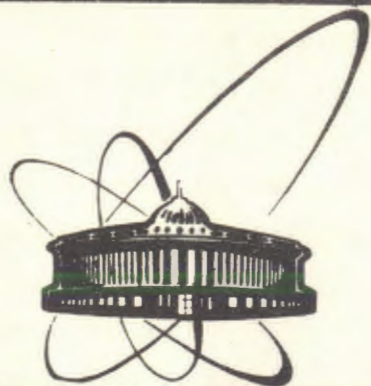


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ОБЪЕДИНЕННЫЙ  
ИНСТИТУТ  
ЯДЕРНЫХ  
ИССЛЕДОВАНИЙ  
ДУБНА

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IMPORTANCE OF THE NEUTRON-PROTON  
INTERACTIONS FOR THE EVEN Ra-Th NUCLEI<sup>2</sup>

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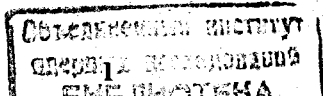
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## I. Introduction

The concept of deformation has extensively been used since 1953<sup>[1]</sup> to describe "collective" properties of heavy and light nuclei, but up to now very little is known about the microscopic structure of deformation. This problem was discussed in a shell-model framework [2-4]. The common conclusion was that the residual neutron-proton interaction could be essential to the development of collectivity and deformation in nuclei. In the light nuclei, for example in  $^{20}\text{Ne}$ , neutrons and protons occupy the same shell model levels, and this idea is commonly accepted. In the heavy nuclei it seems unrealistic; so in the majority of theoretical papers a neutron-proton interaction has not been taken into account.

However, lately <sup>[25]</sup> the importance of the neutron-proton interaction in generating the nuclear deformation is supported by the self-consistent calculations within the Hartree-Fock method with the Skyrme interaction.

Many years ago <sup>[2]</sup> it has been shown that for two nucleons moving in a harmonic oscillator potential and interacting by  $V^{rs} = -\delta(\vec{r}_1 - \vec{r}_2)$ , the energy of the  $J=0$  state is minimum for  $l_1 = l_2$ . One can see from Fig.1 that the difference in the overlap between two orbitals  $(n_1, l_1, j_1)$  and  $(n_2, l_2, j_2)$  in two cases:  $l_1 = l_2$  and  $l_1 = l_2 - 1$  decreases with increasing  $l$ . This means, we can expect relatively large neutron-proton matrix elements in the heavy nuclei too. Experimental matrix elements <sup>[5,6]</sup> confirm this assumption. In addition, the average interaction in  $T=1$  is weaker than in  $T=0$ ; the difference becomes more pronounced in the heavy nuclei.



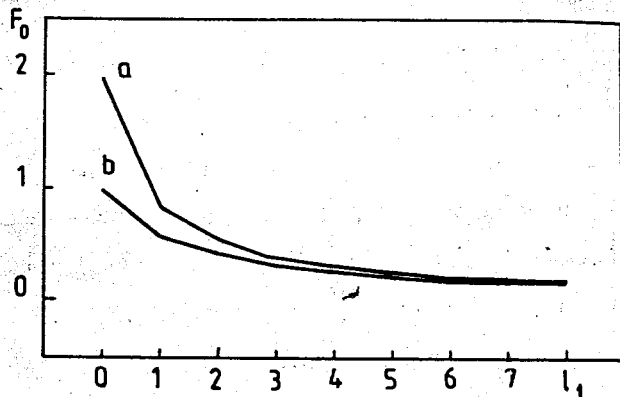


Fig.1. The overlap  $F_0 = \int_0^\infty R_{n_1, l_1}^2 R_{n_2, l_2}^2 dr/r^2$  for  
 a)  $n_1 = n_2 = 1, l_1 = l_2$   
 b)  $n_1 = n_2 = 1, l_1 = l_2 = 1$

