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PARTICLE AND SUBPARTICLE QUANTUM MODELS

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1. Extensions of the Dyson mapping

The Dyson mapping of fermion pairs onto bosons is well known [1]. Its generalisation by Janssen et al. [2] has been applied in an increasing number of publications to the foundation of the interacting boson model (IBM) [3] and interacting boson fermion model (IBFM) [4] by fermion pairs introduced in the shell model (SM) and in the fermion dynamical symmetry model (FDSM) [5], called by us also interacting fermion model (IFM). Recently the possibility of obtaining analogues of the Dyson mapping for boson pairs [6-8], for boson and fermion pairs and for fermion triplets has been shown, and the general possibility for its extensions to any numbers of bosons and fermions has been noticed [7,8]. Here we are going to show the existence of such extensions to any numbers of bosons and fermions and to demonstrate their importance for obtaining the relation between particle and subparticle nuclear models (PSNM), and generally particle and subparticle quantum models (PSQM).

Suppose we consider composite particles with their annihilation $A_{i_1 i_2 .. i_s}$ and creation $A^{(+)}_{i_1 i_2 ... i_s}$ operators, of s fermion subparticles with their annihilation a_i and creation a_i^+ operators, being antysymmetric with respect to any two indices permutation. The index i represents all quantum numbers describing a single fermion subparticle state. Let them satisfy the commutation $[-,-]$ or anticommutation $\{-,-\}$ relations:

$$(A_{i_1 i_2 ... i_s}, A_{j_s j_{s-1} ... j_1}) = 0$$

$$(A^{(+)}_{i_1 i_2 ... i_s}, A^{(+)}_{j_s j_{s-1} ... j_1}) = 0 \qquad (1)$$

$$(A_{i_1 i_2 \dots i_s}, A^{(+)}_{j_s j_{s-1} \dots j_1}) =$$

$$\sum_{p^{js}} p^{js} \delta_{i_1 j_1} \delta_{i_2 j_2} \dots \delta_{i_s j_s}$$

where $(-,-) = [-,-]$ if s is even and $(-,-) = \{-,-\}$ if s is odd.

$$\sum_{p^{js}} p^{js} Q(i_1 j_1, i_2 j_2, \dots, i_s j_s) =$$

$$\sum_{p^{js}} (-)^{p^{js}} Q(i_1 j_{h_1^j}, i_2 j_{h_2^j}, \dots, i_s j_{h_s^j}) \qquad (2)$$

Here p^{js} means a permutation $j_{h_1^j} j_{h_2^j} \dots j_{h_s^j}$ of the $j_1 j_2 \dots j_s$ indices only; $(-)^{p^{js}} = +$ if if the permutation is even, $(-)^{p^{js}} = -$ if the permutation is odd; $\sum_{p^{js}}$ represents the sum over all such permutations. Formulae (1) mean that A, $A^{(+)}$ satisfy "ideal" boson commutation relations if s is even, respectively "ideal" fermion anticommutation relations if s is odd. We are going to call A, $A^{(+)}$ quasibosons, respectively quasifermions, having in mind that $A^{(+)}_{i_s \dots i_1}$ is generally not the hermition conjugate operator of $A_{i_1 \dots i_s}$.

Let us denote the product of s fermion annihilation operators as

$$a_{i_1} a_{i_2} \dots a_{i_s} = A_{i_1 i_2 \dots i_s}$$

Then by using (1) and the fermion a_i, a_i^+ anticommutation relations, we derive the extentions of the Dyson mapping as follows:

$$a^+_{i_1} a^+_{i_2} \dots a^+_{i_r} a_{j_1} a_{j_2} \dots a_{j_r} =$$

$$\sum_{k=1,2,\dots r;\ P_k^{ir},\ P_k^{jr}} (-)^{[k/2]} P_k^{ir} P_k^{jr} a^+_{i_1} a_{j_1} \dots a^+_{i_k} a_{j_k} \delta_{i_{k+1} j_{k+1}} \dots \delta_{i_r j_r} =$$

