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J.Dobes*, S.P.Ivanova, R.V.Jolos, R.Pedrosa

BOSON MAPPING AND THE MICROSCOPIC COLLECTIVE NUCLEAR HAMILTONIAN

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[^0]1. Introduction

In recent years, methods that map many-fermion problems onto many-boson ones have been studied intensively. First established in the theory of spin waves $[1,2]$, the boson representation of the bifermion operators has been extended to general fermion systems [3-6]. Several approaches have been suggested to develop the boson representation techniques [7-12]. In nuclear physics, an aim of such studies is to obtain a microscopic support for the phenomenological models of nuclear collective states, like the interacting boson model (IBM). The bosonization of the fermion problem is thus accompanied by an identification of the subspace of relevant collective degrees of freedom and truncation of the full space to that subspace.

There are three classes of microscopic investigations of nuclear collective structure in terms of boson models. In the first one, the collective Hamiltonian is constructed within a simple underlying symmetry group [13]. Thus, the collective variables are fixed independently of the real physical problem. As a result, a considerable coupling of the collective and noncollective degrees of freedom may occur.

In the second class, one does not demand that the collective Hamiltonian has an exact symmetry group structure. Collective variables are determined so as to minimize the coupling of the collective and noncollective boson variables[14].

The third class ef-studies-represents an intermediate
approach $[8,9,15]$ in which, the coupling between the collective and noncollective variables is not fully minimized. Instead, the structure of the boson collective variables is taken from the lowest fermion pair in the Tamm-Dancoff approximation (TDA). Such an identification seems to give a reasonable definition of the collective subspace.

In this paper, we proceed within the last approach. We start with the BCS quasiparticle representation of the fermion problem. The truncation to the lowest quadrupole TDA pair is made. In the bosonization step, we develop a procedure based on the correspondence of the fermion and boson quadrupole collective states. Our approach goes beyond the $\operatorname{SU}(6)$ symmetry of the IBM which we obtain as a limiting case.

In sect.2, we discuss the mapping of the fermion quadrupole collective states and the pair fermion operator onto the boson ones. In this procedure, the norm matrix plays an important role and an approximate procedure how to calculate it is suggested. The boson image of the particle-hole operator is investigated in sect.3. In sect.4, the collective boson Hamiltonian is derived. Results of some calculations are presented in sect.5. At the present stage, we do not start with the microscopic fermion parameters. We rather inspect whether the present approach is able to give results which differ from the phenomenological IBM. Conclusions are given in sect. 6 .

## 2. Collective pair operator

In the TDA, the lowest quadrupole state is generated by the operatior
$A_{2 M}^{+}=\frac{1}{\sqrt{2}} \sum_{s t} \Psi_{s t}\left(\alpha_{s}^{+} \alpha_{t}^{+}\right)_{2 M}$.
Using $A_{2 M}^{+}$, we construct the subspace of the collective quadrupole states in the fermion space by the recurrence relation $[8,9]$

$$
\begin{align*}
|n v \Omega I M\rangle_{F} & \left.=\frac{1}{\sqrt{n}} \sum_{v_{1} \Omega_{1} I_{1} M_{1} m}\left(n-1 \quad v_{1} \Omega_{1} I_{1} ; 2 \mid\right\} n v \Omega I\right) \times \\
& \times\left(I_{1} M_{1} 2 m \mid I M\right) A_{2 m}^{+}\left|n-1 \quad v_{1} \Omega_{1} I_{1} M_{1}\right\rangle_{F} . \tag{2}
\end{align*}
$$

