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Изучение ангармоничности вибрационных состояний с $K^\pi = 2^+, 0^-, 1^-, 2^-$ в четно-четных ядрах с $228 \leq A \leq 240$

Изучено влияние взаимодействия квазичастиц с фононами на энергию и структуру состояний с $K^\pi = 2^+, 0^-, 1^-, 2^-$ для ядер в области $228 \leq A \leq 240$. Показано, что первые состояния являются, в основном, однофононными, а многие вторые состояния имеют сложную структуру. Если фиксировать значения констант $\kappa^{(\lambda)}$ мультиполь-мультипольных взаимодействий с $\lambda = 2$ и 3 из условия описания экспериментальных данных для первых состояний с указанными K^π, то учет ангармоничности приводит к постоянству $\kappa^{(\lambda)}$ внутри зоны по A.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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On the Anharmonicity of Vibrational States with $K^\pi = 2^+, 0^-, 1^-$ and 2^- in Doubly Even Deformed Nuclei $228 \leq A \leq 240$

The influence of the quasiparticle-phonon interaction on the energy and the structure of states with $K^\pi = 2^+, 0^-, 1^-, 2^-$ for nuclei in the region $228 \leq A \leq 240$ is studied. It is shown that the first states are mainly one-phonon, and most of the second states have complex structure. If the values of the constants $\kappa^{(\lambda)}$ of the multipole-multipole interaction with $\lambda = 2,3$ are fixed so as to reproduce the experimental data on the energies of the first K^π vibrational states, the consideration of the anharmonicity leads to stabilization of $\kappa^{(\lambda)}$ inside the zone in A.

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I. Introduction

The lowest vibrational states in the doubly-even deformed nuclei are calculated, as a rule, in the framework of the ideal vibrator model[1,2]. This is mainly due to the fact that the experimental data on the anharmonicity* of vibrational states and on the "two-phonon" states in deformed nuclei are very scarce[3].

The "two-phonon" states exist among a large number of two-quasiparticle, rotational and one-phonon excitations, and this "masking" makes it difficult to separate them experimentally. Nevertheless, the information on the two-phonon and second one-phonon states is being accumulated, though very slowly.

The theoretical investigations of the anharmonic effects in even deformed nuclei have been attempted long ago[4]. Different terms, which are anharmonic with respect to the phonon operators, naturally appear in the Hamiltonian of the superfluid nuclear model. Thus, the above mentioned effects have been consistently described in the framework of the model taking into account the quasiparticle-phonon interaction.

The first calculations by the model have been performed in ref.[5] with wave functions containing one- and two-phonon terms.

It has been found that admixtures of the two-phonon components in the first vibrational states of strongly deformed nuclei are small but they are increasing for the nuclei of transitional regions. These calculations have been recently repeated[6] making use of the Saxon-Woods potential instead of the Nilson one and with a

*In the present paper by "anharmonicity of vibrational states" we mean the deviation from harmonicity generated by the quasiparticle-phonon interaction.

somewhat complicated wave function as compared with that in ref.[5] By the example of nuclei from the $150 < A < 190$ region, it has been shown that the first vibrational states with $K^{\pi} \neq 0^{+}$ are practically one-phonon. The structure of the second states is, as a rule complicated and this is due to the quasiparticle-phonon interaction.

The aim of the present paper is to investigate the anharmonicity of the vibrational states with $K^{\pi}=2^{+},0^{-},1^{-},2^{-}$ in doubly even nuclei in the region $228 \leqslant A \leqslant 240$. The states with $K^{\pi}=0^{+}$ will be considered separately.

