoted the bound poles, by $C_1$, $C_2$, $C_3$ - the continuum embedded thresholds and by $C_4$ - the complex thresholds of the many body hamiltonian $H$.



replace $\langle 0|G(z)|0\rangle$ with $\langle 0|U^{+}(\theta)\,[z-H(\theta)]^{-1}U(\theta)\,|0\rangle$ (fig. 3) to find [3, 11]

$$a(t) = \qquad e^{-izt}\,\langle 0|G(z)|0\rangle\big|_{z=\epsilon-i\Gamma/2} + r(t)\,, \qquad (12)$$

where $r(t)$ represents the contributions from the other poles or the cuts because we are dealing with a many-body Hamiltonian with the spectrum of the type given in fig. 1 or even more complex.

To compute the residuum in eq. (12) we apply the procedure used in our earlier papers [7-10] namely using the expansion

$$\begin{aligned} G(z) &= PG + QG + qG + AG = \frac{A}{z - H_0 - AH'A} + \\ &+ \Omega_{A_r}(z)\,\frac{q}{z-H_0-qVq}\,\Omega_{A_\ell}(z) + \\ &+ \Omega_{A_r}(z)\,\Omega_{q_r}(z)\,\frac{Q}{z-H_0-QWQ}\,\Omega_{q_\ell}(z)\,\Omega_{A_\ell}(z) + \\ &+ \Omega_{A_r}(z)\,\Omega_{q_r}(z)\,\Omega_{Q_r}(z)\,\frac{P}{z-H_0-PRP}\,\Omega_{Q_\ell}(z)\,\Omega_{q_\ell}(z)\,\Omega_{A_\ell}(z)\,, \end{aligned} \qquad (13)$$

with the "right" and "left" $\Omega$-operators defined by

$$\Omega_{A_r}(z) = 1 + \frac{A}{z-H_0-AH'A}\,H'\,;\quad \Omega_{A_\ell}(z) = H'\,\frac{A}{z-H_0-AH'A} + 1\,, \qquad (14)$$

$$\Omega_{q_r}(z) = 1 + \frac{q}{z-H_0-qVq}\,V\,;\quad \Omega_{q_\ell}(z) = V\,\frac{q}{z-H_0-qVq} + 1\,, \qquad (15)$$

$$\Omega_{Q_r}(z) = 1 + \frac{Q}{z-H_0-QWQ}\,W\,;\quad \Omega_{Q_\ell}(z) = W\,\frac{Q}{z-H_0-QWQ} + 1\,, \qquad (16)$$

$$V = H' + H' \frac{A}{z - H_0 - AH'A} H', \quad (17)$$

$$W = V + V \frac{q}{z - H_0 - qVq} V, \quad (18)$$

$$R = W + W \frac{Q}{z - H_0 - QWQ} W. \quad (19)$$

Here the resolvents corresponding to the projected subspaces must be understood as the solutions for example of the eq.

$$(z - H_0 - AH'A) \frac{q}{z - H_0 - AH'A} = q. \quad (20)$$

and so on.

Inserting eq. (13) into eq. (11) and taking into account the sharp resonance condition as in ref. [8,11] the ratio $P(t)$, defined by eqs. (1,4,10) becomes:

$$P(t) = \exp\{-\Gamma_{00} t\} + F(t), \quad (21)$$

where $\Gamma_{00}$ is the imaginary part of $R_{00} = \langle 0|R(\epsilon)|0\rangle$ and $F(t)$ is a "small" term coming from the contributions from the other poles and cuts. In the following we neglect it. $R(\epsilon)$ is the operator (19) taken in the pole energy

$$\epsilon = E_0 + \mathrm{Real}\langle 0|R(\epsilon)|0\rangle. \quad (22)$$

It is easy to show that [7,9,10]

$$\Gamma_{00} = -i\langle 0|R^{+}+R|0\rangle = \langle 0|\Gamma_R|0\rangle + \langle 0|\Gamma_W|0\rangle + \langle 0|\Gamma'|0\rangle, \quad (23)$$