In the above, $11,1,0,2, M\rangle_{F}=A_{2 M}^{+}|0\rangle$ and ( $\left.n-1 v_{1} \Omega_{1} I_{1} ; 2 \mid\right\} n v \Omega I$ ) are the $\mathrm{L}=2$ boson fractional parentage coefficients (cfp). In the boson space, the $\mathrm{L}=2$ collective states are given analogously to eq.(2)

$$
\begin{align*}
& \left.\mid n v \Omega I M)_{B}=\frac{1}{\sqrt{n}} \sum_{v_{1} \Omega_{1} I_{1} M_{1} m}\left(n-1 v_{1} \Omega_{1} I_{1} ; 2 \mid\right\} n v \Omega I\right) \times \\
& \left.\quad \times\left(I_{1} M_{1} 2 m \mid I M\right) d_{2 m}^{+} \mid n-1 \quad v_{1} \Omega_{1} I_{1} M_{1}\right)_{B}, \tag{3}
\end{align*}
$$

where $\left.11,1,0,2, M)_{B}=d_{2 m}^{+} \mid 0\right)$ and $d_{2 m}^{+}$denotes the creation operator of the collective quadrupole boson.

The boson states (3) form an othonormalized system by definition. That is not true for the fermion states (2) for which the norm matrix is considered

We introduce the boson operator $\exp (-\hat{A})$, which in the boson space reproduces the norm matrix
${ }_{B}\left(n^{\prime} v^{\prime} \Omega^{\prime} I M \mid e^{-A} \operatorname{Inv} \Omega I M\right)_{B}=A_{n^{\prime} v^{\prime} \Omega^{\prime} I M, n v \Omega_{I M}}$
The TDA amplitudes and the $L=2$ boson c.f.p.'s could be chosen real so that the norm matrix $\mathcal{A}$ is real and symmetric. The unitary mapping of the fermion collective subspace (2) onto the boson collective subspace (3) is brought in as $\left.\left|n v \Omega I M_{F} \longrightarrow \exp \left(-\frac{1}{2} \hat{A}\right)\right| n v \Omega I M\right)_{B}$.

One easily sees that the mapping (5) implies the mapping of the TDA operator
$A_{2 m}^{+} \longrightarrow \exp \left(-\frac{1}{2} \hat{A}\right) d_{2 m}^{+} \exp \left(\frac{1}{2} \hat{A}\right)$.

From the unitarity of the mapping it follows
$A_{2 m} \longrightarrow \exp \left(\frac{1}{2} \hat{A}\right) d_{2 m} \exp \left(-\frac{1}{2} \hat{A}\right)$.

To obtain the norm matrix from eq. (4) is not an easy task. We can, however obtain an equation for the norm matrix operator $\exp (-\hat{A})$ starting from the nonunitary Dyson boson mapping. In that, the pair creation and annihilation operators in the general single particle space...s,t,.. are mapped as
$\alpha_{s}^{+} \alpha_{t}^{+} \longrightarrow \mathrm{B}_{\mathrm{st}}^{+}$,
$\alpha_{t} \alpha_{s} \longrightarrow b_{s t}$
where $b_{s t}^{+}=-b_{t s}^{+}$is the ideal boson operator and
$B_{s t}^{+}=b_{s t}^{+}-\sum_{u v} b_{s u}^{+} b_{t v}^{+} b_{u v}=b_{s t}^{+}-\left[\hat{F}, b_{s t}^{+}\right]$,
with
$\hat{F}=\frac{1}{4} \sum_{s t p q} b_{s t}^{+} b_{p q}^{+} b_{s p} b_{t q}$.
Of course, the norm matrix in the collective subspace is expressed through the elements of the norm matrix in the single particle space. For the latter, we have
$\left.<0\left|\ldots \alpha_{t} \alpha_{s} \ldots \alpha_{t}^{+}, \alpha_{s}^{+}, \ldots\right| 0\right\rangle=\left(0\left|\ldots b_{s t} \ldots B_{s, t}^{+}, \ldots\right| 0\right)$.
On the other hand, the matrix elements of any operator in the collective boson space are expressed through the matrix elements of this operator in the full ideal boson space. We thus have for the operator $\exp (-\hat{A})$ in the ideal boson space $\left(0\left|\ldots b_{s t} \ldots B_{s, t}^{+}, \ldots\right| 0\right)=\left(0\left|\ldots b_{s t} \ldots e^{-\hat{A}^{A}} \ldots b_{s, t}^{+}, \ldots\right| 0\right)$.