2. Formulation of the model

As is known, the Hamiltonian of the superfluid nuclear model includes the average field, the interactions, leading to superconducting pairing correlations, and the separable quadrupole-quadrupole and octupole-octupole interactions.

$$H = H_{av} + H_{pair} + H_{Q} . \qquad (1)$$

After carrying out the Bogolubov transformations and introducing the phonon operators, the main part of the Hamiltonian (I), taking into account the solutions of the secular equation for the one-phonon states, is of the form:

$$H = H_{\nu} + H_{\nu q} = \sum_{q} \omega_{q} Q_{q}^{+} Q_{q} - \frac{1}{2} \sum_{q\nu\nu'} [\Gamma^{q}_{(\nu\nu')} \sum_{\sigma} \pm \alpha^{+}_{\nu \pm \sigma} \alpha_{\nu'\sigma} \times \qquad (2)$$

$$\times (Q^{+}_{q} + Q_{q}) + h.c.] + \frac{1}{2} \sum_{\tau} G_{\tau} [\sum_{\nu\nu'} V_{\nu\nu'} U_{\nu\nu'} \times$$

$$\sum_{\sigma} \sigma \alpha^{+}_{\nu-\sigma} \alpha^{+}_{\nu\sigma} \sum_{\sigma'} \alpha^{+}_{\nu'\sigma'} \alpha_{\nu'\sigma'} + h.c.]\tau .$$

The following notations are used: $(\nu\sigma)$, $\sigma = \pm 1$ is a set of quantum numbers characterizing the single-particle states; G_{τ} is the coupling constant for the neutron or proton systems respectively.

$U_{\nu\nu'} = U_{\nu}V_{\nu'} + U_{\nu'}V$; $V_{\nu\nu'} = U_{\nu}U_{\nu'} - V_{\nu}V_{\nu'}$; Q^{+}_{q}, Q_{q} are the creation and annihilation operators the phonon q with moment and projection $q = \lambda\mu$, number i, and frequency ω_{q};

$$\Gamma^{q}_{(\nu\nu')} = \frac{f^{q}(\nu\nu')}{2\sqrt{Y_{q}}} V_{\nu\nu'},$$

where $f^{q}(\nu\nu')$ is the single-particle matrix element of the multipole operator q, Y_{q} is phonon characteristic (see ref.[1]).

The second and third terms in (2) describe the quasiparticle-phonon interaction, and they originate from H_{Q} and H_{pair} respectively. In the Hamiltonian (2) not only the collective but also the one-particle degrees of freedom are taken into account explicitly. Correspondingly their coupling is taken into account in the wave function too:

$$\Psi_n(K^\pi) = \Big[\sum_{i=1}^{3} C^n_{q_0 i} Q^+_{q_0 i} + \sum_{g_1 g_2} \Delta^{g_1}_{g_2}(q_0 n) Q^+_{g_1} Q^+_{g_2}\Big]\Psi_0 . \quad (3)$$

Here n is the number of the excited state, $Q_g \Psi_0 = 0$.

The coefficients C and Δ obey the normalization condition:

$$\sum_{i=1}^{3} (C^n_{q_0 i})^2 + 2 \sum_{g_1 g_2} [\Delta^{g_1}_{g_2}(q_0 n)]^2 = 1 . \quad (4)$$

In the wave function (3) the three-phonon and so on admixtures are not included, and this limits the range of applicability of the model. Obviously, the wave function of the type (3) will not be valid in the case of high excitation energies.

Under these assumptions the model permits an exact analytical solution.

The energies of the excited states are found from the secular equation:

$$\det \| (\omega_{q_0 i} - \eta_n)\delta_{ii'} - K_{ii'}(q_0 n) \| = 0 , \quad (5)$$

where

$$K_{ii'}(q_0 n) \equiv \frac{1}{2} \sum_{g_1 g_2} \frac{U^{g_1}_{g_2}(q_0 i) U^{g_1}_{g_2}(q_i')}{\omega_{g_1} + \omega_{g_2} - \eta_n} .$$