$$\sum_{\substack{k=1,2,\dots r;\ P_k^{ir},\ P_k^{jr} \\ l_1^{(1)} \dots l_{s-1}^{(1)} \\ \vdots \\ l_1^{(k)} \dots l_{s-1}^{(k)}}} \frac{(-)^{[k/2]}}{[(s-1)!]^k} P_k^{ir} P_k^{jr} \tag{3}$$

$$A^{(+)}_{i_1 l_1^{(1)} \dots l_{s-1}^{(1)}} A_{l_{s-1}^{(1)} \dots l_1^{(1)} j_1} \dots A^{(+)}_{i_k l_1^{(k)} \dots l_{s-1}^{(k)}} A_{l_{s-1}^{(k)} \dots l_1^{(k)} j_k} \delta_{i_{k+1} j_{k+1}} \dots \dots \delta_{i_r j_r}$$

$r = 1,2,\dots,s$.

Here $[k/2]$ is the integer part of $k/2$ and P_k^{ir}, P_k^{jr} are obtained by (2) with the following limitations on their permutations

$$h_1^{\,i} < \dots\dots < h_k^{\,i}\ ,\ h_{k+1}^{\,i} < \dots\dots < h_r^{\,i}$$
$$h_1^{\,j} < \dots\dots < h_k^{\,j}\ ,\ h_{k+1}^{\,j}\ ,\ \dots\dots\ ,\ h_r^{\,j}\ . \tag{4}$$

It means that P_k^{ir} induces only the $\binom{r}{k}$ combinations of the i indices: r elements k-th klass. P_k^{jr} induces the same combinations of the j indices together with the $(r-k)!$ permutations of the last $r-k$ indices $h_{k+1}^{\,j} \dots h_r^{\,j}$. We see that the number of sums in (3) of given r and k but different P_k^{ir}, P_k^{jr} type *is* obtained by the number of combinations P_k^{ir} being $\binom{r}{k}$ times the number of combinations with permutations P_k^{jr} being $\binom{r}{k}(r-k)!$, i.e. it is $\binom{r}{k}^2(r-k)!$.

Let us notice that if we had composite particles $B_{k_1 \ldots k_s}$, $B^{(+)}_{k_1 \ldots k_s}$ instead of $A_{i_1 \ldots i_s}$, $A^{(+)}_{i_1 \ldots i_s}$ of boson subparticles b_k, b_k^+ instead of a_i, a_i^+ , being symmetric with respect to indices permutations, we would obtain the same formulae (1-4) with the following additional changes: $(-,-)$ into $[-,-]$, and the signs $(-)^{[k/2]}$ into $(-)^{r-k}$, $(-)^{P_k^{ir}}$, $(-)^{P_k^{jr}}$ into + . This means that B, $B^{(+)}$ would be quasibosons for any s . Let us also notice that the general extensions to composite particles of s^f fermion and s^b boson, or altogether s subparticles, are straightforward. This will be evident from the particular case s = 2 in section 4. A particle of $s = s^f+s^b$ fermions and bosons will be a quasiboson if s^f is even or quasifermion if s^f is odd.

The usual form of the Dyson mapping can be obtained from (3) by transforming all the A, $A^{(+)}$ products into normal form by the well-known Wick's theorem, which states that their usual product is the sum of all normal products (all $A^{(+)}$ to the left, all A to the right-hand side) with any contractions. This will be shown in the particular cases of fermion and boson pairs in section 4 and of fermion triplets in section 5. The advantage of the last transformation is that it is more convenient to transform the physical operators, e.g. hamiltonian and transition operators, from subparticle into particle form. Its drawback is that a greater number of different type terms appears.

2. Hermiticity problem

As we have seen in section 1, the well-known advantage of the Dyson mapping being finite is preserved in its extended analogues, although the number of terms increases with increasing number of subparticles. On the other hand its applications have been hindered by its known disadvantages, e.g. of transforming the hamiltonian matrix into a nonhermitian one. The mentioned drawback is due to the fact that the operator $A^{(+)}_{i_s \ldots i_1}$ is not the hermitian conjugate to $A_{i_1 \ldots i_s}$. Many publications have been devoted to this problem. Recently ways to its solution have been presented [9 - 12]. A possible development of [9] has been mentioned in [8] and is left to be realised in a future paper.