The operator $\exp (-\hat{A})$ which satisfies for every $s, t$ the relation
$e^{-\hat{A}} b_{s t}^{+}=B_{s t}^{+} e^{-\hat{A}}$
together with the normalization condition
$e^{-\hat{A}}|0|=(0)$,
satisfies also eq.(8). We have shown [16] that the above equations determine the operator $\exp (-\hat{A})$ uniquely.

The relation between the ideal boson space $b_{s t}$ and the transformed boson space $b_{L M}(1)$ with the collective quadrupole boson $d_{2 M}=b_{2 M}(1)$ is
$b_{s t}^{+}=\sqrt{2} \sum_{L M}\left(j_{s} m_{s} j_{t} m_{t} \mid L M\right) b_{L M}^{+}(s t)$,
$b_{L M}^{+}(s t)=\sum_{i} \Psi_{s t}^{1 L} b_{L M}^{+}(i)$.

Note, that for $i=1, L=2$ the superscripts $i$ and $L$ are not written in eq. (1) and also in the following treatment. Equation (9) is transformed into the collective boson subspace as
$e^{-\hat{A}} d_{2 M}^{+}=\left\{d_{2 M}^{+}\left[\hat{F}, d_{2 M}^{+}\right]\right\} e^{-\hat{A}}$.
Here, the operator $\hat{F}$ is expressed generally through the collective quadrupole bosons $d_{2 M}$ as well as through the , other transformed bosons $b_{L M}$. At this stage, we made an approximation by restricting $\hat{F}$ in eq.(10) only to the collective quadrupole bosons. Using the definition (7), we have
$\hat{F}=\frac{1}{2} \sum_{L=0,2,4} C_{L} \hat{H}_{L}$,
where $c_{L}=50 \sum_{r s t u}\left\{\begin{array}{lll}j_{t} & j_{u} & 2 \\ j_{r} & j_{s} & 2 \\ 2 & 2 & L\end{array}\right\} \quad \Psi_{s r} \quad \Psi_{r t} \quad \Psi_{s u} \quad \Psi_{u t}$,
$\hat{H}_{L}=\sum_{M}\left(d_{2}^{+} d_{2}^{+}\right)_{L M}\left(d_{2} d_{2}\right)_{L M}$,
$\hat{H}_{0}=\frac{1}{5}\left[3 \hat{N}+\hat{N}^{2}-\hat{T}^{2}\right]$,
$\hat{H}_{2}=\frac{1}{7}\left[-4 \hat{N}+2 \hat{N}^{2}-\hat{\mathrm{I}}^{2}+2 \hat{\mathrm{~T}}^{2}\right]$,
$\hat{H}_{4}=\frac{1}{35}\left[-36 \hat{N}+18 \hat{\mathrm{~N}}^{2}+5 \hat{\mathrm{I}}^{2}-3 \hat{\mathrm{~T}}^{2}\right]$.
Here $\hat{N}$ is the $d$ - bosons number operator and $\hat{I}^{2}$ is the square of the angular momentum. The eigenvalue of the operator $\hat{T}^{2}$ in the $S U(5)$ basis is equal to $v(v+3)$, where $v$ is the
seniority of the state.
From eq.(10), one gets the recurrence relation for the matrix: elements of the operator $\exp (-\hat{A})$ in the $\operatorname{SU}(5)$ basis

$$
\left(n v_{1} \Omega_{1} I\left|e^{-\hat{A}}\right| n v \Omega I\right)=\sum_{v_{2} \hat{\Omega}_{2} v_{3} \Omega_{3} L}\left(n-1 v_{2} \Omega_{2} L\left|e^{-\hat{A}^{\prime}}\right| n-1 v_{3} \Omega_{3} L\right) x
$$

$$
\times\left(1-F\left(n v_{1} I\right)+F\left(n-1 \quad v_{2} L\right)\right) \times
$$