Here $U^{g_1}_{g_2}(q_0)$ is the matrix element Hvq between one-phonon and two-phonon $(\lambda_0 \mu_0 i_0);(\lambda_1 \mu_1 i_1; \lambda_2 \mu_2 i_2)$ states, and its explicit form is given in ref.[8]. The expressions for $C^n_{q_0 i}$ and $\Delta^{g_1}_{g_2}(q_0 n)$ are the following:

$$C^n_{q_0 i} = \frac{M^i}{\sqrt{\sum_{i'=1}^{3} (M^{i'})^2 + \frac{1}{2} \sum_{g_1 g_2} \Big(\sum_{i=1}^{3} \frac{U^{g_1}_{g_2}(q_0 i) M^{i'}}{\omega_{g_1} + \omega_{g_2} - \eta_n} \Big)^2}} = \frac{M^i}{N} \quad (6)$$

$$\Delta^{g_1}_{g_2}(q_0 n) = \frac{1}{2N} \sum_{i=1}^{3} \frac{U^{g_1}_{g_2}(q_0 i) M^i}{\omega_{g_1} + \omega_{g_2} - \eta_n} , \quad (7)$$

where M^i is the i-th minor of the matrix (5) ($i = 1,2,3$).

If $U^{g_1}_{g_2}(q_0) = 0$, then $\Delta^{g_1}_{g_2}(q_0 n) = 0$, $\eta_n = \omega_{q_0 n}$, $C^n_{q_0 i} = \delta_{in}$, i.e., the harmonic model is obtained.

3. Parameters of the model

The considered mass region has been divided into 2 zones of nuclei according to the chosen single- particle scheme:

a) ^{228}Th, ^{230}Th, ^{232}Th ^{232}U (A=229)

b) ^{234}U, ^{236}U, ^{238}U, ^{238}Pu, ^{240}Pu . (A=239)

The parameters of the Saxon-Woods potential and the constants C_τ have been taken from ref.[8]. The constants of the multipole-multipole interaction $\varkappa^{(\lambda)}$ are the only varying parameters in the calculations. They are chosen so that the first roots of the secular equation (5) describe correctly the experimental energies of the first vibrational states.(The experimental data on the energies of the low-lying vibrational states are systematized in reviews[9]).For each of the considered nuclei it was possible to find a set of constants $\{\varkappa^{(\lambda)}_{anharm.}\}$ allowing us to describe the energies of all the first vibrational states.The values of the constant $\varkappa^{(\lambda)}_{anharm.}$ are plotted in figs.1-8 versus A. They are compared with the values of the constants of the multipole-multipole interaction $\varkappa^{(\lambda)}_{harm.}$ for which the energies of the

first vibrational states, calculated in the harmonic approximation, coincide with the experimental ones. The isotopes belonging by the same single-particle scheme are given on the horizontal axis of each figure, and the vertical axis represents the values of $\varkappa^{(\lambda)}_{harm.}$ (connected by dashed line) and of $\varkappa^{(\lambda)}_{anharm.}$ (connected by solid line). It should be noted that the values $\{\varkappa^{(\lambda)}_{anharm.}\}$ as a function of A behave more smoothly than $\{\varkappa^{(\lambda)}_{harm.}\}$. " A tendency" of straightening" the solid broken line is clearly seen on figs.1,2 and 6.

It means that in the model by one set of constants $\{\varkappa^{(\lambda)}_{anharm.}\}$ a group of nuclei is described.

The effect of stabilization of constants $\{\varkappa^{(\lambda)}_{anharm}\}$ may be interpreted in the following way: the Hamiltonian (2) is more close to the "true" one than $H_\nu = \sum_q \omega_q Q^+_q Q_q$, and thus the coupling constants are determined better from the Hamiltonian (2).

IV. The results and discussion

The calculation results for the energies and wave functions of the vibrational nuclear states with $K^\pi = 2^+, 0^-, 1^-, 2^-$ in the region $228 \leqslant A \leqslant 240$ are given in Tables 1-9. In the column "structure" each line represents the one- and two-phonon components ($\lambda_0 K i$), ($\lambda_1 \mu_1 i_1; \lambda_2 \mu_2 i_2$) and their contributions $(C^n_{\lambda_0 K i})^2$, $4[\Delta^{\lambda_1 \mu_1 i_1}_{\lambda_2 \mu_2 i_2}(\lambda_0 K n)]^2$ (see eqs.(6) and (7)) to the state $\Psi_n(K^\pi)$ ($\pi = (-1)^{\lambda_0}$).