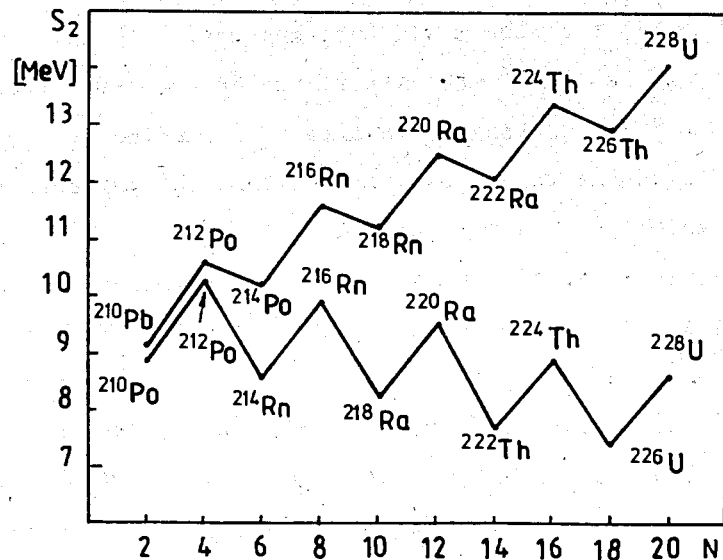


Fig.2. The separation energy  $S_2^{[24]}$  of two neutrons (upper line) and two protons (down line) against the number of valence nucleons.  $\bar{S}_2(4k, T=0) - \bar{S}_2(4k-2, T=1) \approx 1.31$  MeV. In the light and medium nuclei this effect is a few times stronger.

The competition between  $T=1$  and  $T=0$  interactions leads to often discussed  $\alpha$ -clusters [7-10]. They can be seen, for example, in the so-called odd-even pair effect, Fig.2. The separation energy  $S_2$  from a " $\alpha$ -clustered" nucleus (a nucleus with  $n = 4k$ ,  $T=0$  valence nucleons) is higher than from a nucleus with  $n = 4k \pm 2$ ,  $T=1$ . The difference  $\bar{S}_2(n=4k, T=0) - \bar{S}_2(n=4k \pm 2, T=1)$  can be a measure of effective four-nucleon interactions. This and other experimental data [7,8] together with the conclusion of the paper [11] that low-lying states ( $E \leq 1.5$  MeV) in  $^{218}\text{Ra}$  can be built mainly from  $S$  and  $\rho$  bosons suggest applying the model [12] in the actinide region.

In § 2 we give a sketch of the model, § 3 includes the results for even  $^{222-226}\text{Ra}$  and  $^{226-232}\text{Th}$ , § 4 presents summary and conclusions.

## 2. Model

In the actinide region the correlation energy [7] of the nucleon pairs is of about a few hundred KeV. Then, for low excitation energy ( $E \leq 1$  MeV), we will treat the nucleon pairs as building blocks of a nucleus. With the above assumption, the nucleon pair with quantum numbers of the total angular momentum  $J$ , the parity  $\pi$  and the isospin  $T$  corresponds to the boson with the same quantum numbers. Taking into account the strongest interactions we have six bosons:  $S_\mu^+$  with  $J^\pi = 0^+$ ,  $T=1$ ,  $\mu = 0, \pm 1$  and  $P_\mu^+$  with  $J^\pi = 1^-$ ,  $\mu = 0, \pm 1$ ,  $T=0$ . The boson  $S^+$  corresponds to a pair of nucleons coupled by pairing forces, the boson  $\rho^+$  substitutes for a neutron-proton pair found on single particle shell model levels with  $|l_1 - l_2| = 1$ .

The most general Hamiltonian  $H$  for a system of interacting  $s$  and  $p$  bosons is:

$$H = \varepsilon_1 \hat{n}_s + \varepsilon_2 \hat{n}_p + \varepsilon_3 [p^+ p^+]^{J=0, T=0} [\tilde{p} \tilde{p}]^{00} + \varepsilon_4 [s^+ s^+]^{00} [\tilde{s} \tilde{s}]^{00} + \varepsilon_5 [p^+ p^+]^{20} \cdot [\tilde{p} \tilde{p}]^{20} + \varepsilon_6 [s^+ s^+]^{02} \cdot [\tilde{s} \tilde{s}]^{02} + \varepsilon_7 [p^+ s^+]^{11} [\tilde{s} \tilde{p}]^{11} \quad (I) + \varepsilon_8 ([p^+ p^+]^{00} [\tilde{s} \tilde{s}]^{00} + [s^+ s^+]^{00} [\tilde{p} \tilde{p}]^{00}).$$

Square brackets denote spin and/or isospin coupling and

$$T^k \cdot T^k = (-1)^k (2k+1)^{1/2} [T^k T^k]^0; \quad \tilde{b}_\mu = (-1)^{l-\mu} b_{-\mu}.$$