In principle this problem can be solved if one finds a nonunitary transformation:

$$A \longrightarrow ZAZ^{-1} = \bar{A} \qquad A^{(+)} \longrightarrow ZA^{(+)}Z^{-1} = \bar{A}^{+} \qquad (5)$$

such that $\bar{A}^{+}$ becomes the hermitian conjugate to $\bar{A}$. A very nice algebraic way to do it avoiding the additional difficulty with the infinite transformation Z has been suggested in a particular case in [9]. The problem is if one can extend it to a more general case.

3. Orthosymplectic supersymmetry

The unitary supersymmetry $U^{bf}(n/m)$ in n boson and m fermion dimensions has been suggested and shown to describe spectra of adjacent nuclei [13]. It has been judged to be the first evidence of existence of supersymmetry in nature, although not a fundamental one. Its generators are the upper rows of each square in figure 1.

$k,l = 1,2,\dots n$	
$\frac{1}{2}(b_k^+ b_l + b_l b_k^+)$	$b_k^+ a_i$
$b_k^+ b_l^+$, $b_k b_l$	$b_k a_i$
$a_i^+ b_k$	$\frac{1}{2}(a_i^+ a_j - a_j a_i^+)$
$a_i^+ b_k^+$	$a_i^+ a_j^+$, $a_i a_j$
	$i,j = 1,2,\dots m$

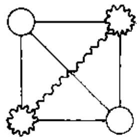

Figure 1. Supergroup $SpO^{bf}(2n/2m)$ generators in the l.h.s.; its superalgebra operation scheme in the r.h.s.: ○ —— commutation, ✲ ~~~ anticommutation.

A more general orthosympletic supersymmetry with the upper supergroup $OSp^{fb}(2m/2n)$ is known from elementary particle physics [14]. It has been proposed to be used in nuclear physics recently [15,7,8]. Its upper supergroup has been denoted also $SpO^{bf}(2n/2m)$ and generalised to a semidirect product with a Heisenberg-Weyle type supergroup [7,8]. Its generators and their superalgebra operations are shown in figure 1. Its subgroups $Sp^b(2n,R)$ denoted here by $Sp^b(2n)$ [16], respectively $O^f(2m)$ [17], have been considered about 20 years ago. Their generators are shown in the left upper, respectively right lower squares of the same figure. The embedding of their representations has already been discussed [18]. Thus its important chains are the following:

$$SpO^{bf}(2n/2m) \supset \begin{matrix} Sp^b(2n) \times O^f(2m) \\ \\ U^{bf}(n/m) \end{matrix} \supset U^b(n) \times U^f(m) \qquad (6)$$

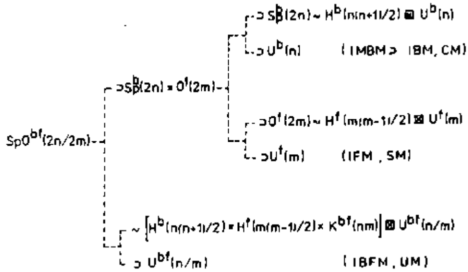

Figure 2. Supergroup $SpO^{bf}(2n/2m)$ lattice, including the upper (supper)groups of the known DSNM.

Let us notice that this suppersymmetry gives a group-theoretical method to classify and unify the well-known dynamic symmetry nuclear models (DSNM): IBM, IFM and IBFM, as shown schematically by its lattice in figure 2.

4. Interacting spinor model

The SM single particle states can be denoted by their quantum numbers as follows: $|nlm_l sm_s tm_t\rangle$ or $|nlsjm_j tm_t\rangle$. They have been transformed for the purpose of IFM into $|nkm_k im_i tm_t\rangle$, and the quantum numbers separated in a pseudoorbital part nkm_k which will be denoted shortly by k and pseudospin-isospin part $im_i tm_t$ denoted by i [5]. We have suggested a further step in this separation by avoiding the usual nucleon nm fermion a_{ki} operators and introducing instead subnucleon n pseudoorbital boson b_k and m pseudospin-isospin fermion a_i operators. Thus the nucleon is described by $b_k a_i$, i.e. by a subnucleon boson-fermion pair. This has given the name interacting spinor model (ISM) [7,8].