It is seen from the Tables that except for the first

$K^\pi = 2^+$ in ^{240}Pu all the first states are practically one-phonon ones, the total contribution of admixtures does not exceed 10%, i.e., the situation is the same as in the region $150 < A < 190$. As to the second vibrational states, it should be noted that just in their structure the anharmonic effects become evident. In most cases, the structure of the second states is very complicated some different components give a considerable contribution to the wave function. For instance, in ^{230}Th the second $K^\pi = 0^-$ state has the contribution of 87% of the component (201,301), 9% of (302), 3% of (301) and 1% of (303). The quasiparticle-phonon interaction distorts essentially the picture of harmonic nuclear vibrations mixing the collective degrees of freedom with the non-collective and two-phonon ones. Such a mixing is observed, e.g., in ^{232}Th (Table 3): the second $K^\pi = 2^+$, the second $K^\pi = 0^-$, the second $K^\pi = 2^-$ states.

In a number of cases the model predicts almost pure two-phonon structure of the second vibrational states. For instance, in ^{228}Th the component (201,301) gives 98% contribution to the second state.

It is difficult to compare the calculation results of the energies and the structure of the second vibrational states with the experiment due to scanty experimental data. When the energies are experimentally measured the theoretical energy values agree with them.

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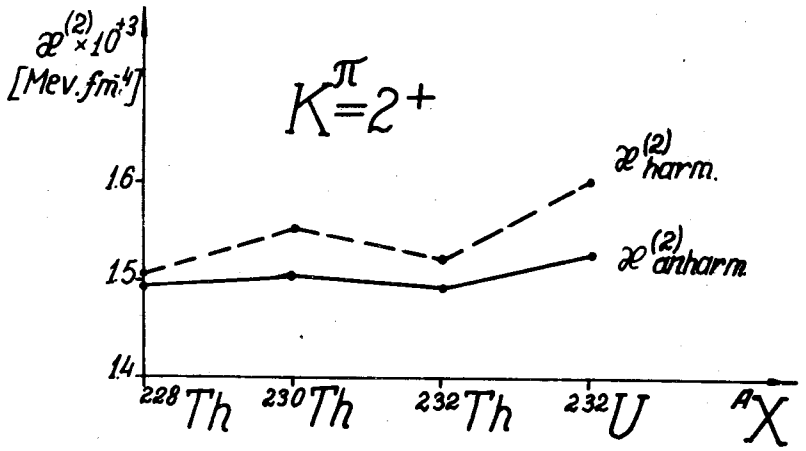

Fig. 1.

