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UNIFIED DESCRIPTION OF ELECTRIC AND MAGNETIC EXCITATIONS IN DEFORMED NUCLEI

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1. Introduction

The microscopic theory of deformed nuclei expounded in many monographs and reviews [1-5] has gained recognition. It is used to treat experimental data on nonrotational states. As is known, the system of coordinates coupled to a deformed nucleus is specified by an axial symmetry, and the angular momentum projection onto the symmetry axis of a nucleus, K, and parity π are good quantum numbers. We shall not consider the Coriolis interaction mixing states with different K and fixed parity. Thus, we shall restrict our consideration to the internal wave function with a good quantum number K.

The quasiparticle-phonon nuclear model (QPNM) uses one-phonon states: quadrupole, octupole, hexadecapole and others, as a basis. It has been shown in [6] that in some cases high multipole interactions with $\lambda = 5$-9 play an important role and they should be taken into account. The specific feature of deformed nuclei is that one-phonon states with a fixed K^{π} can be determined by different multipole and spin-multipole interactions. Thus, the $K^{\pi}=0^{+}$ states are determined by the monopole pairing and quadrupole particle-particle interactions. To them multipole interactions with $\lambda\mu = 40, 60$, etc. must be added. One-phonon states of the electric type with fixed K^{π} can be described by the multipoles $\lambda\mu = KK$, $K{+}2\ K$, $K{+}4\ K$, etc. and by the spin-multipoles $\lambda\lambda\mu = KKK$, $K{+}2\ K{+}2\ K$, etc. Introducing one phonon for λK and another for $\lambda\lambda K$ we shall have a double number of states. To avoid this, a common phonon is introduced, and taking account of different λ the corresponding secular equation is derived (see [5]). Thus, the influence of hexadecapole interactions with $\lambda\mu = 42$ on the $K^{\pi}=2^{+}$ states is studied by a simultaneous inclusion of $\lambda\mu =$

22+42 interactions. In [9], interactions with $\lambda=1$ and $\lambda=3$ were taken into account in studying E1 transitions from octupole to ground states.

One-phonon states of the magnetic type are described by the spin-multipole interactions $\lambda' L K$ with $\lambda' = L-1$ and $L+1$. In spherical nuclei, one-phonon states of the electric type with $I^{\pi}=2^+, 3^-, \ldots$ and magnetic type with $I^{\pi}=1^+, 2^-, 3^+, \ldots$ are described independently. In deformed nuclei, for instance, the $K^{\pi}=2^-$ state can be treated as an electric octupole one with $\lambda\mu=32$ and a magnetic quadrupole one with $\lambda' L K=122$ and 322. The states with $K^{\pi}=1^+$, which are described excluding spurious states [10,11], connected with rotations, are treated with the spin-spin and quadrupole interactions. If in deformed nuclei, as in spherical nuclei, one introduces independent phonons of the electric and magnetic type, the number of states will be doubled. Therefore, it is necessary to construct a common phonon for a state with a fixed K^{π}. The construction of a phonon consisting of the electric and magnetic parts, derivation of the corresponding RPA equations and the inclusion of the new phonon into the general scheme of the QPNM are just the aim of the present paper.

2. The QPNM Hamiltonian

The QPNM Hamiltonian for nonrotational states of deformed nuclei contains an average field of neutron and proton systems in the form of the axial-symmetric Saxon-Woods potential, monopole pairing, isoscalar and isovector particle-hole (ph) and particle-particle (pp) multipole and spin-multipole interactions between quasiparticles. The wave functions of excited states of deformed nuclei have the form

$$\Psi^{I}_{MK}(\nu)=\sqrt{\frac{2I+1}{16\pi^2}}\left\{D^{I}_{MK}\Psi_{\nu}(K^{\pi}\sigma=+)+(-)^{I+K}D^{I}_{M-K}\Psi_{\nu}(K^{\pi},\sigma=-)\right\}. \quad (1)$$

In this paper we study the internal wave functions $\Psi_\nu(K^{\pi}\sigma)$ of excited nonrotational states of doubly even deformed nuclei.

Interactions between quasiparticles in the separable form, usually of the rank $n_{max}=1$, are used for calculations in the QPNM. As is known, separable interactions of the rank $n_{max}>1$ are widely used in describing nucleon-nucleon interactions, three-body nuclear systems and light nuclei, i.e. they are used in the cases where the results of calculations are more sensitive to the form of radial dependence of forces in comparison with the QPNM calculations of the properties of complex nuclei. Therefore, the use of separable interactions of the rank $n_{max}\geq 1$ in the QPNM calculations is justified.