$\left.\left.\times\left(\mathrm{n}-1 \mathrm{v}_{2} \Omega_{2} \mathrm{~L} ; 2 \mid\right\} \mathrm{nv} \Omega_{1} \mathrm{I}_{1}\right) \times\left(\mathrm{n}-1 \mathrm{~V}_{3} \Omega_{3} \mathrm{~L} ; 2 \mid\right\} \mathrm{nv} \Omega \mathrm{I}\right)$.

Note that the matrix elements from eq.(12) are not symmetric due to the approximation made for $\hat{F}$. We symmetrize them by neglecting the antisymmetric part of the $\exp (-\hat{A})$ matrix in the recurrence relation (12). In fact, we have found in actual calculations that the antisymmetric part from full eq.(12) is well smaller than the symmetric part.

Another feature which calculations reveal is the smallness of the nondiagonal matrix elements of $\exp (-\hat{A})$ in comparison with the diagonal ones. We neglect the nondiagonal elements in the following; which simplifies the calculation of the square root operator $\exp \left(-\frac{1}{2} \hat{A}\right)$ and its inverse.

If in eq. (11) the equality of $C_{0}=C_{2}=C_{4}=C$ is assumed, the operator $\hat{F}$ reduces to $\hat{F}=\frac{1}{2} \hat{C N}(\hat{N}-1)$. Then the elements $F(n v I)$ of $\hat{F}$ depend only on the boson number $n$. Substituting this result in the recurrence relation (12), we have $\left(n v \Omega I\left|e^{-\hat{A}}\right| n v \Omega I\right)=(1-(n-1) C)\left(n-1 v_{1} \Omega_{1} I_{1}\left|e^{-\hat{A}}\right| n-1 v_{1} \Omega_{1} I_{1}\right)$, or alternatively
$e^{-A(n)+A(n-1)}=1-(n-1) C$.
Since $\hat{A}$ depends only on $\hat{N}$, we write eq. (6) as
$A_{2 M}^{+} \longrightarrow \exp \left(-\frac{1}{2} A(\hat{N})\right) d_{2 M}^{+} \exp \left(\frac{1}{2} A(\hat{N})\right)=d_{2 K}^{+} \exp \left[\frac{1}{2}(A(\hat{N})-A(\hat{N}+1))\right]$.
We obtain the final result
$\mathrm{A}_{2 M}^{+} \longrightarrow \mathrm{d}_{2 M}^{+} \sqrt{1-\mathrm{C} \mathrm{\hat{N}}}$.
Thus, in the case of $C_{0}=C_{2}=C_{4}=C$, the boson representation (6) of the quadrupole TDA operator reduces to the SU(6) IBM expression with $\mathrm{C}^{-1}$ being the maximum number $\mathrm{N}_{\max }$ of bosons in the IBM.

We introduce the cutoff in the boson number when the corresponding matrix element of $\exp (-\hat{A})$ are less than zero. In the case of different parameters $C_{0}, C_{2}$, and $C_{4}$, this cutoff depends not only on the $d$ boson number $n$ but also on the other $S U(5)$ quantum numbers. In this way, we go beyond the $\mathrm{SU}(6)$ symmetry.