where

$$\Gamma_R = 2\pi R^{+} Q\delta(\epsilon - H_0)\, QR, \quad (24)$$

$$\Gamma_W = 2\pi W^{+} g\delta(\epsilon - H_0)\, qW, \quad (25)$$

$\Gamma'$=third and higher order in $W$ terms. Considering that all the intensities of the $\alpha$-transitions to the $q$-subspace states are small quantities of the second and higher order* we can neglect the last two terms in eq. (23) and write

$$\Gamma_{00} \cong \langle 0|\Gamma_R|0\rangle = 2\pi \sum_c |\langle \phi_{c\epsilon}|R(\epsilon)|0\rangle|^2, \quad (26)$$

## 3. "RESONANCE" MANY BODY STATES OR "SHELL MODEL" STATES FOR THE MOTHER NUCLEUS

Following the treatment of ref. [11] the $R(\epsilon)$ operator from eq. (26) of the $\alpha$-decay width can be written as follows

$$R(\epsilon) = H'\Omega_{A_r}(\epsilon)\Omega_{q_r}(\epsilon)\Omega_{Q_r}(\epsilon) = H'F(\epsilon). \quad (27)$$

Goldberger and Watson [11] showed that the function $F(\epsilon)|0\rangle$ has a resonance boundary condition [12-16]. The same condition imposed on the wave function of the mother nuc-

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*The hindrance factors corresponding to the $\alpha$-transitions to excited states increase much with the complexity of the states, the penetrability decreases very much with the excitation energy. We can choose the $q$-subspace in concordance with the magnitudes of the $\alpha$-intensities.

leus in the $\alpha$-decay width expression were obtained by Mang[2]. In practical calculations the $F(\epsilon)|0>$ - wave function is replaced by the bound state "shell model" wave function. It is not possible to obtain a "rigorous" or at least approximate many body "resonance wave function at the moment because we have to run into very hard mathematical difficulties, much computer time and so on.

It is better to use another procedure, namely to factorize the $R(\epsilon)$ operator as follows

$$R(\epsilon) = \Omega_{Q_\ell}(\epsilon) H'\Omega_{A_r}(\epsilon)\Omega_{q_r}(\epsilon) = \tag{28}$$

$$= \Omega_{Q_\ell}(\epsilon) H'\Omega_{A_\ell}(\epsilon) + \Omega_{Q_\ell}(\epsilon) H'\Omega_{A_r} \frac{q}{\epsilon - H_0} V\Omega_{q_\ell}(\epsilon).$$

We can neglect the second term in the right-hand side as we have done in eq. (26) and the $\alpha$-decay width becomes:

$$\Gamma_{00}(\epsilon) = 2\pi \sum_c |<\phi_{c\epsilon}|\Omega_{Q_\ell}(\epsilon) H'\Omega_{A_\ell}(\epsilon)|0>| \ . \tag{29}$$

Defining the new initial and final states as follows

$$|\phi_i> = \Omega_{A_\ell}(\epsilon)|0> \tag{30}$$

and

$$|\psi_{c\epsilon}> = \Omega^+_{Q_\ell}(\epsilon)|\phi_{c\epsilon}> \tag{31}$$

or

$$\{\epsilon - H_0 - QW^+(\epsilon)Q\}|\psi_{c\epsilon}> = 0 \tag{32}$$



we find that

$$\Gamma_{00}(\epsilon) \approxeq 2\pi \sum_c |<\psi_{c\epsilon}|H'|\phi_i>|^2 . \tag{33}$$

The new initial state (30) has bound state boundary conditions at large distances. The operator $\Omega_{A\ell}(\epsilon)$ is equal, in the first approximation of the perturbation theory, to the projector [17,18] onto the bound state of the exact hamiltonian H having the eigenenergy in the nearest vicinity of the $E_0$. The $\phi_i$ function can be very well approximated by the usual "shell model with residual interaction" wave functions. The wave function (31) is an enriched $\alpha$-channel wave function due to the residual $QW^+Q$ complex interaction potential. The imaginary part of this interaction $\frac{1}{2}iQ\Gamma_W Q$ cannot be very large due to the approximations above made. The real part, however, plays very important role. Part of it, has been already taken into account in the Mang theory [2] using the Froman matrix method for the case of the $\alpha$-transitions to the rotational levels. This part includes the electrostatic interaction of the $\alpha$-particle and the quadrupole moment of the daughter nucleus. It is possible also to include interactions involving other degrees of freedom as, for instance, in ref. [19], or coming from the nonviolation of the Pauli principle [20,27].