The first two terms in (I) represent familiar pairing and isospin pairing interactions, the subsequent terms - different effective four-nucleon interactions. Hamiltonian (I) conserves the total number of bosons  $N = n_p + n_s$ , total angular momentum  $J$  and isospin  $T$ . It can be rewritten in the generators

$$[p^+ \tilde{p}]_{\mu}^{J=0,1,2; T=0}, [s^+ \tilde{s}]_{\nu}^{J=0; T=0,1,2}, [p^+ \tilde{s}]_{\mu\nu}^{11}, [s^+ \tilde{p}]_{\mu\nu}^{11} \quad (2)$$

of the unitary group  $U(6)$ . One from two possible complete chains of subgroups of this  $U(6)$

$$U(6) \supset U_{n_p}(3) \otimes U_{n_s}(3) \supset SO_3(3) \otimes SO_3(3) \supset SO_{M_J}(2) \otimes SO_{M_T}(2) \quad (3)$$

provides the basis

$$|N n_p J M_J T M_T\rangle \quad (4)$$

in which  $H$  is diagonalized.

### 3. Results

The actinide nuclei, containing a  $^{208}\text{Pb}$  core with valence protons filling the  $1h_{9/2}$ ,  $2f_{7/2}$ ,  $1i_{13/2}$  and  $2f_{5/2}$  orbitals and valence neutrons in  $2g_{7/2}$ ,  $1i_{11/2}$ ,  $1j_{15/2}$  and  $2d_{5/2}$  orbitals, are suitable for verifying the importance of the effective neutron-proton interaction. Even nuclei of Ra with  $E_{4_1^+}/E_{2_1^+} < 3.2$  and Th with  $E_{4_1^+}/E_{2_1^+} > 3.2$  are chosen for the study.

The Hamiltonian (I) was diagonalized in the basis (4) for the boson numbers equal to  $1/2$  of the number of nucleons over the core  $^{208}\text{Pb}$  and for the isospin numbers  $T = T_z$  for the valence nucleons. From 8 one- and two-boson energies  $\varepsilon_i$  only 6 parameters  $k_i = f_i(\varepsilon_1 \dots \varepsilon_8)$  are independent<sup>[12]</sup>. They were fitted in order to obtain low-lying spectra of searching nuclei (Fig.3 and Fig.4).

The eigenstates of  $H$

$$|N J M_J T M_T, E\rangle = \sum_{n_p=J \text{ step } 2}^{N \text{ or } N-1} a_{n_p}(J T, E) |n_p J M_J\rangle |N - n_p, T M_T\rangle \quad (5)$$

make it possible to find the reduced E1 and E2 transitions, defined as usual

$$B(E\lambda; J_1 - J_2) = (2J_1 + 1)^{-1} |\langle J_2 || \hat{B}(E\lambda) || J_1 \rangle|^2$$

with:

$$\hat{B}_{\mu\nu}(E1) = c_1 [s^+ \tilde{p} + p^+ \tilde{s}]_{\mu\nu}^{11}, \quad (6)$$

