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THE INFLUENCE OF COMPLEX CONFIGURATION ON THE PROPERTIES OF LOW-LYING STATES

[^0]The development of the experimental equipment makes it possible to obtain detailed information on the structure of low-lying states in heavy nuclei. The (e,e') experiments performed at NIKHEF [1] and MIT [2,3] allowed one to extract the transition densities of not only the first $2^{+}$state but of many others with different $J^{x}$ from the energy range $E_{x}=0-3 \mathrm{MeV}$. The comparison of experimentally extracted transition densities of the first three $2^{+}$states in ${ }^{142} N d$ with the theoretically calculated ones (within the Quasiparticle-phonon model (QPM) [4]) has shown a good agreement of the theory with the experiment [5].

Recently, the QPM has been extended for even-even spherical nuclei in a way to take into account three-phonon terms in the excited state wave function [6,7]. In this paper, we aim to investigate the influence of the three-phonon terms on the properties of low-lying states. To achieve this we will make a comparison between the results obtained in the QPM with and without three-phonon components taken into account.

We take the QPM Hamiltonian in the form:

$$
\begin{equation*}
H=H_{s . p .}+H_{\text {pair }}+H_{\text {res.int. }}, \tag{1}
\end{equation*}
$$

where $H_{s . p .}$ accounts for the average field and is taken in the form of the Woods-Saxon potential; $H_{\text {pair }}$ accounts for the monopole pairing; $H_{\text {resint. }}$ is a multipole-multipole separable interaction. The general QPM Hamiltonian can be found in Refs.[4].


The excited state wave function is taken to be

$$
\begin{align*}
& \Psi_{\nu}(J M)=\left\{\sum_{i} R_{i}(J \nu) Q_{J M i}^{+}+\sum_{\lambda i \lambda^{\prime} i^{\prime}} P_{\lambda i}^{\lambda^{\prime} i^{\prime}}(J \nu)\left[Q_{\lambda \mu i}^{+} Q_{\lambda^{\prime} \mu^{\prime} i^{\prime}}^{+}\right]_{J M}\right. \\
& \left.+\sum_{\lambda i \lambda^{\prime} i^{\prime}} \sum_{I \lambda^{\prime \prime} i^{\prime \prime}} T_{I}^{\lambda i \lambda_{\lambda^{\prime \prime}}^{\prime \prime i^{\prime \prime}}}(J \nu)\left[\left[Q_{\lambda \mu i}^{+} Q_{\lambda^{\prime} \mu^{\prime} i^{\prime}}^{+}\right]_{I k} Q_{\lambda^{\prime \prime} \mu^{\prime \prime} i^{\prime \prime}}^{+}\right]_{J M}\right\} \Psi_{g . s .}, \tag{2}
\end{align*}
$$

where $Q_{\lambda \mu i}^{+}$denotes the $i^{\text {th }}$ RPA phonon with momentum $\lambda$ and projection $\mu$, $\Psi_{g . s .}$ is the wave function of the ground state treated here as a phonon vacuum. $[\ldots]_{J M}$ denotes momentum coupling, and $R, P, T$ are unknown coefficients. The normalization condition for (2) reads [6]:

$$
\begin{align*}
& <\Psi_{\nu}(J M) \mid \Psi_{\nu}(J M)>=\sum_{i}\left[R_{i}(J \nu)\right]^{2}+2 \sum_{\lambda i \lambda^{\prime} i^{\prime}}\left[P_{\lambda i}^{\lambda^{\prime} i^{\prime}}(J \nu)\right]^{2} \bar{K}^{I}\left(\lambda i \lambda^{\prime} i^{\prime}\right) \\
& \quad+6 \sum_{\lambda_{1} i_{1} \lambda_{2} i_{2}} \sum_{I \lambda_{3} i_{3}}\left[T_{I}^{\lambda_{1} i_{1} \lambda_{\lambda_{3} i_{2}}}(J \nu)\right]^{2} C_{I}^{J}\left(\lambda_{1} i_{1}, \lambda_{2} i_{2}, \lambda_{3} i_{3}\right)=1 \tag{3}
\end{align*}
$$