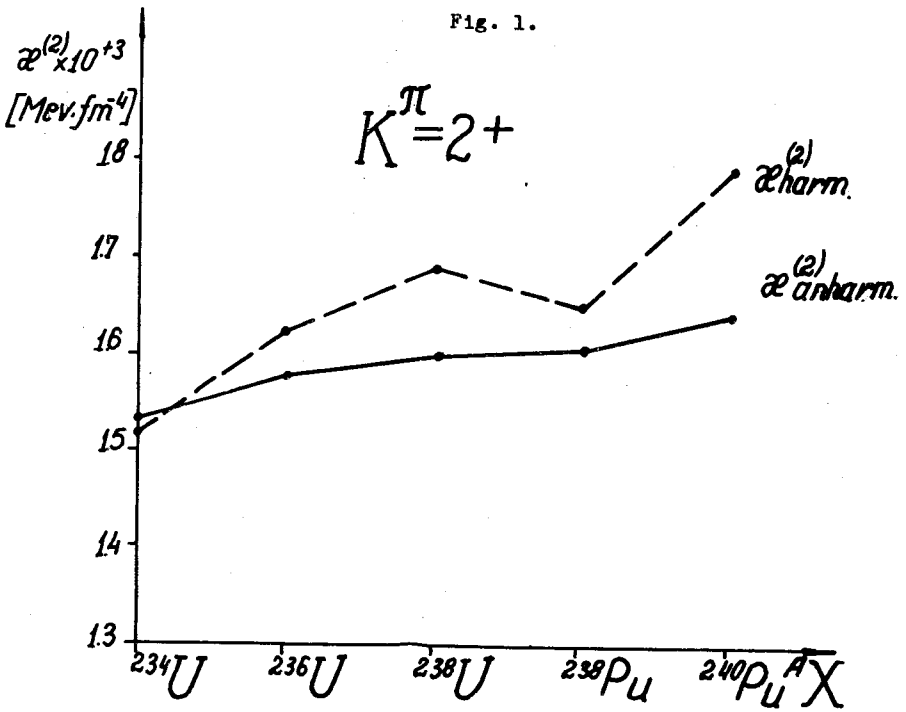

Fig. 2.

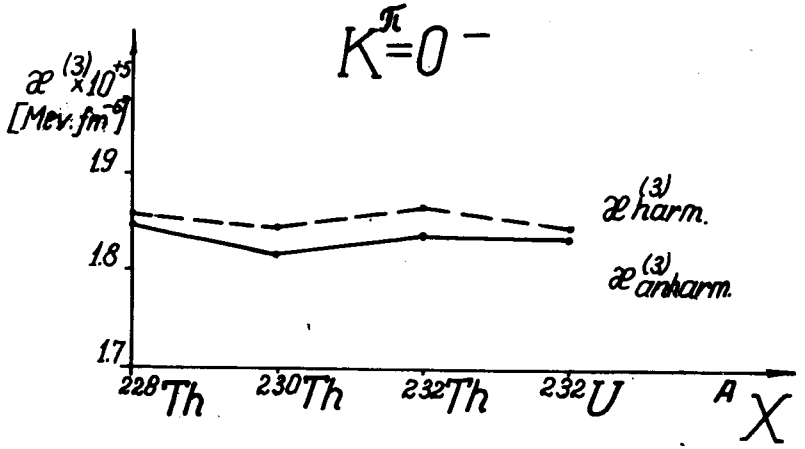

Fig. 3 .

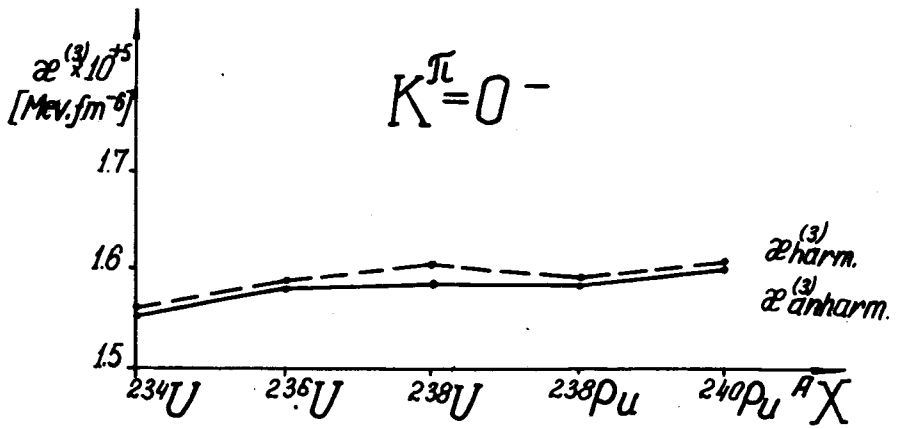

Fig. 4.

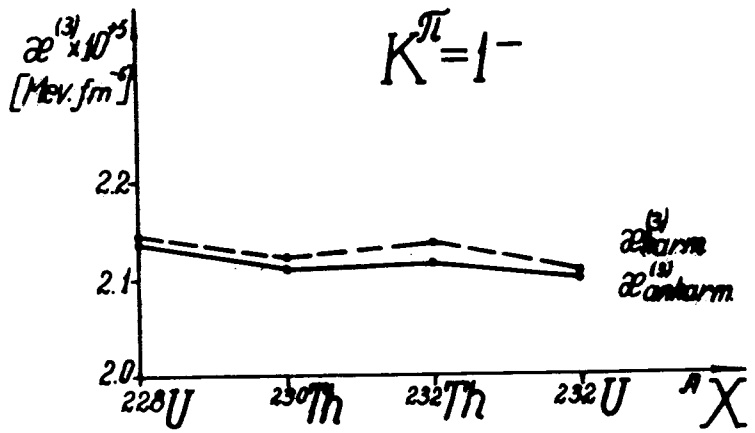

Fig. 5.