Assuming the following expansion of the channel function (31)

$$<x|\psi_{c\epsilon}> = \sum_{c'} \Omega_{c'c} <x|\phi_{c\epsilon}> = \sum_{c'} \Omega_{c'c} \frac{1}{r}\sqrt{\frac{2m}{\pi \hbar^2 k}} F_c(\epsilon, r)\eta_c(\xi) . \tag{35}$$

and also

$$\langle x|\Phi_i\rangle = \sum_{c''} \left(\frac{2mr}{h^2}\right)^{1/2} \gamma_{ic''}(r)\frac{1}{r}\eta_{c''}(\xi)\,, \tag{36}$$

where

$$\gamma_{ic}(r) = \left(\frac{h^2}{2mr}\right)^{1/2} \langle\Phi_i|r^{-1/2}\,\delta(r_\alpha - r)|\eta_c\rangle \tag{37}$$

is the usual amplitude of the reduced width[2], the $\alpha$-decay width (33) becomes

$$\Gamma_{00}(\epsilon) = \frac{8m^2}{h^2k}\sum_c \Big|\sum_{c'c''}\Omega_{cc'}\int_0^\infty dr\, r^{1/2}\gamma_{ic''}(r)\,H'_{c''c'}(r)\,F_{c'}(\epsilon,r)\Big|^2 \tag{38}$$

where

$$H'_{c''c'}(r) = \int d\xi \langle\eta_{c''}(\xi)|H'|\eta_{c'}(\xi)\rangle\,. \tag{39}$$

Here the $F_c(\epsilon,r)$ is the regular solution of the scattering, on a central (nuclear plus coulomb) potential, problem. $\eta_c(\xi)$ is the channel spin function[1]. The $\Omega_{c'c}$ represents a large part of the virtual excitation occuring after the formation process of the $\alpha$-particle and before its penetration through the barrier (fig. 4).

## 4. RELATIONS TO EARLIER APPROACHES

The general expression (38) of the $\alpha$-decay width contains as approximations all earlier approaches

a) Mang's R-matrix approach. Taking for the matrix element (39) a kind of a surface delta interaction

$$H'_{c''c'}(r) = h^2k\{2mF_{c'}(\epsilon r)\,G_{c'}(\epsilon r)\}^{-1}\;\delta(r-R)\delta_{c''c'} \tag{40}$$

we obtain for the expression (33) the well-known R-matrix formula[1,2]

$$\Gamma_{00}(\epsilon) = \sum_c P_c(\epsilon,R)\,\tilde{\tilde{\gamma}}^2_{ic}(R)\,, \tag{41}$$

where

$$P_c(\epsilon,R) = 2kRG_c^{-2}(\epsilon,R) \tag{42}$$

is the barrier penetrability and

$$\tilde{\tilde{\gamma}}_{ic}(R) = \sum_{c'}\Omega_{c'c}G_c(\epsilon,R)\,G_{c'}^{-1}(\epsilon,R)\,\gamma_{ic'}(R) \tag{43}$$

is the "correlated" amplitude of the reduced width. The matrix $\Omega_{c'c}G_c(\epsilon,R)G_{c'}^{-1}(\epsilon,R)$ does not depend on the energy at least in the first order[21]. In the three-dimensional WKB-approximation Froman has obtained for this matrix element the simple expression given in ref[2].