where

$$
\bar{K}^{I}\left(\lambda i \lambda^{\prime} i^{\prime}\right)=1+\frac{1}{2} K^{J}\left(\lambda^{\prime} i^{\prime} \lambda i \mid \lambda i \lambda^{\prime} i^{\prime}\right)
$$

and

$$
C_{I}^{J}\left(\lambda_{1} i_{1}, \lambda_{2} i_{2}, \lambda_{3} i_{3}\right)=1+\frac{3}{2} K^{I}\left(\lambda_{2} i_{2} \lambda_{1} i_{1} \mid \lambda_{1} i_{1} \lambda_{2} i_{2}\right)
$$

$+\frac{1}{2} K^{I}\left(\lambda_{2} i_{2} \lambda_{1} i_{1} \mid \lambda_{1} i_{1} \lambda_{2} i_{2}\right) \sum_{I^{\prime}} \bar{U}\left(\lambda_{1} \lambda_{2} J \lambda_{3} ; I, I^{\prime}\right) K^{I^{\prime}}\left(\lambda_{3} i_{3} \lambda_{2} i_{2} \mid \lambda_{2} i_{2} \lambda_{3} i_{3}\right)$.
Here $\bar{U}\left(\lambda_{1} \lambda_{2} J \lambda_{3} ; I, I^{\prime}\right)$ denotes the Racah coefficient. Since in our approach the phonon operators are not ideal bosons, we have to take into account the Pauli principle violation. The method has been developed in Ref.[8] and applied here in the diagonal approximation to the quantities $K^{J}\left(\lambda^{\prime} i^{\prime} \lambda i \mid \lambda i \lambda^{\prime} i^{\prime}\right)$. Taking into account the Pauli principle leads to a renormalization of the interaction matrix element $U_{\lambda_{2} i_{2}}^{\lambda_{1} i_{1}}(J i)=\left\langle Q_{J i}\right| H\left|\left[Q_{\lambda_{1} i_{1}}^{+} Q_{\lambda_{2} i_{2}}^{+}\right]_{J}\right\rangle$ by the values $\bar{K}^{I}\left(\lambda i \lambda^{\prime} i^{\prime}\right)$ and to a shift of the two-phonon poles by the values
$\Delta \omega^{J}\left(\lambda_{1} i_{1} \lambda_{2} i_{2}\right)$. The exact expressions for $K^{J}\left(\lambda^{\prime} i^{\prime} \lambda i \mid \lambda i \lambda^{\prime} i^{\prime}\right)$ and $\Delta \omega^{J}\left(\lambda_{1} i_{1} \lambda_{2} i_{2}\right)$ can be found in Ref.[8] and for $U_{\lambda_{2} i_{2}}^{\lambda_{1} i_{1}}(J i)$ in Ref.[9].

The equations for the excited state energy $\eta_{J \nu}$ and the wave function structure coefficients are obtained by means of a variational procedure taking into account (3)

$$
\begin{equation*}
\delta\left\{\left\langle\Psi^{\nu}(J M)\right| H\left|\Psi^{\nu}(J M)>-\eta_{J \nu}<\Psi^{\nu}(J M)\right| \Psi^{\nu}(J M)>\right\}=0 \tag{4}
\end{equation*}
$$

The system of equations reads [7]