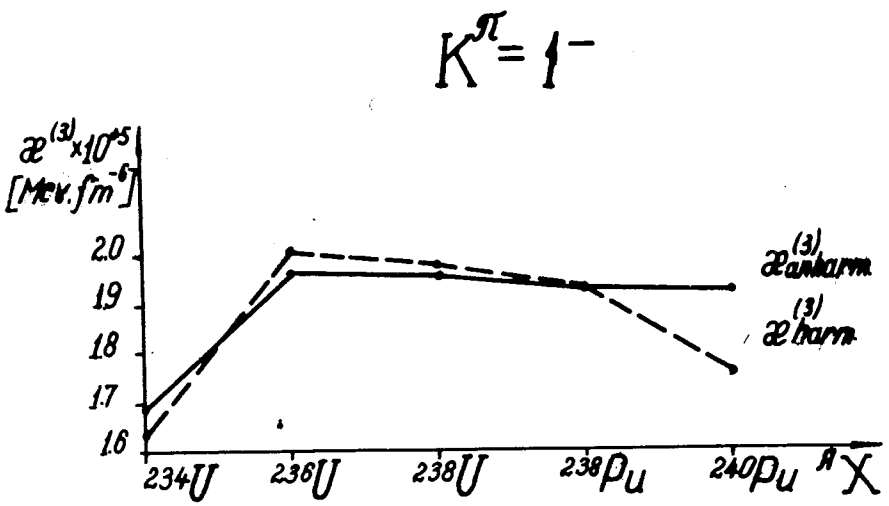

Fig. 6.

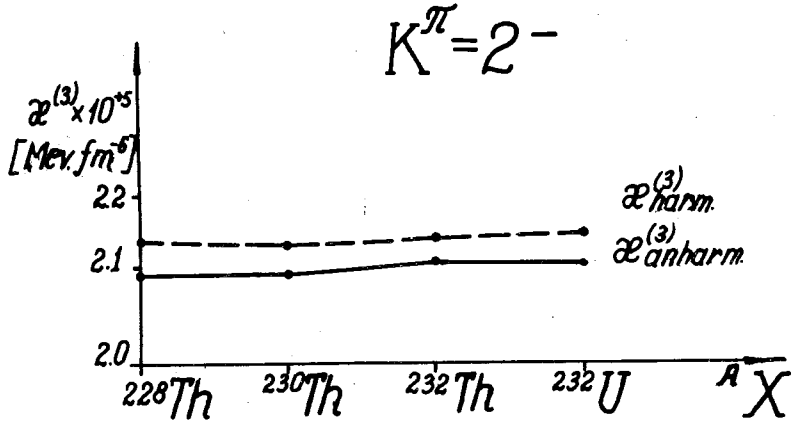

Fig. 7.