$$\hat{B}_{\mu}(E2) = c_2 [p^+ \tilde{p}]_{\mu}^2. \quad (7)$$

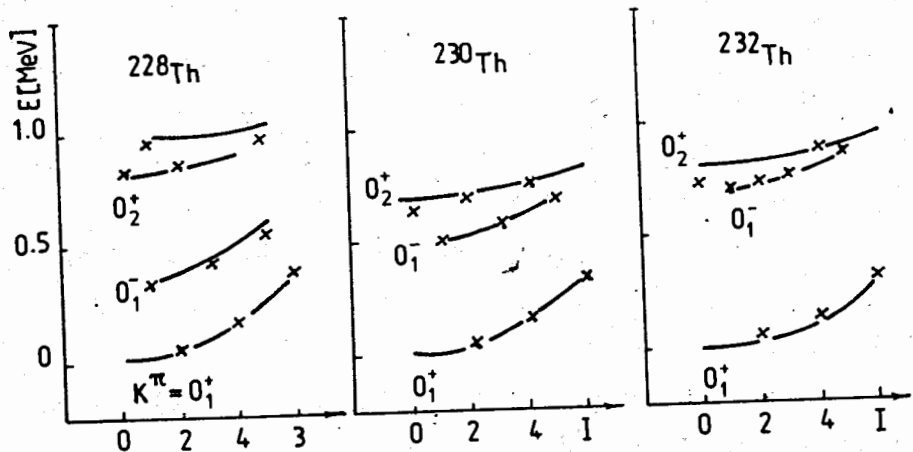


Fig.3. The calculated (lines) with parameters (in MeV):  $H_0 = 0$ ,  $k_1 = 1.6252$ ,  $k_2 = -0.0100$ ,  $k_3 = -0.0098$ ,  $k_4 = 0.1317$ ,  $k_5 = 0.0129$ ,  $k_6 = 0.0581$  and experimental (crosses) spectra of  $^{228}\text{Th}$  [13],  $^{230}\text{Th}$  [14] and  $^{232}\text{Th}$  [15].

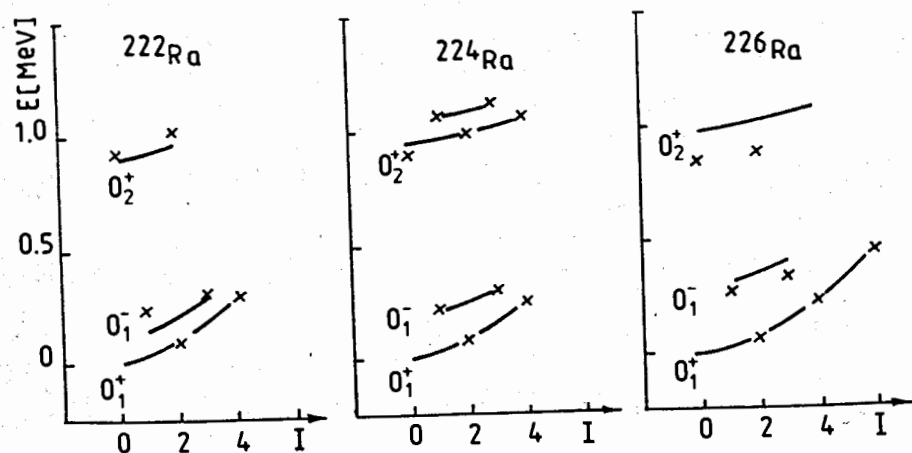


Fig.4. The calculated (lines) with parameters (in MeV):  $H_0 = 0$ ,  $k_1 = 3.1722$ ,  $k_2 = -0.0146$ ,  $k_3 = -0.0218$ ,  $k_4 = 0.2946$ ,  $k_5 = 0.0212$ ,  $k_6 = 0.0333$  and experimental (crosses) spectra of  $^{222}\text{Ra}$  [16],  $^{224}\text{Ra}$  [13] and  $^{226}\text{Ra}$  [14].

Table I. The ratio for reduced probabilities of the electric dipole transitions from the  $1^-$  state to the yrast band states

$\frac{B(E1, 1^- \rightarrow 2^+)}{B(E1, 1^- \rightarrow 0^+)}$	$^{222}\text{Ra}$	$^{224}\text{Ra}$	$^{226}\text{Ra}$	$^{236}\text{Th}$	$^{228}\text{Th}$	$^{230}\text{Th}$
exp. [ref]	2.03 [16]	2.18 [13]	1.85 [14]	1.76 [14]	2.08 [13]	2.44 [18] 1.72 [19]
calc.	2.37	2.17	2.60	2.13	2.24	2.30

Table 2. Reduced probabilities of E2 transitions  $0_{g.s.}^+ \rightarrow 2_1^+$  in  $^A\text{Ra}$  nuclei

$B(E2, 0^+ \rightarrow 2^+)$ ( $e^2 b^2$ )	$^{222}\text{Ra}$	$^{224}\text{Ra}$	$^{226}\text{Ra}$	$^{228}\text{Ra}$
exp [17]	$4.52_{-38}$	$3.99_{-16}$	$5.13_{-28}$	$6.01_{-49}$
calc.	3.34	4.09	6.43	7.01

Table 3. Reduced probabilities of E2 transitions  $0_{g.s.}^+ \rightarrow 2_1^+$  in  $^A\text{Th}$  nuclei

$B(E2, 0^+ \rightarrow 2^+)$ ( $e^2 b^2$ )	$^{226}\text{Th}$	$^{228}\text{Th}$	$^{230}\text{Th}$	$^{232}\text{Th}$
exp [17]	$6.85_{-40}$	$7.07_{-27}$	$8.04_{-10}$	$9.28_{-9}$
calc.	4.80	6.58	8.18	10.14