This DSNM will be described by the same $SpO^{bf}(2n/2m)$ orthosymplectic supergroup or by the semidirect product $\boxtimes$ of the Heisenberg-Weyle type and orthosympletic supergroups discussed in section 3. However the boson and fermion meaning will be quite different: the boson is related not to a pair of nucleons, but to a quasiorbital subnucleon, the fermion is related not to a nucleon but to quasispin-isospin subnucleon. Correspondingly the numbers of bosons n and of fermions m will be lower. If we consider the IFM $U^f(nm)$ subgroup and compare with it the ISM $U^{bf}(n/m)$ subsupergroup, we will see the advantage of ISM with respect to IFM by the lower number of dimensions and much lower of generators as follows:

Model	Nucleon operators	(Super) group	Dimensions	Generators
IFM	a_{ki}	U(nm)	nm	$(nm)^2$
ISM	$b_k a_i$	U(n/m)	n+m	$(n+m)^2$

We are going to obtain the EDM to boson and fermion pairs. We will use the results of section 1 for antisymmetric fermion pair quasibosons A_{ij} and symmetric boson pair quasibosons B_{kl}, and generalise them to include boson-fermion pair quasifermions as well: $C_{ki} = b_k a_i$, $C^{(+)}_{ik}$, satisfying the fermion anticommutation relations:

$$\begin{aligned} &\left\{ C_{ki}, C_{lj} \right\} = 0 \\ &\left\{ C^{(+)}_{ik}, C^{(+)}_{jl} \right\} = 0 \\ &\left\{ C_{ki}, C^{(+)}_{jl} \right\} = \delta_{ij}\delta_{kl} \end{aligned} \qquad (7)$$

Extending (3) of section 1 for A_{ij} , B_{kl} to include C_{ki}, and applying the Wick's theorem to the result, we obtain the following EDM:

$$a_i a_j = A_{ij} \qquad b_k b_l = B_{kl} \qquad b_k a_i = C_{ki}$$

$$a^+_i a^+_j = A^{(+)}_{ij} - \sum_{gh} A^{(+)}_{ig} A^{(+)}_{hj} A_{gh} + \sum_{mn} C^{(+)}_{im} C^{(+)}_{jn} B_{mn} - $$

$$- \sum_{hm} \left[A^{(+)}_{ih} C^{(+)}_{jm} + C^{(+)}_{im} A_{hj} \right] C_{mh}$$

$$b^+_k b^+_l = B^{(+)}_{kl} + \sum_{mn} B^{(+)}_{km} B^{(+)}_{nl} B_{mn} - \sum_{gh} C^{(+)}_{gk} C^{(+)}_{hl} A_{gh} + $$

$$+ \sum_{hm} \left[B^{(+)}_{mk} C^{(+)}_{hl} + C^{(+)}_{hk} B^{(+)}_{lm} \right] C_{mh} \qquad (8)$$

$$a_i^+b_k^+ = C_{ik}^{(+)} - \sum_{gh} A_{ig}^{(+)}C_{hk}^{(+)}A_{gh} + \sum_{mn} C_{im}^{(+)}B_{nk}^{(+)}B_{mn}$$
$$+ \sum_{hm} \left[A_{ih}^{(+)}B_{mk}^{(+)} + C_{im}^{(+)}C_{hk}^{(+)}\right]C_{mh}$$

$$a_i^+a_j = \sum_h A_{ih}^{(+)}A_{hj} + \sum_m C_{im}^{(+)}C_{mj}$$

$$b_k^+b_l = \sum_m B_{km}^{(+)}B_{ml} + \sum_h C_{hk}^{(+)}C_{lh}$$

$$a_i^+b_k = \sum_h A_{ih}^{(+)}C_{kh} + \sum_m C_{im}^{(+)}B_{km}$$

$$b_k^+a_i = \sum_m B_{mk}^{(+)}C_{mi} + \sum_h C_{hk}^{(+)}A_{hi}$$

It is constructed so that $A, A^{(+)}$ satisfy the boson commutation relations (1), B , $B^{(+)}$ satisfy the boson commutation relations (1) with the modification $(-)^{p^{js}} \longrightarrow +$, but $C, C^{(+)}$ satisfy the fermion anticommutation relations (7). However operators of different $A,A^{(+)}$ or $B,B^{(+)}$ or $C,C^{(+)}$ type commute. Nevertheless the relations of the $SpO^{bf}(2n/2m)$ supergroup generators of figure 1 are reproduced.