Let us introduce, as in [12] for spherical nuclei, a separable interaction of the rank $n_{max}>1$ for deformed nuclei. Expand over multipoles the central spin independent interaction and write it as

$$\sum_{\substack{q_1 q_2 q_1' q_2' \\ \sigma_1 \sigma_2 \sigma_1' \sigma_2'}} \langle q_1\sigma_1, q_2\sigma_2 \Big| \sum_{\lambda\mu} \left(\varkappa_0^{\lambda\mu} + \varkappa_1^{\lambda\mu}(\tau^{(1)}\tau^{(2)})\right) R^{\lambda\mu}(r_1, r_2)$$

$$\sum_{\sigma=\pm 1} Y_{\lambda\sigma\mu}(\theta_1\varphi_1) Y_{\lambda-\sigma\mu}(\theta_2\varphi_2) \Big| q_2'\sigma_2', q_1'\sigma_1' \rangle a^+_{q_1\sigma_1} a^+_{q_2\sigma_2} a_{q_2'\sigma_2'} a_{q_1'\sigma_1'}.$$

If a separable interaction of the rank $n_{max}>1$ is taken in the form

$$R^{\lambda\mu}(r_1, r_2) = \sum_{n=1}^{n_{max}} R_n^{\lambda\mu}(r_1) R_n^{\lambda\mu}(r_2),$$

then the expansion over multipoles becomes

$$\sum_{\lambda\mu} \sum_{n=1}^{n_{max}} \Big\{ \sum_{\tau\rho=\pm 1} (\varkappa_0^{\lambda\mu} + \rho\varkappa_1^{\lambda\mu}) \sum_{\sigma} M^+_{n\lambda\mu\sigma}(\tau) M_{n\lambda\mu\sigma}(\rho\tau) +$$

$$+ \sum_{\tau\sigma} G^{\lambda\mu} P^+_{n\lambda\mu\sigma}(\tau) P_{n\lambda n\sigma}(\tau) + \dots \Big\}.$$

Introduction of a separable interaction of the finite rank $n_{max} > 1$ in comparison with $n_{max} = 1$ leads to summation over n. Introduction of a separable interaction of the rank n_{max} is meaningful if n_{max} is much smaller than the rank of determinant of the RPA secular equation for a nonseparable interaction.

The starting Hamiltonian of the QPNM is

$$H = \sum_{\tau} \Big\{ \sum_{q\sigma}^{\tau} [E'(q) - \lambda_\tau] a^+_{q\sigma} a_{q\sigma} - G_\tau \sum^{\tau}_{qq'} a^+_{q+} a^+_{q-} a_{q'-} a_{q'+} -$$

$$- \frac{1}{2} \sum_{\lambda\mu\sigma} \sum_{n=1}^{n_{max}} \Big[\sum_{\rho=\pm 1} (\varkappa_0^{\lambda\mu} + \rho \varkappa_1^{\lambda\mu}) M^+_{n\lambda\mu\sigma}(\tau) M_{n\lambda\mu\sigma}(\rho\tau) +$$

$$+ G^{\lambda\mu} P^+_{n\lambda\mu\sigma}(\tau) P_{n\lambda\mu\sigma}(\tau) \Big] - \frac{1}{2} \sum_{LK\sigma} \sum_{\lambda'=L, L\pm 1} \sum_{n=1}^{n_{max}} \Big[\sum_{\rho=\pm 1} (\varkappa_0^{\lambda' LK} + \qquad (2)$$

$$+ \rho \varkappa_1^{\lambda' LK}) (S^{\lambda'}_{nLK\sigma}(\tau))^+ S^{\lambda'}_{nLK\sigma}(\rho\tau) + G^{\lambda' LK} (P^{\lambda'}_{nLK\sigma}(\tau))^+ P^{\lambda'}_{nLK\sigma}(\tau) \Big] \Big\} .$$

Here $q\sigma$ are quantum numbers of single-particle states, q equals to K^{π} and asymptotic quantum numbers $Nn_z\Lambda\uparrow$ at $K = \Lambda + 1/2$ and $Nn_z\Lambda\downarrow$ at $K = \Lambda - 1/2$, $\sigma = \pm 1$; $E(q)$ are the single-particle energies, λ_τ is the chemical potential; $\sum^{\tau}_{qq'}$ means summation over single-particle states of the proton at $\tau = p$ and neutron at $\tau = n$ systems. Then, G_τ are the monopole pairing constants, $G^{\lambda\mu}$ and $G^{\lambda' LK}$ are the constants of pp interactions; $\varkappa_0^{\lambda\mu}$, $\varkappa_0^{\lambda' LK}$ and $\varkappa_1^{\lambda\mu}$, $\varkappa_1^{\lambda' LK}$ are the isoscalar and isovector constants of ph multipole and spin-multipole interactions.