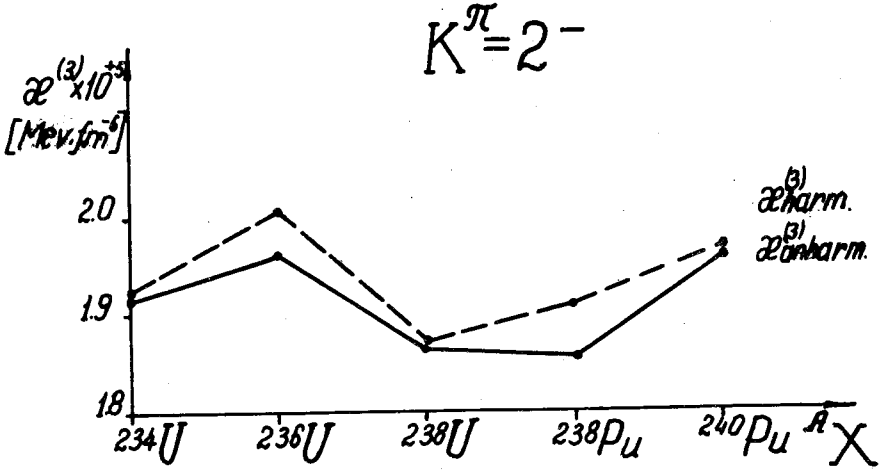

Fig. 8.

Table 1.

Lowlying vibrational levels in ^{228}Th

K^{π}_{n}	Energy, MeV exp.	Energy, MeV calc.	Structure, %
2^{+}_{1}	0,977	I,0	(22I)95(30I,32I)2
2^{+}_{2}	I,620	I,6	(3II, 3II) 98
0^{-}_{1}	0,328	0,4	(30I) 97 (20I, 30I) I
0^{-}_{2}		I,7	(20I, 30I) 99
I^{-}_{1}		0,7	(3II) 97 (20I, 3II) 2
I^{-}_{2}		I,8	(3I2) 90 (20I, 3II) 9
2^{-}_{1}	I.I23	I.2	(32I) 90 (22I, 30I) I0
2^{-}_{2}		I,5	(22I, 30I) 90 (32I) I0

Table 2.

Lowlying vibrational states in ^{230}Th

K^{π}_{n}	Energy, exp.	Energy, MeV calc.	Structure, %
2^{+}_{1}	0.782	0.9	(22I) 97 (30I, 32I) I
2^{+}_{2}		I.9	(30I, 32I) 98 (22I) I
0^{-}_{1}	0,508	0.6	(30I) 96 (20I, 30I) 3
0^{-}_{2}		I.9	(20I, 30I) 87 (302) 9(30I)3(303)I
I^{-}_{1}	0.954	0.9	(3II) 96 (20I, 3II) 2
I^{-}_{2}		I.8	(3I2) 97 (20I, 3I2) I
2^{-}_{1}		I.I	(32I) 94 (22I, 30I) 5
2^{-}_{2}		I.6	(22I, 30I) 95 (32I) 5

Table 3.

Lowlying vibrational levels in ^{232}Th

K^{π}_{n}	Energy, MeV exp.	Energy, MeV calc.	Structure, %
2^{+}_{1}	0.785	0.8	(22I)94(20I.22I)4(30I,32I)I
2^{+}_{2}		2.0	(222)67(30I,32I)29(20I,22I)I
0^{-}_{1}	0,7I3	0.8	(30I)94(20I,30I)4(202,30I)I
0^{-}_{2}		2.0	(22I,32I)95(303)2(302)2
I^{-}_{1}	I.0-I.I	I.0	(3II)96(20I,3II)2(22I,3II)I
I^{-}_{2}		I.9	(3I2)94(20I,3II) 4
2^{-}_{1}		I.0	(32I)95(22I,30I)3(20I,32I)2
2^{-}_{2}		I.8	(22I,30I)95(32I)3(322)I(323)I

Table 4.

Lowlying vibrational states in ^{232}U

K^{π}_{n}	Energy, exp.	Energy, MeV calc.	Structure, %
2^{+}_{1}	0.868	I.0	(22I)98 (20I,22I)I
2^{+}_{2}		2.0	(222)98 (20I,22I)I
0^{-}_{1}	0.564	0.8	(30I)98 (20I,30I)I
0^{-}_{2}		2.0	(302)95 (20I,30I)I
I^{-}_{1}		I.3	(3II) 99
I^{-}_{2}		I.8	(3I2) 99
2^{-}_{1}	(I.0I9)	I.3	(32I)98 (22I,30I) 2
2^{-}_{2}		I.9	(22I,30I)98 (32I) 2

Table 5.