In the particular case of the known Dyson mapping for fermion pairs only (8) is reduced to its first, second and fifth formulae with terms including $A,A^{(+)}$ only. For its extension to boson pairs only (8) is reduced to its first, third and sixth formulae with terms including $B,B^{(+)}$ only. For its extension to boson-fermion pairs only (8) is reduced to its first, fourth, fifth and sixth formulae with terms including $C,C^{(+)}$ only.

We are going to show that ISM is able to yield foundation of the known DSNM: IBM, IFM, IBFM. Let us consider the physical operators, e.g. hamiltonian and transition operators. If they are r-body subparticle operators, they transform into r-body particle operators according to formula (3) and its transformation into normal form by the Wick's theorem (section 1).

A simple and known example is the transformation of the nucleon two-body fermion hamiltonian of the SM into a nucleon pair two-body quasiboson hamiltonian of the IBM. It can be obtained by the usual Dyson mapping with the only A, $A^{(+)}$ including terms of formulae (8). Thus it will give a known way to derive the IBM from the SM.

We illustrate the situation by a slightly more complicated example of a subnucleon two-body one-boson and one-fermion operator T of the ISM in a nucleon two-body boson and fermion operator of an IBFM. Let the original one of ISM be:

$$T = \sum_{\substack{ik\\lj}} \tau_{ik,lj} a_i^+ b_k^+ b_l a_j \tag{9}$$

The transformed one is obtained directly by substituting the $b_k a_i$ and $a_i^+ b_k^+$ formulae of (8) into (9):

$$T = \sum_{\substack{ik\\lj}} \tau_{ik,lj} \Big\{ C_{ik}^{(+)} C_{lj} + \sum_{hm} \Big[C_{im}^{(+)} C_{hk}^{(+)} + A_{ih}^{(+)} B_{mk}^{(+)} \Big] C_{mh} C_{lj} - \sum_{gh} A_{ig}^{(+)} C_{hk}^{(+)} A_{gh} C_{lj} + \sum_{mn} C_{im}^{(+)} B_{nk}^{(+)} B_{mn} C_{lj} \Big\} \tag{10}$$

The pseudofermions C, $C^{(+)}$ describe separate nucleons as in the SM. The pseudobosons B, $B^{(+)}$ are analogues to the quasiorbital nucleon pairs of the IFM. The pseudobosons A, $A^{(+)}$ are analogues to the quasispin-isospin nucleon pairs of the IFM. This means that we can obtain the foundation of an extension of the IFM, giving a new type of IBFM.

5. Quark nuclear-plasma model

There is a high number of quark models in the elementary particle physics, some of them applied in nuclear physics as well [19], including the quark bag model. An attempt to create a quark shell model of the nucleus has also been performed. References about it, as well as its comparison with the classical nucleon shell model, can be found in [20]. Their drawback from the view point of nucleus description is that they do not give a direct dynamical mechanism of the quark condensation into triplets representing nucleons, except by some imposed conditions, e.g. of allowed colour combinations. Quark confinement interactions alone do not give a clear solution of the same problem either. They can do it indirectly if they contain some colour hidden dynamics to make them saturating [21,22].

In this situation the idea to construct a quark triplet analogue to the nucleon pairing operator seems an important one [23]. We have used this idea to suggest a colour explicit dynamics by a quark triplet interaction, analogue to the nucleon pairing interaction, which might solve dynamically the problem of obtaining a quark condensate into nucleons in the nucleus [7,8]. As we have mentioned in section 4 and as we are going to show here, the EDM will lead generally up to a three body nucleon interaction. The necessity of such interactions in quark models starts to be recognised [24].