Let us perform the canonical Bogolubov transformation

$$a_{q\sigma} = u_q \alpha_{q\sigma} + \sigma v_q \alpha^+_{q-\sigma}$$

and get

$$M_{n\lambda\mu\sigma}(\tau) = \frac{1}{2}\sum_{q_1 q_2}^{\tau} f^{\lambda\mu}(q_1 q_2)\Big\{ u^{(+)}_{q_1 q_2}\big[A^+(q_1 q_2;\mu\sigma) + A(q_1 q_2;\mu-\sigma)\big] + 2v^{(-)}_{q_1 q_2} B(q_1 q_2;\mu\sigma)\Big\},$$

$$P_{n\lambda\mu\sigma}(\tau) = \frac{1}{2}\sum_{q_1 q_2}^{\tau} f_n^{\lambda\mu}(q_1 q_2)\Big\{ v^{(+)}_{q_1 q_2}\big[A(q_1 q_2;\mu\sigma) - A^+(q_1 q_2;\mu-\sigma)\big] + v^{(-)}_{q_1 q_2}\big[A(q_1 q_2;\mu\sigma) + A^+(q_1 q_2;\mu-\sigma)\big] - 4u_{q_2} v_{q_1} B(q_1 q_2;\mu-\sigma)\Big\},$$

$$S^{L\pm 1}_{nLK\sigma}(\tau) = \frac{1}{2}\sum_{q_1 q_2}^{\tau} f_n^{L\pm 1 LK}(q_1 q_2)\Big\{ u^{(-)}_{q_1 q_2}\big[\mathcal{A}^+(q_1 q_2;K\sigma) + \mathcal{A}(q_1 q_2;K-\sigma)\big] + 2v^{(+)}_{q_1 q_2}\mathcal{B}(q_1 q_2;K\sigma)\Big\}$$

and other formulae. Here

$$A^+(q_1 q_2;\mu\sigma) = \sum_{\sigma'} \delta_{\sigma'(K_1-K_2),\sigma\mu}\, \sigma' \alpha^+_{q_1\sigma'} \alpha^+_{q_2-\sigma'} \quad \text{or} \quad \delta_{K_1+K_2,\mu}\, \alpha^+_{q_2\sigma} \alpha^+_{q_1\sigma},$$

$$\mathcal{A}^+(q_1 q_2;\mu\sigma) = \sum_{\sigma'} \delta_{\sigma'(K_1-K_2),\sigma\mu}\, \alpha^+_{q_1\sigma'} \alpha^+_{q_2-\sigma'} \quad \text{or} \quad \delta_{K_1+K_2,\mu}\, \sigma \alpha^+_{q_2\sigma} \alpha^+_{q_1\sigma},$$

$$B(q_1q_2;\mu\sigma)=\sum_{\sigma'}\delta_{\sigma'(K_1-K_2),\sigma\mu}\alpha^+_{q_1\sigma'}\alpha_{q_2-\sigma'} \quad \text{or} \quad \delta_{K_1+K_2,\mu}\,\sigma\,\alpha^+_{q_1\sigma}\alpha_{q_2-\sigma},$$

$$\mathcal{B}(q_1q_2;\mu\sigma)=\sum_{\sigma'}\delta_{\sigma'(K_1-K_2),\sigma\mu}\,\sigma'\alpha^+_{q_1\sigma'}\alpha_{q_2\sigma'} \quad \text{or} \quad \delta_{K_1+K_2,\mu}\alpha^+_{q_1\sigma}\alpha_{q_2-\sigma},$$

$$u^{(\pm)}_{q_1q_2}=u_{q_1}v_{q_2}\pm u_{q_2}v_{q_1}, \quad v^{(\pm)}_{q_1q_2}=u_{q_1}u_{q_2}\pm v_{q_1}v_{q_2}.$$

The matrix elements of the multipole and spin-multipole operators are expressed through

$$f_n^{\lambda\mu}(q_1q_2)=\langle q_1|R_n^{\lambda\mu}(r)Y_{\lambda\mu}(\theta,\varphi)|q_2\rangle,$$
$$f_n^{\lambda' LK}(q_1q_2)=\langle q_1|R_n^{\lambda' LK}(r)\{\sigma Y_{\lambda'}(\theta\varphi)\}_{LK}|q_2\rangle, \tag{3}$$

their characteristics are given in [1,5].