Table 1 shows the ratios for reduced probabilities of the electric dipole transitions from the  $1^-$  state ("octupole") to the yrast band states. Tables 2 and 3 contain the reduced probabilities of E2 transitions  $0_{g.s.}^+ \rightarrow 2_1^+$  in

$^{A}\text{Ra}$  and  $^{A}\text{Th}$  nuclei. The calculated values were obtained with the parameter  $C_2^2 = 0.200 e^2 b^2$ . Table 4 and Table 5 show the ratios for the reduced E2 and E1 probabilities but only for  $^{230}\text{Th}$  compared with the last experimental data [19].

Table 4. The ratios for reduced probabilities of the quadrupole transitions from some  $\beta$ -band states to the yrast-band states in  $^{230}\text{Th}$

$J_\beta$	$J_g$	$J'_g$	$B(E2, J_\beta - J_g) / B(E2, J_\beta - J'_g)$		Alaga
			exp. [19]	Calc.	
2	2	0	$2.31 \pm 0.31$	1.58	1.42
2	2	4	$0.58 \pm 0.10$	0.35	0.55
2	4	0	$3.44 \pm 0.51$	4.33	2.57
4	4	2	$2.06 \pm 1.41$	1.01	0.91

Table 5. The ratios for reduced probabilities of the electric dipole transitions from some octupole states to the yrast-band states in  $^{230}\text{Th}$

$J^{\text{oct}}$	$J_g$	$J'_g$	$B(E1, J^{\text{oct}} \rightarrow J_g) / B(E1, J^{\text{oct}} \rightarrow J'_g)$		
			exp. [ref. J]	Calc.	ref. [10]
1	2	0	2.44 [18]	2.30	2.30
			1.72 [19]		
3	4	2	2.15 [20]	1.86	1.78
			1.95 [18]		
			1.61 [19]		
5	6	4	3.17 [18]	2.08	2.04
			2.08 [19]		

Then, we calculate the average number of  $\rho$  and  $s$  bosons in any state

$$\bar{n} = \sum_i |a_{n_i}(J, T, E)|^2 n_i \quad (8)$$

One can estimate also the average number of  $\alpha$ -like clusters

$$\bar{n}_\alpha = 1/2 (N - \bar{\omega}) \quad (9)$$

where  $\bar{\omega}$  means the average number of bosons not coupled in  $J=0, T=0$  pairs and it can be extracted from a given eigenenergy of (1) [12]. Figs. 5 and 7 show that the ground state and octupole  $1^-$  state in the studied  $^{A}\text{Th}$  nuclei have very similar neutron-proton and  $\alpha$ -like structure. This is consistent with the remark [21] that the ground states of heavier  $^{A}\text{Th}$  can be octupole-deformed. The fact that reduced width for  $\alpha$ -decay is nearly constant in the well deformed actinides implies that the number of " $\alpha$ -clusters" in the ground state is constant [10]. The calculations confirm this

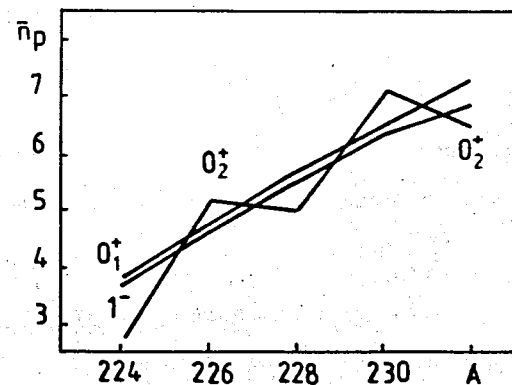


Fig. 5. The average number of  $J=1, T=0$  pairs in the ground state  $1^-$  state and  $0_2^+$  state against the mass of  $^{A}\text{Th}$ .