Now we formulate the idea of a nonrelativistic quark nuclear-plasma model (QNPM). Let us denote the quark single particle states by their quantum numbers as follows: $|nlm_l sm_s tm_t c\rangle = |i\rangle$, where to the orbital nlm_l and spin-isospin $sm_s tm_t$ we have added the colour quantum number c of the u and d quarks, having as usual three values: red (r), green (g) and blue (b). We also represent $|i\rangle$ as $|i^o\, i^c\rangle$ where i^o is only the orbital and spin-isospin part of the quantum numbers $nlm_l sm_s tm_t$, and i^c – their colour part c.

We introduce the quark triplet operator Q by analogy with the nucleon pairing operator P:

$$Q_{i_1^o i_2^o i_3^o} = \sum_{i_1^c\, i_2^c\, i_3^c} q_{i_1^c i_2^c i_3^c}\, a_{i_1} a_{i_2} a_{i_3} \qquad (11)$$

It is chosen so that it has white colour, i.e. all the three quarks have different colour, and it corresponds to the nucleon representations of the $U^c(3)$ colour group.

Finally we write the quark up to three body hamiltonian as follows:

$$H = \sum_{ij} \xi_{i,j} a_i^+ a_j + \sum_{i_1 i_2} \eta_{i_1 i_2, j_1 j_2}\, a_{i_1}^+ a_{i_2}^+ a_{j_1} a_{j_2}$$
$$+ \sum_{\substack{i_1^o i_2^o i_3^o \\ j_1^o j_2^o j_3^o}} \zeta_{i_1^o i_2^o i_3^o, j_1^o j_2^o j_3^o}\, Q^+_{i_3^o i_2^o i_1^o} Q_{j_1^o j_2^o j_3^o} . \qquad (12)$$

We notice the possibility of including gluons in the formalism as well.

We are going to obtain the EDM to fermion triplets. We apply directly the EDM of section 1 to the particular case of $s = 3$ antisymmetric quasifermion triplets $A_{i_1 i_2 i_3}$, $A^{(+)}_{i_1 i_2 i_3}$, satisfying the fermion anticommutation relations (1) with $(-,-) = \{-,-\}$. Applying the Wick's theorem to (3), we obtain the following EDM:

$$a_{i_1} a_{i_2} a_{i_3} = A_{i_1 i_2 i_3}$$

$$a^+_{i_1} a^+_{i_2} a^+_{i_3} = A^{(+)}_{i_1 i_2 i_3}$$

$$- \frac{1}{2} \sum_{l_1 l_2 l_3} \Big[A^{(+)}_{i_1 l_1 l_2} A^{(+)}_{i_2 i_3 l_3}$$

$$+ A^{(+)}_{i_2 l_1 l_2} A^{(+)}_{i_3 i_1 l_3} + A^{(+)}_{i_3 l_1 l_2} A^{(+)}_{i_1 i_2 l_3} \Big] A_{l_1 l_2 l_3}$$

$$- \frac{1}{8} \sum_{\substack{l_1 l_2 l_3 \\ l_4 l_5 l_6}} A^{(+)}_{i_1 l_1 l_2} A^{(+)}_{i_2 l_3 l_4} A^{(+)}_{i_3 l_5 l_6} A^{(+)}_{l_1 l_2 l_3} A^{(+)}_{l_4 l_5 l_6} \qquad (13)$$

$$a^+_{i_1} a^+_{i_2} a_{j_1} a_{j_2} = \sum_{l} A^{(+)}_{i_1 i_2 l} A^{(+)}_{l j_1 j_2}$$

$$+ \frac{1}{4} \sum_{\substack{l_1 l_2 \\ l_3 l_4}} A^{(+)}_{i_1 l_1 l_2} A^{(+)}_{i_2 l_3 l_4} A_{l_1 l_2 j_1} A_{l_3 l_4 j_2}$$

$$a^+_i a_j = \frac{1}{2} \sum_{l_1 l_2} A^{(+)}_{i l_1 l_2} A_{l_2 l_1 j} .$$

Applying the EDM (13) to the three-body quark hamiltonian (12) of the QNPM, we obtain its transformation into the following generally three-body hadron hamiltonian:

$$H = \sum_{ij} \xi_{i,j} \frac{1}{2} \sum_{l_1 l_2} A^{(+)}_{i l_1 l_2} A_{l_2 l_1 j}$$

$$+ \sum_{\substack{i_1 i_2 \\ j_1 j_2}} \eta_{i_1 i_2, j_1 j_2} \Big\{ \sum_{l} A^{(+)}_{i_1 i_2 l} A_{l j_1 j_2}$$

$$+ \frac{1}{4} \sum_{\substack{l_1 l_2 \\ l_3 l_4}} A^{(+)}_{i_1 l_1 l_2} A^{(+)}_{i_2 l_3 l_4} A_{l_1 l_2 j_1} A_{l_3 l_4 j_2} \Big\}$$

$$+ \sum_{\substack{i_1 i_2 i_3 \\ j_1 j_2 j_3}} \zeta_{i_1^o i_2^o i_3^o, j_1^o j_2^o j_3^o} q^*_{i_3^c i_2^c i_1^c} q_{j_1^c j_2^c j_3^c} \Big\{ A^{(+)}_{i_1 i_2 i_3} A_{j_1 j_2 j_3} \qquad (14)$$

$$- \frac{1}{2} \sum_{l_1 l_2 l_3} \Big[A^{(+)}_{i_1 l_1 l_2} A^{(+)}_{i_2 i_3 l_3}$$

$$+ A^{(+)}_{i_2 l_1 l_2} A^{(+)}_{i_3 i_1 l_3} + A^{(+)}_{i_3 l_1 l_2} A^{(+)}_{i_1 i_2 l_3} \Big] A_{l_1 l_2 l_3} A_{j_1 j_2 j_3}$$

$$- \frac{1}{8} \sum_{\substack{l_1 l_2 l_3 \\ l_4 l_5 l_6}} A^{(+)}_{i_1 l_1 l_2} A^{(+)}_{i_2 l_3 l_4} A^{(+)}_{i_3 l_5 l_6} A_{l_1 l_2 l_3} A_{l_4 l_5 l_6} A_{j_1 j_2 j_3} \Big\} .$$

Choosing the quark interaction to include confinement, we can see that in the limit of high density and temperature the first sum in (12) (and (14)) will prevail and describe free quarks. On this sum will be imposed the second and third sums including interactions yielding corrections. Thus we obtain quark chaotic behaviour or plasma. On the other hand in the limit of low density and temperature the third sum of (14) (and (12)), with coefficients chosen so as to make the nucleon representations of the $U^c(3)$ colour group energetically favourable, will prevail and make the phase transition with quark condensation into nucleons predominant. In this way we obtain quark condensate into nucleons or nucleus.

Now we can see that in the second nuclear limit we obtain a relation between the nuclear SM and the QNPM. The first term of the third sum of (14) gives the single nucleon shell model, and the second term of the same sum: the nucleon -nucleon two body interaction. This yields a method to obtain a foundation of the nucleon SM on a quark level of the QNPM, or an opposite method to get some additional information about quark interactions of the QNPM from the well developed nucleon SM of the nucleus.

6. Relation of particle to subparticle quantum models

In conclusion let us notice that the method suggested in this work, based on the extensions of the Dyson mapping developed in sections 1 and 4, and illustrated by the relation between PSNM for the ISM in section 4 and for the QNPM in section 5, may have rather wider applications to the relations between PSQM in any many-body problem. This is due to the generality of the EDM as shown in sections 1 and 4, and to their applicability to any physical operator as demonstrated in sections 4 and 5.

In fact, any quantum model for particles consisting of s^f fermion and s^b boson subparticles can be derived by the EDM from the subparticle model. It will become a quasiboson model if s^f is even or a quasifermion model if s^f is odd. This situation is well known if the particle intrinsic degrees of freedom are considered to be frozen. The EDM give a method to take them into account. When we freeze them, we can see, e.g. from the first term of (10) or from the first term in the third sum of (14) that we return to free particles.

The point left for a future publication, which has been recently developed rather much in literature, as noticed in section 2, is the transformation of the physical operator matrices, so that hermitian are transformed again into hermitian.

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