$$
\begin{gather*}
\left(\omega_{J i}-\eta_{J \nu}\right) R_{i}(J \nu)-\sum_{\lambda_{1} i_{1} \lambda_{2} i_{2}} P_{\lambda_{2} i_{2}}^{\lambda_{1} i_{1}}(J \nu) U_{\lambda_{2} i_{2}}^{\lambda_{1} i_{1}}(J i) \bar{K}^{J}\left(\lambda_{1} i_{1} \lambda_{2} i_{2}\right)=0 \\
\left(\omega_{\lambda_{1} i_{1}}+\omega_{\lambda_{2} i_{2}}+\Delta \omega^{J}\left(\lambda_{1} i_{1} \lambda_{2} i_{2}\right)-\eta_{J \nu}\right) P_{\lambda_{2} i_{2}}^{\lambda_{1} i_{1}}(J \nu)-\frac{1}{2} \sum_{i^{\prime}} U_{\lambda_{2} i_{2}}^{\lambda_{1} i_{1}}\left(J i^{\prime}\right) R_{i^{\prime}}(J \nu) \\
-3 \sum_{\lambda_{1}^{\prime} i_{1}^{\prime} \lambda_{2}^{\prime} i_{2}^{\prime}} T_{\lambda_{1}}^{\lambda_{1}^{\prime} i_{1}^{\prime} \lambda_{\lambda_{2}^{\prime} i_{2}^{\prime} i_{2}^{\prime}}(J \nu) U_{\lambda_{2}^{\prime} i_{2}^{\prime}}^{\lambda_{1}^{\prime} i_{1}^{\prime}}\left(\lambda_{1} i_{1}\right) C_{\lambda_{1}}^{J}\left(\lambda_{1}^{\prime} i_{1}^{\prime}, \lambda_{2}^{\prime} i_{2}^{\prime}, \lambda_{2} i_{2}\right)=0} \\
\left(\omega_{\lambda_{1}^{\prime} i_{1}^{\prime}}+\omega_{\lambda_{2}^{\prime} i_{2}^{\prime}}+\omega_{\lambda_{3}^{\prime} i_{3}^{\prime}}+\Delta \omega^{I}\left(\lambda_{1}^{\prime} i_{1}^{\prime}, \lambda_{2}^{\prime} i_{2}^{\prime}, \lambda_{3}^{\prime} i_{3}^{\prime}\right)-\eta J \nu\right) T_{I}^{\prime_{1}^{\prime} i_{1}^{\prime} \lambda_{2}^{\prime} i_{2}^{\prime}} \lambda_{3}^{\prime} i_{3}^{\prime} \\
-\sum_{i_{1}^{\prime \prime}} P_{\lambda_{3}^{\prime} i_{3}^{\prime}}^{I i_{1}^{\prime \prime}}(J \nu) U_{\lambda_{2}^{\prime} i_{2}^{\prime}}^{\lambda_{1}^{\prime} i^{\prime}}\left(I i_{1}^{\prime \prime}\right) \bar{K}^{J}\left(I i_{1}^{\prime \prime} \lambda_{3}^{\prime} i_{3}^{\prime}\right)=0, \tag{5}
\end{gather*}
$$

where

$$
\Delta \omega^{I}\left(\lambda_{1}^{\prime} i_{1}^{\prime}, \lambda_{2}^{\prime} i_{2}^{\prime}, \lambda_{3}^{\prime} i_{3}^{\prime}\right)=\Delta \omega^{I}\left(\lambda_{1}^{\prime} i_{1}^{\prime} \lambda_{2}^{\prime} i_{2}^{\prime}\right)+\Delta \omega^{I}\left(\lambda_{1}^{\prime} i_{1}^{\prime} \lambda_{3}^{\prime} i_{3}^{\prime}\right)+\Delta \omega^{I}\left(\lambda_{2}^{\prime} i_{2}^{\prime} \lambda_{3}^{\prime} i_{3}^{\prime}\right)
$$

If we neglect the ground state correlations only one- $\rho_{i}^{J}(r)$ and two-phonon $\rho_{\lambda i \lambda^{\prime} i^{\prime}}^{J}(r)$ densities will contribute to the transition density of the excited state

$$
\begin{equation*}
\rho_{\nu}^{J}(r)=\sum_{i} R_{i}(J \nu) \rho_{i}^{J}(r)+\sum_{\lambda i \lambda^{\prime} i^{\prime}} P_{\lambda i}^{\lambda^{\prime} i^{\prime}}(J \nu) \rho_{\lambda i \lambda^{\prime} i^{\prime}}^{J}(r) \tag{2}
\end{equation*}
$$

The explicit expressions for $\rho_{i}^{J}(r)$ and $\rho_{\lambda_{i} \lambda^{\prime} i^{\prime}}^{J}(r)$ can be found in Ref.[10].

