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LIGHT NUCLEI AS CHIRAL SOLITON STATES AND NUCLEAR MATTER INCOMPRESSIBILITY PHENOMENOLOGY

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In[1] an approach is proposed by G. Brown, I. Durso and M. Johnson in which the MIT - bag radius is treated as a dynamical variable and the Roper- resonance energy as a first excited state in the quantized radial motion of the surface. The energy of the bag as a function of its radius R plays the role of the potential energy of a motion along the collective variable R. In such model some more difficult is the determination of the kinetic energy operator in the Schredinger equation. It is natural to assume that this operator is proportional to the second derivative with respect to R acting on the wave function. The last gives the amplitude of the size fluctuation of the system. Similar equation may be obtained in Hill-Wheeler-Griffin approach[2] and has been proposed by Dirac in the theory of the electrone[3].

The calculation of the effective mass in the kinetic operator is a common difficulty for these approaches. Usually the effective mass is a phenomenological parameter for the model or can be calculated from euristic assumptions.

For example let us consider the breathing mode of a piece of nuclear matter. There is no unique definition of the incompressibility K_A of a finite nucleus. It might be obtained as the second derivative of the energy per particle with respect to the radius parameter R at the equalibrium value R_0

$$K_A = R_0^2 \frac{\partial^2 (E/A)}{\partial R^2}\bigg|_{R_0} \tag{1}$$

In order to evaluate this expression, however, one has to known the dependence of the energy E on the radius R. Alternatively, a scaling transformation $r \rightarrow \lambda r$ applied to the single-particle wave function of the ground state, leads to the so called "scaling incompressibility"

$$K_A = \frac{\partial^2 (E/A)}{\partial \lambda^2}\bigg|_{\lambda=1} \tag{2}$$

The kinetic density around the point r is given by

$$\frac{mv^2}{2} = \frac{m}{2}\left(\frac{r}{R}\right)^2 \dot{R}^2 \tag{3}$$

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if the breathing mode conserves the homogeneity of the system. Then the total kinetic energy of a system with particle density distribution

$$\rho(r) = \frac{3A}{4\pi R^3} \cdot \theta(R-r) \tag{4}$$

containing A particles, is equal to

$$T = \int \rho(r)T(r)d^3r = \frac{1}{2}\dot{R}^2 \cdot \frac{3}{5}m_N A \tag{5}$$

The last equation corresponds to the effective mass

$$m_{eff} = \frac{3}{5}m_N A \tag{6}$$

Here m_N is the nucleon mass. The value (6) at $A=1$ has been accepted for the effective mass of the nucleon breathing mode in[1].

For the energy of the monopole vibrations we have

$$h\omega = \sqrt{\frac{K}{m_{eff}}} = \frac{1}{R_0}\sqrt{\frac{K_\infty}{\frac{3}{5}m_N}} = \frac{1}{r_0 A^{1/3}}\sqrt{\frac{K_\infty}{\frac{3}{5}m_N}} \tag{7}$$

which is proportional to $A^{-1/3}$.Consequently one can think that the incompressibility of the infinite homogeneous nuclear matter must be equal to the incompressibility of a single nucleon. This is seen from (7) if $r_0 \sim r_N$ and $A = 1$. These phenomenological speculations lead to the idea that the nucleon breathing mode must transform to nuclear breathing mode or, in other words, these two events are of the same nature. It seams that the Skyrme model is namely the model in the framework of which this idea can be realized. The dynamical variables, in terms of which both nucleons and nuclei are described, are boson fields obeying the Euler-Lagrange equations of motion corresponding to a chiral-invariant Lagrangian[4]

$$\mathcal{L} = \frac{F_\pi^2}{16} \cdot Tr(L_\mu L_\mu) + \frac{1}{32e^2} \cdot Tr\left[L_\mu, L_\nu\right]^2. \tag{8}$$

Here $L_\mu = U^+\partial_\mu U$ are the left currents and $U(\vec{x}) = exp(i\vec{\tau}\cdot\vec{\pi})$ is an $SU(2)$-matrix given by the pion isotriplet $\vec{\pi}(\vec{x})$ and $\vec{\tau}$ are the Pauli matrix.

F_π in (8) is the pion decay constant and its empirical value is 186.4 MeV. The constant e is a phenomenological parameter.

We can introduce a scaling transformation[5]

$$U(\vec{r}) \to U\left(\vec{r}e^{\lambda(t)}\right) \tag{9}$$

for the field U in the Skyrme Lagrangian. This transformation does not change the baryon charge and takes into account the size fluctuations of the system. Now the Lagrangian can be presented in the form:

$$L = \frac{1}{2}\dot{\lambda}^2 e^{-3\lambda}Q_2 + \frac{1}{2}\dot{\lambda}^2 e^{-\lambda}Q_4 - M_2 e^{-\lambda} - M_4 e^{\lambda} \tag{10}$$

After the canonical transformation and quantization procedure we have a Hamiltonian

$$\hat{H} = \frac{\hat{P}_\lambda^2}{2(Q_2+Q_4)} + \frac{1}{2}\lambda^2(M_2+M_4) + M_2 + M_4 \tag{11}$$

in harmonic approximation. The definition (2) gives for the incompressibility K of the soliton

$$K = M_2 + M_4 = M \tag{12}$$

which is equal to the soliton mass. The functionals M_2 and M_4 of the solution of the equations of motion are calculated in [6]. The integrals Q_2 and Q_4 determine the effective mass of the breathing mode vibration in the Skyrme model and are given in [6].

The frequances $h\omega$ of the breathing mode in light systems with baryon charge B are given in Table (in $F_\pi e$ units).

B	1	2	3	4	6	8	9	12
$h\omega$	0.31	0.27	0.24	0.23	0.20	0.18	0.17	0.15

In conclusion we can note that if the constants in the Skyrme model are chosen so that the mass difference $M_N - M_{N^*}$ (N^*-Roper) is equal to the experemental one the monopole vibration frequencies in nuclei essentially overestimate the known experimental results in light nuclei.

If we choose constants corresponding to the nucleon-Δ-isobar splitting then $h\omega$ are from 3 to 5 time greater then the experimental ones in this region of mass numbers.

The numerical results reproduce the $B^{-1/3}$-dependence, analogous to the $A^{-1/3}$ fenomenological dependence for more heavy nuclei.

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Легкие ядра как киральные солитоны
и феноменология ядерной сжимаемости

Вычислены частоты дыхательной моды легких ядер в SU(2)-модели Скирма. Для ядер с массовыми числами $A \leq 12$ расчет приводит к $\hbar\omega \sim A^{-1/3}$ - зависимости.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Light Nuclei as Chiral Soliton States and
Nuclear Matter Incompressibility Phenomenology

The frequences of the breathing mode have been calculated in framework of the SU(2)-Skyrme model. The calculations lead to $\hbar\omega_{breath} \sim A^{-1/3}$ behavior for light nuclei with mass number $A \leq 12$.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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