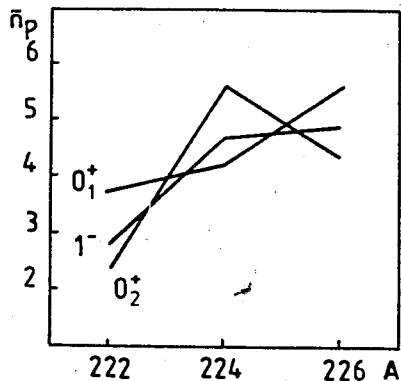


Fig. 6. The same as in Fig. 5 but for  $^A\text{Ra}$  nuclei.

mention too (Fig. 7). We can see in Figs. 8 and 9 that within a given band ( $K^\pi = 0^+$ ,  $0_1^-$  and  $0_2^+$ ) of  $^{230}\text{Th}$  the average number of neutron-proton pairs  $\bar{n}_p$  slowly increases with

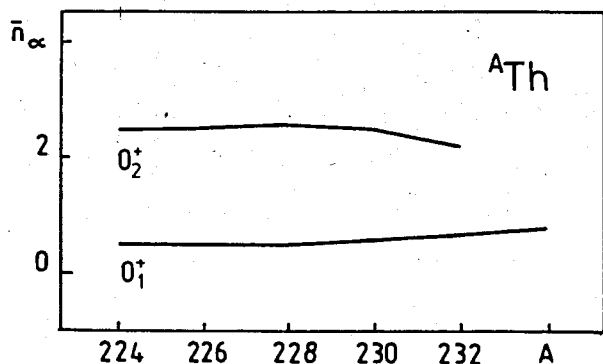


Fig. 7. The average number of  $\alpha$ -clusters in the ground state and  $0_2^+$  state. We obtain in  $1^-$  state the same values of  $\bar{n}_\alpha$  as in  $0_1^+$  state.

$\bar{n}_p$ ; simultaneously,  $\bar{n}_\alpha$  slowly decreases. In every case we obtain more deuteron-like pairs than neutron-neutron and proton-proton pairs. A similar result was obtained in the paper<sup>[9]</sup> and in our earlier calculations for the light<sup>[10,22]</sup> and rare earth<sup>[23]</sup> nuclei.

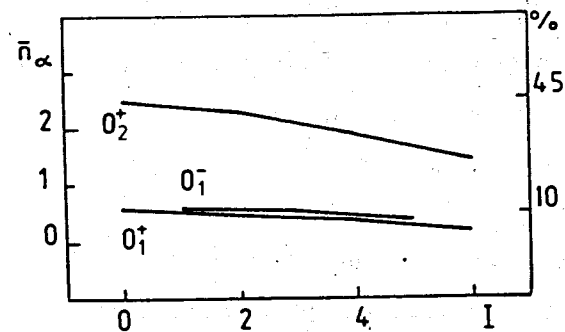


Fig. 8.  $\alpha$ -clustering in some states of the  $K^\pi = 0_1^+$ ,  $0_1^-$  and  $0_2^+$  bands in  $^{230}\text{Th}$ .

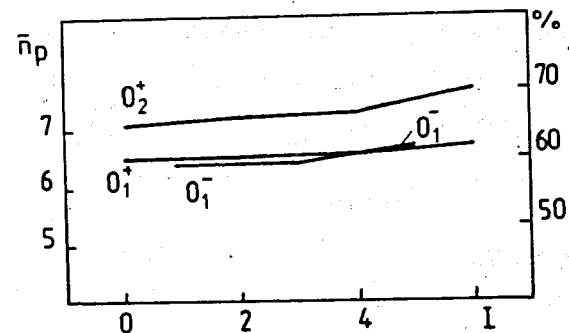


Fig. 9. Percentage of neutron-proton pairs in some states of  $^{230}\text{Th}$ .

### Summary

Experimental data and many successful calculations in different versions of the interacting boson model confirm that for low excitation energy the nucleon pairs can be treated as building blocks of a nucleus. By the above assumption and taking into account the most interacting pairs with  $J^\pi$ ,  $T=0^+$ ,  $1^-$ ,  $0$  we are able to reproduce the experimental energies and the reduced probabilities of E2 and E1 transitions in even Ra-Th nuclei. It is interesting and unexpected that in every

case of deformed nucleus (light <sup>[22]</sup>), rare earth <sup>[23]</sup> and actinide nuclei), we obtain over 50% of neutron-proton pairs, even in the ground states! This suggests that the residual neutron-proton interactions can be of an origin of collectivity and deformation in any nucleus. These aspects (collectivity and deformation) are the most pronounced in the actinide region where valence neutrons and protons can fill many single particle levels with large  $j$  and  $j_n = j_p \pm 1$ .

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