b) Harada - Rausher approach[22]. The matrix element (39) of the $H'$-interaction, in this approach is taken to be of the form

$$H'_{c''c'}(r) = \delta_{c''c'}\Big\{\sum_{i=1}^{4} V^{opt}_{iN}(r) - U_{\alpha N}(r)\Big\} \tag{44}$$

and $\Omega_{c'c} = \delta_{c'c}$. Thus the $\alpha$-decay width (38) becomes

$$\Gamma_{00}(\epsilon) \cong \frac{8m^2}{kh^4}\sum_c\Big|\int_0^\infty r^{1/2}\,dr\,\gamma_{ic}(r)\{V^{opt}_{iN}(r) - U_{\alpha N}(r)\}F_c(\epsilon,r)\Big|^2 \tag{45}$$

Here the $\alpha$-decay probability is determined by the transition matrix element between the bound state shell model wave function and the noncorrelated alpha-channel wave function. The perturbation which causes the alpha transition is given by the difference between four times the nucleon nucleus and alpha-nucleus potentials. The formula (45) depends strongly on the optical parameters involved in the Saxon-Woods alpha nucleus $U_{\alpha N}$-potential, because the potential used in the scattering problem is not quite good for the alpha decay problem.

c) Kadmensky et al approach. In refs.[23,24] and the others cited therein an expression for the alpha decay width is used that can be obtained from eqs. (33) and (38) using the following approximations:

1°. $\Omega_{c'c} = \delta_{c'c}$ ,

2°. $F_c(\epsilon, r) = F_c^{coul}(\epsilon, r)$ ,

3°. The interaction H′ is taken to be of the following form

$$H' \cong \sum_{i=1}^{4} V_{iN}^{opt}(r_i) \cong \tilde{\tilde{V}}_{\alpha N}(r) , \qquad (46)$$

where

$$\tilde{\tilde{V}}_{\alpha N}(r) = \langle \phi_\alpha | \sum_{i=1}^{4} V_{iN}^{opt}(r_i) | \phi_\alpha \rangle \qquad (47)$$

is the averaged potential, $\phi_\alpha$ - being the intrinsic alpha particle wave function. Using the approximation 1° - 2° - 3°, the alpha decay width becomes

$$\Gamma_{00}(t) \cong \frac{8m^2}{kh^4} \sum_c |\int_0^\infty r^{1/2}\, dr\, \gamma_{ic}(r) \tilde{\tilde{V}}_{\alpha N}(r) F_c^{coul}(\epsilon, r)|^2 . \qquad (48)$$

This expression however, assumes for the asymptotic interaction "free" hermitean hamiltonian $H_0$ (see eqs. (7) and above) the following expression



$$H_0 = H_\alpha(in) + H_d(in) + T_{\alpha d} + \sum_{i=1}^{4} \sum_{j=5}^{A} V_{ij}^{coul} , \qquad (49)$$

where $H_{\alpha(d)}(in)$ labels the internal alpha (daughter) hermitean many body hamiltonian, $T_{\alpha d}$ denotes the relative motion (alpha-daughter) kinetic energy operator and the last term of eq. (49) represents the Coulomb interaction between the decay products. Until now we knew only two mechanisms of generating the many body resonance (quasi-stable) states [6]: a) the existence in the initial (mother) state wave function a component described by one (or even many) bound state embedded in the continuum (BSEC) having the eigenenergy lying in the nearest vicinity of the decay energy or b) a single particle resonance (SPR) in the same energy interval. The hamiltonian (49) is not able to produce any of these possibilities. Therefore we believe[10] that no theory can explain eq. (48), except if we neglect, maybe, important terms[24].

d) Other approaches. The approaches from refs.[25,26] may be related to the approaches above mentioned. In ref.[25] the expression (41) has been calculated with such an alpha-nucleus potential that generates a SPR near the decay energy $\epsilon$. The treatment from ref.[26] assumes the following in eqs. (33) and (38)

1°. $\Omega_{c'c} = \delta_{c'c}$

2°. The form of the interaction H′ is the following

$$H' = \sum_{i>j}^{4} V_{ij} + \sum_{i=1}^{4} V_{iN}^{opt}(r_i) - U_{\alpha N}^{opt}(r) . \qquad (50)$$

One can easily see that such an interaction never allows the formation of the alpha particle in the ground state.

e) Possible rigorous treatments. Starting from eq. (30) the alpha decay width (33), in the framework of the short range correlations theory [10,28-31], can be performed[27].