## 3. Particle-hole operator

In order to define the boson mapping completely, we have to find the boson image of the particle-hole operator. The matrix elements of this operator in the single-particle basis are given in the Dyson boson mapping by

$$
<0\left|\ldots \alpha_{t} \alpha_{s} \ldots \alpha_{u}^{+} \alpha_{v} \ldots \alpha_{t}^{+}, \alpha_{s}^{+}, \ldots 10\right\rangle
$$

$$
\begin{equation*}
=\left(0\left|\ldots b_{s t} \ldots \hat{\rho}_{u v} \ldots B_{s, t}^{+}, \ldots\right| 0\right) \tag{13}
\end{equation*}
$$

where
$\hat{\rho}_{u v}=\sum_{x} b_{v x}^{+} b_{u x}$
Using the properties of the operator $\exp (-\hat{A})$, we write the right- hand side of eq. (13) as

$$
\begin{aligned}
& \left(0\left|\ldots b_{s t} \ldots \hat{\rho}_{u v} \exp (-\hat{A}) \ldots b_{s, t}^{+}, \ldots\right| 0\right)= \\
& \quad=\left(01 \ldots b_{s t} \ldots \exp \left(-\frac{1}{2} \hat{A}\right) \hat{\rho}_{u v} \exp \left(-\frac{1}{2} \hat{A}\right) \ldots b_{s, t}^{+}, \ldots \mid 0\right) .
\end{aligned}
$$

Here, the commutativity of the operator $\exp (-\hat{A})$, and consequently of any function of $\exp (-\hat{A})$, with the $\hat{\rho}_{u v}$ has been used [15]. The relation (5) generalized to the single particle basis is

$$
\left.\ldots \alpha_{s}^{+}, \alpha_{t}^{+}, \ldots|0\rangle=\exp \left(-\frac{1}{2} \hat{A}\right) \ldots b_{s, t}^{+}, \ldots \mid 0\right)
$$

In order to make the equality of the particle-hole matrix elements consistent with this mapping, we set

$$
\alpha_{u}^{+} \alpha_{v} \longrightarrow \hat{\rho}_{u v}
$$

or in the transformed boson space

$$
\begin{aligned}
\alpha_{u}^{+} \alpha_{v} \longrightarrow & 2 \sum_{L_{L M i}}\left(j_{v} m_{v} j_{x} m_{x} \mid L M\right)\left(j_{u} m_{u} j_{x} m_{x} \mid L^{\prime} M^{\prime}\right) \\
& \times \Psi_{v x}^{1 L} \quad \Psi_{u x}^{1^{\prime} L^{\prime} *} b_{L M}^{+}(i) b_{L^{\prime} M},\left(i^{\prime}\right)
\end{aligned}
$$

Of course, in the discussion of the following section we restrict ourselves in the above relation only to the collective quadrupole boson terms $d_{2 M}^{+} d_{2 M}$.

## 4. Collective Hamiltonian

The fermion quasiparticle representation of the nuclear

Hamiltonian is written as follows:

$$
\hat{H}_{f}=\sum_{s} \varepsilon_{s} \alpha_{s}^{+} \alpha_{s}+\sum_{r s t u I} G_{r s t u}^{I(22)}\left[\left(\alpha_{r}^{+} \bar{\alpha}_{s}\right)_{I}\left(\alpha_{t}^{+} \bar{\alpha}_{u}\right)_{I}\right]_{00}
$$

$$
+\sum_{r s t u I} G_{r s t u}^{I(40)}\left\{\left[\left(\alpha_{r}^{+} \alpha_{s}^{+}\right)_{I}\left(\alpha_{t}^{+} \alpha_{u}^{+}\right)_{I}\right]_{00}+h . c .\right\}
$$

$$
\begin{equation*}
+\sum_{r s t u I} G_{r s t u}^{I(31)}\left\{\left[\left(\alpha_{r}^{+} \alpha_{s}^{+}\right)_{I}\left(\alpha_{t}^{+} \bar{\alpha}_{u}\right)_{I}\right]_{00}+h \cdot c \cdot\right\} \tag{14}
\end{equation*}
$$