Lowlying vibrational states in ^{234}U

K^{π}_{n}	Energy, exp.	MeV calc.	Structure, %
2^{+}_{I}	0.927	0.9	(22I)96 (30I,32I)2 (202,22I) I
2^{+}_{2}		I.7	(222)85(223)8(30I,32I)4(20I,222)2(20I,223)I
0^{-}_{I}	0.785	0.8	(30I)95(20I,30I)I(22I,32I)I
0^{-}_{2}		I.7	(302)9I(202,30I)3(20I,302)2(20I,30I)2
I^{-}_{I}	(I.436)	I.2	(3II)97 (20I,3I3) 2
I^{-}_{2}		I.6	(3I2) 98
2^{-}_{I}	0.989	I.0	(32I)96 (22I,30I) 3
2^{-}_{2}		I.7	(322)62 (22I,30I)37 (32I)I

Table 6.

Lowlying vibrational states in ^{236}U

K^{π}_{n}	Energy, exp.	MeV calc.	Structure, %
2^{+}_{I}	0.959	0.9	(22I)90 (30I,32I)5 (20I,22I) 4
2^{+}_{2}		I.4	(222) 98
0^{-}_{I}	0.685	0.7	(30I)96 (20I,30I)I (22I,32I)I
0^{-}_{2}		I.8	(302)82 (2I0,30I)I3 (202,30I)I
I^{-}_{I}	0.970	0.8	(3II) 98
I^{-}_{2}		I.6	(3I2)96 (20I,3II) 3
2^{-}_{I}		0.8	(32I)94(20I,32I)3 (22I,30I) 3
2^{-}_{2}		I.7	(322)64(22I,30I)24(323)4(20I,32I)3

Table 7.

Lowlying vibrational states in ^{238}U

K^{π}_{n}	Energy, exp.	MeV calc.	Structure, %
2^{+}_{I}	I.06I	I.I	(22I)84(20I,22I)9(222)4(20I,222)I
2^{+}_{2}		I.4	(222)93 (22I)6
0^{-}_{I}	0.680	0.8	(30I)97 (20I,30I) 2
0^{-}_{2}		I.7	(302)94 (20I,30I) 2
I^{-}_{I}	0.93I	0.9	(3II)97 (20I,3II) 2
I^{-}_{2}		I.7	(3I2)87 (20I,3II)I0(20I,3I2) 2
2^{-}_{I}		I.I	(32I)97 (20I,32I) I
2^{-}_{2}		I.8	(322)97 (20I,32I) 2

Table 8.

Lowlying vibrational states in ^{238}Pu

K^{π}_{n}	Energy, exp.	MeV calc.	Structure, %
2^{+}_{I}	I.028	I.0	(22I)93 (20I,22I)4 (30I,32I) 2
2^{+}_{2}		I.4	(222) 99
0^{-}_{I}	0.605	0.6	(30I) 98
0^{-}_{2}		I.6	(302) 94
I^{-}_{I}	0.963	I.0	(3II) 99
I^{-}_{2}		I.6	(3I2) 99
2^{-}_{I}	(I.3I0)	I.3	(32I)88 (22I,30I)9 (20I,32I) 2
2^{-}_{2}		I.7	(22I,30I)6I (322)30 (32I)8

Table 9.

Lowlying vibrational states in ^{240}Pu

K^{π}_{n}	Energy, MeV exp.	calc.	Structure, %
2^{+}_{I}	0.938	I.0	(22I)74(30I,32I)8(20I,22I)8(222)7(20I,222)2
2^{+}_{2}	I.559	I.4	(222)88 (22I)I0 (20I,222) I
0^{-}_{I}	0.597	0.6	(30I)96 (20I,30I) 2
0^{-}_{2}		I.5	(302)8I (20I,30I) I8
I^{-}_{I}		I.0	(3II)99 (20I,3II) I
I^{-}_{2}		I.7	(3I2) 98
2^{-}_{I}	0.959	0.9	(32I)92(20I,32I)4(22I,30I) 2
2^{-}_{2}		I.9	(22I,30I)69(322)I4(323)I3(32I)I(20I,32I)I

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