## 5. NUMERICAL CALCULATIONS. ALPHA DECAY OF $^{210}Po$

To fulfil the conditions resulting from the theory presented in this paper, we have chosen the case of the $^{210}Po-\alpha$-decay. For both mother and daughter nuclei we have used the shell model with the residual interactions. The $^{210}Po$ is in its ground state and has $N=126$ and $Z=82$ shell closure structure plus two protons in $(1h_{9/2})$ - s.p. state. The $^{206}Pb$ - ground and excited ($0^+$ or $2^+$) states have been obtained by adding to the shell model Hamiltonian the multipole pairing interaction:

$$H_p = \sum_\lambda H_p(\lambda)\ ;\quad H_p(\lambda) = -G_\lambda \pi(2\lambda+1)\sum_\mu P^+_{\lambda\mu} P_{\lambda\mu}, \tag{51}$$

where

$$P^+_{\lambda\mu} = -\frac{2}{\sqrt{2\lambda+1}}\{\sum_{k_1\geq k_2}(-1)^{\ell_2}<k_1||r^\lambda Y_\lambda||k_2>[a^+_{k_1}a^+_{k_2}]^\lambda_\mu\{1+\delta_{12}\}^{-1}$$
$$-(-1)^\mu\sum_{i_1\geq i_2}(-1)^{\ell_2}<i_1||r^\lambda Y_\lambda||i_2>[a^+_{i_1}a^+_{i_2}]^\lambda_{-\mu}\{1+\delta_{12}\}^{-1}\}\ . \tag{52}$$



The operator $a^+_s$ creates a particle in an orbital $s$. The label $k$ denotes a state above the Fermi sea while $i$ denotes a state below the Fermi sea. The properties of this interaction are well discussed in ref.[32]. Particles interacting through this force have a high degree of spatial correlation in the lowest state of each spin $\lambda$ and the parity $(-1)^\lambda$. These lowest levels display large $0s$ relative state of motion components as compared to excited levels of the same spin and parity. They are specifically excited in the $\alpha$-decay and $(t,p)$ processes. The calculations have been performed using a a surface delta interaction (40) and inserting such a $QW^+Q$ interaction in eq.(32) that the decay width (38,41) does not depend on the channel radius parameter (see eqs. 40-43), at least in the nuclear surface region. This condition is not fulfiled in the usual Mang[2] theory. In figs. 5-7 we show the independence of $\Gamma_\alpha$ of the channel radius $R$ in a large region 6.5 - 8.5 fm. The $\alpha$-decay width $\Gamma_\alpha$ is stable to the variation of the optical parameters too, if they fulfil the following eq.:

$$V = V_0\ \{1+\exp\frac{R-r_0A^{1/3}}{a}\}^{-1} \tag{53}$$

with $V=-96$ MeV and $R=7.63$ fm.

The amplitude of the reduced width (43) has been computed rigorously as in Mang's works[2]. For the channel - channel coupling (eq. 32) we have taken only the diagonal matrix elements that fulfil the conditions shown in the figures 5-7.