Using the boson representation of the fermion operators as discussed in the preceding section, we obtain after truncation to the quadrupole collective degree of freedom the collective boson Hamiltonian in the form
$\hat{H}=\varepsilon \hat{n}+\frac{1}{2} \sum_{L=0,2,4} g_{L} \sqrt{2 L+1}\left[\left(d_{2}^{+} d_{2}^{+}\right)_{L}\left(d_{2} d_{2}\right)_{L}\right]_{00}+$
$+V\left\{\exp \left(-\frac{1}{2} \hat{A}\right)\left[d_{2}^{+} \exp \left(\frac{1}{2} \hat{A}\right)\left(d_{2}^{+} d_{2}\right)\right]_{00}+h \cdot c \cdot\right\}$
$+W\left\{\exp \left(-\frac{1}{2} \hat{A}\right)\left({d_{2}^{+}}_{d_{2}^{+}}\right)_{00} \exp \left(\frac{1}{2} \hat{A}\right)+h \cdot c \cdot\right\}$.
The coefficients of the boson Hamiltonian (15) are expressed in terms of the parameters of the original fermion Hamiltonian (14) and of the TDA amplitudes $\Psi_{s t}[17]$

$$
\left.\varepsilon=\sum_{s t}\left(\varepsilon_{s}+\varepsilon_{t}\right) \Psi_{s t} \Psi_{s t}+20\right\rangle_{\text {Iprstuv }}^{I(22)}(2 I+1) \Psi_{s v}^{I} \Psi_{v r} \Psi_{u p} \Psi_{p t}\left\{\begin{array}{lll}
I & 2 & 2 \\
j_{v} & j_{r} j_{s}
\end{array}\right\}\left\{\begin{array}{lll}
I & 2 & 2 \\
j_{p} j_{t} & j_{u}
\end{array}\right\},
$$

$$
g_{L}=200 \sum_{r s t u p q I} G_{r s t u}^{I(22)} \Psi_{s p} \Psi_{p r} \Psi_{u q} \Psi_{q t} \sqrt{2 I+1}\left\{\begin{array}{ll}
I .2 & 2 \\
j_{p} j_{r} j_{s}
\end{array}\right\}\left\{\begin{array}{lll}
I & 2 & 2 \\
j_{q} & j_{t} & j_{u}
\end{array}\right\}\left\{\begin{array}{l}
I 22 \\
L 2
\end{array}\right\}
$$

$V=-10 \sqrt{2} \sum_{r s t u v} G_{r s t u}^{2(31)} \Psi_{r s} \Psi_{u v} \Psi_{v t}\left\{\begin{array}{lll}2 & 2 & 2 \\ j_{v} & j_{t} & j_{u}\end{array}\right\}$,
$W=2 \sum_{r s t u} G_{r s t u}^{2(40)} \quad \Psi_{r s} \Psi_{t u}$.

We see immediately another feature in which the boson Hamiltonian (15) differs from the IBM Hamiltonian. Namely, the presence of the $\exp \left(-\frac{1}{2} \hat{A}\right)$ and $\exp \left(\frac{1}{2} \hat{A}\right)$ operators implicitly introduces many-body effects. These are not considered within the ordinary IBM which deals only with one- and two-body terms in the Hamiltonian.

## 5. Calculations

The procedure of calculations with the fully microscopically determined collective quadrupole Hamiltonian should start with the fermion quasiparticle Hamiltonian from which the TDA amplitudes, the norm matrix operator and the boson Hamiltonian would be derived. It appears, however, in different microscopic studies of the collective nuclear structure that the truncation only to the lowest collective pair is restrictive in most cases [18]. other degrees of freedom, as for example the $L=4$ collective pair, play an important role. These degrees of freedom can be included either explicitly or implicitly through the renormalization of the parameters of the quadrupole boson Hamiltonian.

We do not perform the microscopic calculations in the present paper in which the truncation to the quadrupole
woson $1 s$ only considered. Rather we want to investigate the
possible aspects of the deviations of the boson Hamiltonian (15) from the $S U(6)$ symmetry and from the ordinary IBM Hamiltonian. For this purpose, we fix the Hamiltonian parameters so as to get for $C_{0}=C_{2}=C_{4}=N_{\max }^{-1}$ the $S U(3)$ or the O(6) limits of the IBM. Then, the variation of spectra depends only on one of the parameters $c_{0}, c_{2}$, and $c_{4}$ while two other being still fixed.