The calculations have been performed for the nonmagic isotope ${ }^{142} \mathrm{Ce}$ and half-magic isotope ${ }^{142} \mathrm{Nd}$. The first isotope was chosen for the following reasons. At first, the interaction of modes with different number of phonon operators in the wave function (2) is strong enough to believe that three-phonon components will contribute to the structure of low-lying states. On the other hand, it is not very strong that allows us to cut the wave function of excited states on the three-phonon terms. And the third reason is that there are experimental data obtained in the ( $\mathrm{e}, \mathrm{e}$ ) reaction [3]. The calculations in ${ }^{142} \mathrm{Nd}$ have been performed in order to see how the results of paper'[5] would change if we take into account three-phonon terms. We truncate the phonon space in our calculations as follows. The first six one-phonon states for the first term of the wave function (2), twenty two-phonon components with the largest matrix elements $U_{\lambda_{2} i_{2}}^{\lambda_{1} i_{1}}(J i)\left(\lambda^{\pi}\right.$ are taken to be $\left.1^{-}, 2^{+}, 3^{-}, 4^{+}, 5^{-}, 6^{+}\right)$and fifty three-phonon components from the energy range $E_{x}=0-6.5 \mathrm{MeV}$ are taken into account. The parameters of the residual interaction have been chosen to reproduce the experimental position and $B(E \lambda)$ values of the lowest state for each $\lambda^{\pi}$ in the calculations with the wave function (2). To see the role of the three-phonon terms in (2) calculations without these terms have been performed with the same set of parameters.

Consider low-lying $2^{+}$states in ${ }^{142} \mathrm{Ce}$. The first $2^{+}$state has the excitation energy $E_{x}=.62 \mathrm{MeV}$ and $B(E 2 \uparrow)=4.2 \cdot 10^{3} e^{2} \cdot f m^{4}$. The main contribution $(74 \%)$ to its structure comes from the first one-phonon term $Q_{2+1}^{+}$, the other components with a visible contribution to (3) are $\left[Q_{2+1}^{+} Q_{4+1}^{+}\right]_{2+}-11 \%$ and $\left[Q_{3-1}^{+} Q_{3-1}^{+}\right]_{2+}-2 \%$. If we neglect the three-phonon terms in (2) the first $2^{+}$ state lies a little higher at $E_{x}=.72 \mathrm{MeV}$ and the changes in the structure are not significant: $81 \%$ comes from $Q_{2+1}^{+}, 7 \%$ - from $\left[Q_{2+1}^{+} Q_{4+1}^{+}\right]_{2+}$ and $2 \%$ - from $\left[Q_{3-1}^{+} Q_{3-1}^{+}\right]_{2+}$. A little increase in the $Q_{2+1}^{+}$component contribution leads to a little increase in the $\mathrm{B}(\mathrm{E} 2)$ value ( $4.5 \cdot 10^{3} e^{2} \cdot f m^{4}$ ) and in the amplitude of the charge transition density which we tried to plot in the left part of Fig. by the dashed curve. .


Figure: Charge transition densities of the $2_{1}^{+}, 2_{2}^{+}, 1_{1}^{-}$states in ${ }^{142} \mathrm{Ce}$ calculated with the wave function (2).