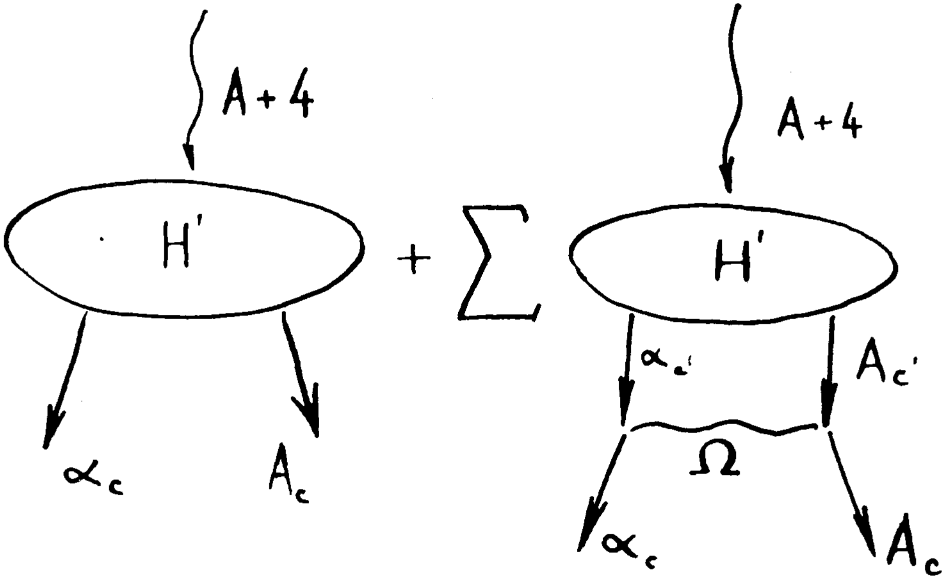


Fig. 4. The coupling terms that are necessary to be introduced in the total $\alpha$-decay width. The wave line represents the $A$-nucleons coming into $H'$-interaction, the $A_c$ and $\alpha_c$ lines denote the nucleons of the decay products that go out from the interaction $H'$, the $\Omega$-wave line denotes the channel - channel interaction propagator.



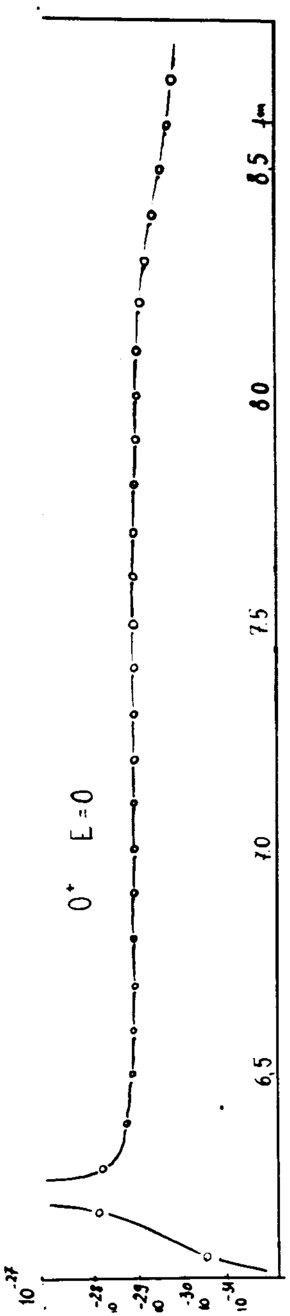


Fig. 5. The $\alpha$-decay width as a function of the channel radius. Case of the $\alpha$ transition to $0^+$(0 KeV) state.

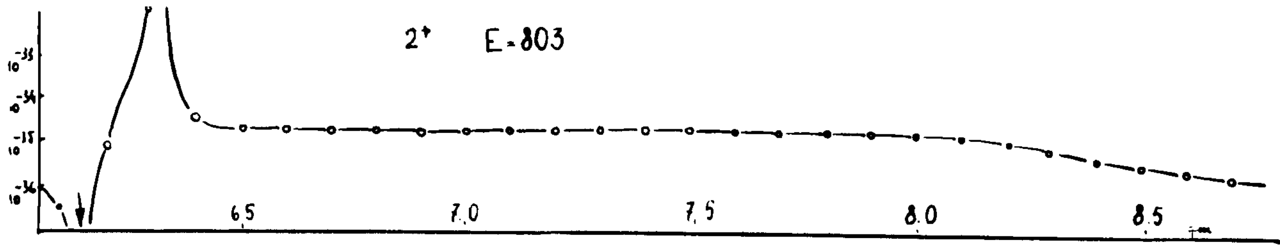


Fig. 6. The same as in fig. 5. Case of the $\alpha$ transition to $2^+$ (803 KeV) state.