The results obtained are shown in figures 1 and 2 . It is seen that the variations in the wide limits of the values of the $C_{0}^{-1}, C_{2}^{-1}$ and $C_{4}^{-1}$ near $N_{\max }$ do not change drastically the spectrum of the collective states.

## 6. Conclusions

We have presented an approach to the description of the low-lying nuclear collective states in which the fermion problem has been transformed into the boson one. Starting with the mapping of the quadrupole collective states in the fermion space onto the boson space, we have found the boson images of the bifermion operators and fermion Hamiltonian. In this mapping, the operator reproducing the fermion norm matrix in the boson space plays an essential role. We have suggested recurrence relations for this operator and solved them in the truncated quadrupole boson space.

The resulting Hamiltonian contains the terms which go beyond the ordinary $S U(6)$ IBM Hamiltonian. However, as far as one stands on the phenomenological level, the deviations from the IBM results are not drastic and could likely be reproduced by changing the phenomenological IBM parameters.


Fig. 1 The spectrum of the collective states for different values of $C_{0}^{-1}, C_{2}^{-1}$ and $C_{4}^{-1}$. In every graph, one of the parameters $C_{0}^{-1}, C_{2}^{-1}$, and $C_{4}^{-1}$ is changed and the others are equal to 10. The parameters of the Hamiltonian are fixed so that the $S U(3)$ limit is realized when $C_{0}^{-1}=C_{2}^{-1}=C_{4}^{-1}=10$. For convenience, the ground state quasirotational band (left part) and the excited bands (right part) are shown separately.


Fig. 2 The same as fig.1, but for the $o(6)$ limit.

The norm matrix operator $\exp (-\hat{A})$ is closely related to the projection operator onto the physical boson space [16]. We have to note, however, that the problem of mixture of the nonphysical boson components is not solved in the present approach and remains to be an important challenge to the boson models of nuclear collective structure.

Another problem is the inclusion of the degrees of freedom other than the lowest quadrupole collective states. on the phenomenological level, such an inclusion increases the number of model parameters enormously. Therefore, the microscopically motivated models should be developed. In principle, the present approach can be extended easily to include them. Then the norm matrix operator $\exp (-\hat{A})$ could contain large nondiagonal elements which might influence the results significantly. However, handling the square root operator $\exp \left(-\frac{1}{2} \hat{A}\right)$ and its inverse should be more difficult. References
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Довеш Я. и др.
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Бозонное представление и микроскопический
коллективный ядерный гамильтониан
Получено бозонное представление фермионного гамильтониана на основе бозонного представления бифермионных квадрупольных коллективных операторов. С помощью приближенного рекуррентного соотношения вычислена матрица нормы бозонного представления. Полученный бозонный гамильтониан содержит члены, обобщающие обычную SU(6)-симметрию гамильтониана модели взаимодействующих бозонов. Результаты расчета показывают, что отличия между полученным гамильтонианом и гамильтонианом модели взаимодействующих бозонов несущественны.

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## Dobes J. et al.

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Boson Mapping and the Microscopic
Collective Nuclear Hamiltonian
Starting with the mapping of the quadrupole collective states in the fermion space onto the boson space, the fer mion nuclear problem is transformed into the boson one. The boson images of the bifermion operators and of the fermion Hamiltonian are found. Recurrence relations are used to obtain approximately the norm matrix which appears in the boson-fermion mapping. The resulting boson Hamiltonian contains terms which go beyond the ordinary $\operatorname{SU}(6)$ symmetry Hamiltonian of the interacting boson model. Calculations, however, suggest that on the phenomenological level the differences between the mapped Hamiltonian and the $S U(6)$ Hamiltonian are not too important.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.


[^0]:    *Institute of Nuclear Physics, CS-25068 Rez, Czechoslovakia