The main contribution to the structure of the second $2^{+}$state comes from the two-phonon component $\left[Q_{2+1}^{+} Q_{2+1}^{+}\right]_{2+}-51 \%$, but the $B(E 2)=2.1$ $10^{2} e^{2} \cdot f m^{4}$ value and the shape of the charge transition density is determined mainly by the contribution of weak one-phonon components ( $25 \%$ from $Q_{2^{+}}^{+}$ and $2 \%$ from $\left.Q_{2+5}^{+}\right)$. To demonstrate this statement we plot by the dashed curve in the central part of Fig. the contribution to the full density coming from $\rho_{\left[Q_{2+1}^{+} Q_{2+1}^{+}\right]_{2+}}(r)$. We have to point out that the two-phonon pole $\left(\omega_{2+1}+\right.$ $\omega_{2}+1$ ) lies at energy 2.8 MeV and a large contribution of the two-phonon term $\left[\dot{Q}_{2+1}^{+} Q_{2+1}^{+}\right]_{2+}$ to the second $2^{+}$state is caused by the interaction with the three-phonon components of the wave function (2). This second $2^{+}$state has the excitation energy 1.65 MeV which is very close to the experimental one $E_{x}=1.54 \mathrm{MeV}$ [11]. If we neglect three-phonon components the contribution from the component $\left[Q_{2+1}^{+} Q_{2+1}^{+}\right]_{2+}$ to the structure of the second $2^{+}$state is much less, only $12 \%$ (the main component becomes the one-phonon $Q_{2+2}^{+}$ component-80\%).

Another interesting example of the role of the three-phonon terms is the first $1^{-}$state. Since the first one-phonon $1^{-}$component has the excitation energy 7.3 MeV , the main contribution to its structure comes from the two-

Table: Excitation energies and B(E2) values for the low-lying $2^{+}$in ${ }^{142} \mathrm{Nd}$ calculated without the three-phonon terms ( 2 ph ) and with them ( 3 ph ) compared with the experimental data from Ref.[5]

|  | 2ph |  | 3ph |  | Experiment |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $E_{x}$, | $\mathrm{B}(\mathrm{E} 2)$, | $E_{x}$, | $\mathrm{B}(\mathrm{E} 2)$, | $E_{x}$, | $\mathrm{B}(\mathrm{E} 2)$, |
| $\nu$ | MeV | $e^{2} \cdot \mathrm{fm}^{4}$ | MeV | $e^{2} \cdot \mathrm{fm}^{4}$ | MeV | $e^{2} \cdot \mathrm{fm}^{4}$ |
| 1 | 1.58 | 5220 | 1.58 | 5210 | 1.58 | 2806 |
| 2 | 2.46 | 167 | 2.46 | 166 | 2.39 | 230 |
| 3 | 2.53 | 62 | 2.53 | 52 | $2.55(?)$ |  |
| 4 | 3.05 | 862 | 3.04 | 836 | 2.85 | 452 |
| 5 | 3.73 | 53 | 3.53 | 81 |  |  |

phonon term $\left[Q_{2^{+}}^{+} Q_{3^{-1}}^{+}\right]_{1-}$, as was shown in Ref.[12]. This two-phonon component lies too high at $E_{x}=4.1 \mathrm{MeV}$, but due to the interaction with the three-phonon terms the first $1^{-}$state appears at $E_{x}=2.3 \mathrm{MeV}$, close to the experimental value [3]. It has $B(E 1)=0.034 e^{2} \cdot f m^{2}$ and the main components are $\left[Q_{2+1}^{+} Q_{3-1}^{+}\right]_{1-}-65 \%$ and $\left[\left[Q_{2+1}^{+} Q_{3-1}^{+}\right]_{3}-Q_{2+1}^{+}\right]_{1}--21 \%$. The charge transition of this state is presented in the right part of Fig.

If we turn now to half-magic or magic isotopes, the role of the threephonon terms in forming the low-lying state structure will be different. In these isotopes the matrix elements $U_{\lambda_{2 i 2}}^{\lambda_{1} i_{1}}(J i)$ are much smaller and practically no changes in the structure of the low-lying states appear due to the threephonon terms. This is clearly seen from the Table where excitation energies and $\mathrm{B}(\mathrm{E} 2)$ values in ${ }^{142} \mathrm{Nd}$ are presented.

Concluding, we would like to point out that the three-phonon terms in the wave function of excited low-lying states will be important for the description of the two-phonon multiplet splitting in nonmagic isotopes.

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