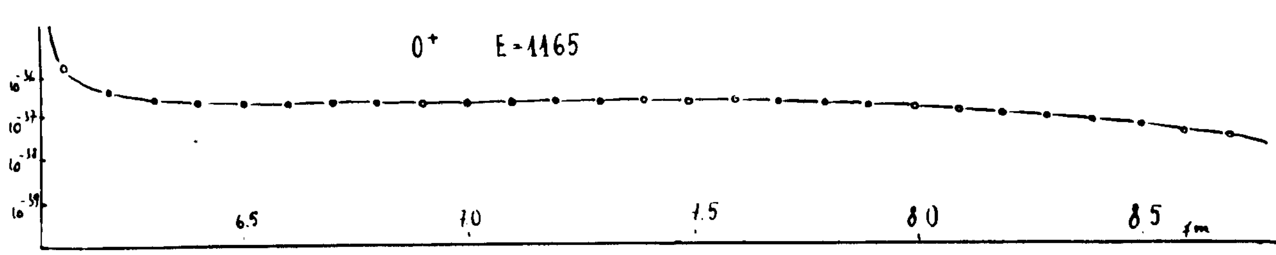


Fig. 7. The same as in fig. 5. Case of the $\alpha$ transition to $0^+$ (1165 KeV) state.

Table

The $\alpha$ -decay widths for the $\alpha$ -transitions to some positive parity states of $^{206}$Pb.

| $I^{\pi}$ | $E_{exp}$ (Kev) | $E_{th}$ (KeV) | $\Gamma_{exp}$ (KeV) | $\Gamma_{th}$ (KeV) | $\Gamma^{Mang}_{R_c=8.2 fm}$ (KeV) | $HF_{exp}$ | $HF_{th}$ | $-V_0$ (MeV) | $a$ (fm) | $r_0$ (fm) |
|---|---|---|---|---|---|---|---|---|---|---|
| $0^+$ | 0 | 0 | $3.3.IO^{-29}$ | $5.68.IO^{-30}$ | $8.7.IO^{-31}$ | I | I | IO7.2 | 0.57 | I.I8 |
| $2^+$ | 803 | 802 | $2.2.IO^{-33}$ | $I.3I7.IO^{-35}$ | $9.8.IO^{-37}$ | I.4 | 5.7 | III.9 | 0.57 | I.I8 |
| $0^+$ | II65 | IO70 | - | $I.49.IO^{-37}$ | - | - | 2.8 | 92.42 | 0.75 | I.I5 |
| $2^+$ | I460 | I427 | - | $6.73.IO^{-42}$ | - | - | 78.7 | II7 | 0.75 | I.I5 |
| $2^+$ | I784 | I7I8 | - | $4.07.IO^{-46}$ | - | - | 70.0 | 73 | 0.75 | I.I5 |
| $2^+$ | 2I49 | 208I | - | $I.44.IO^{-48}$ | - | - | II7 | IIO | 0.75 | I.I5 |
| $0^+$ | 23I4 | 2208 | - | $7.45.IO^{-49}$ | - | - | III | I27 | 0.75 | I.I5 |



## 6. CONCLUSIONS

One may ask what such a rather complex theory as represented by eq. (38) is good for. The answer is that the experiments can yield the informations (applying this theory) concerning:

- complicated structure of the excited nuclear states,
- complicated mechanism of the $\alpha$ -decay process, involving even the complicated nucleon-nucleon interactions, responsible for the clustering phenomenon and to the emission of the $\alpha$ -particle,
- complicated virtual excitations of other $\alpha$ -channels after the fragment breaking.

All the problems together can be compared with the recent nuclear matter and shell model calculations, which are also becoming more and more complicated. In any case our treatment can answer in the future the question which forces are predominant in the decay process, hence, the $\alpha$-decay becomes a tool for studying the nuclear forces.

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