Using the operators $A^+(qq';\mu\sigma)$ and $A(qq';\mu\sigma)$ to construct phonons of the electric type, as in [1,4,5], and the operators $\mathcal{O}\!\iota^+(qq';\mu\sigma)$ and $\mathcal{O}\!\iota(qq';\mu\sigma)$ to construct phonons of the magnetic type, then in contrast with the spherical nuclei we shall have a doubled number of states. Consider, for instance, the $K^{\pi}=2^-$ states shown in fig. 1. They can be described as one-phonon octupole states with $\lambda\mu$=32 and as a rule, with the enhanced E3 transition from the $I^{\pi}K_i=3^-2_1$ to the ground states. Between the first and second poles there is a second $K_i^{\pi}=2_2^-$ state whose energy is determined as the second i=2 root of the RPA secular equation. At the same time, these $K_i^{\pi}=2_1^-$ and 2_2^- states can be described as one-phonon quadrupole states of the magnetic type and with the enhanced M2 transition from the $I^{\pi}K_i=2^-2_1$ to the ground states. If the set of one-

-phonon states is chosen as a basis, as in the QPNM, then the number of one-phonon states with a fixed value of K^{π} equals the number of two-quasiparticle poles. If phonons of the electric and magnetic type are introduced in the deformed nuclei, then the number of states will be doubled. Therefore, it is necessary to construct a common phonon operator consisting of the electric and magnetic parts. The phonon creation operator consisting of the electric and magnetic parts and with a fixed value of K^{π} can be written as follows:

$$Q^{+}_{Ki_2\sigma} = \frac{1}{2}\sum_{qq'}\{\psi^{Ki_2}_{qq'}[f^{\lambda K}(qq')u^{(+)}_{qq'}A^{+}(qq';K\sigma) +$$
$$+ if^{LK}(qq')u^{(-)}_{qq'}\mathcal{O}l^{+}(qq';K\sigma)] - \tag{4}$$
$$-\varphi^{Ki_2}_{qq'}[f^{\lambda K}(qq')u^{(+)}_{qq'}A(qq';K-\sigma) +$$
$$+ if^{LK}(qq')u^{(-)}_{qq'}\mathcal{O}l(qq';K-\sigma)]\}.$$

Here $\psi^{Ki_2}_{qq'} = \psi^{Ki_2}_{q'q}$ and $\varphi^{Ki_2}_{qq'} = \varphi^{Ki_2}_{q'q}$ are common for the electric and magnetic parts, which indicates the existence of a one-phonon state with a fixed value of the state number i_2 where $i_2 = 1, 2, 3, \ldots$. By $f^{LK}(qq')$ we denote the matrix element (3) with $\lambda' LK = L\text{-}1LK$ or $L+1LK$. When we use a separable interaction with $n_{max} > 1$ by $f^{\lambda K}(qq')$ and $f^{LK}(qq')$ we denote the matrix elements with $n = 1$. It is seen from formula (4) that the electric part of the phonon creation operator $Q^{+}_{Ki_2\sigma}$ is chosen to be real and the magnetic part to be imaginary.

The one-phonon state in the RPA is described by the wave function

$$Q^{+}_{Ki_2\sigma}\Psi_0, \tag{5}$$

where Ψ_0 is the ground state wave function of a doubly even nucleus determined as a phonon vacuum. The normalisation condition of the wave function (5) has the form

$$\frac{1}{2}\sum_{qq'}\left[(\psi^{Ki_2}_{qq'})^2-(\varphi^{Ki_2}_{qq'})^2\right]\gamma^K(qq')=1,$$

$$\gamma^K(qq')=\left|f^{\lambda K}(qq')u^{(+)}_{qq'}+if^{LK}(qq')u^{(-)}_{qq'}\right|^2= \quad (6)$$

$$=\left[f^{\lambda K}(qq')u^{(+)}_{qq'}\right]^2+\left[f^{LK}(qq')u^{(-)}_{qq'}\right]^2,$$

$$A^+(qq';K\sigma)=f^{\lambda K}(qq')u^{(+)}_{qq'}\sum_{i_2}\left[\psi^{Ki_2}_{qq'}Q^+_{Ki_2\sigma}+\varphi^{Ki_2}_{qq'}Q_{Ki_2-\sigma}\right], \quad (7)$$

$$\mathcal{A}(qq';K\sigma)=-if^{LK}(qq')u^{(-)}_{qq'}\sum_{i_2}\left[\psi^{Ki_2}_{qq'}Q^+_{Ki_2\sigma}-\varphi^{Ki_2}_{qq'}Q_{Ki_2-\sigma}\right].$$

One can easily show that the phonon operators $Q^+_{Ki_2\sigma}$ and $Q_{Ki_2\sigma}$ obey the conditions which are usually imposed on the RPA phonons.

Using formulae (7) we express the operators $M_{n\lambda\mu\sigma}(\tau)$, $P_{n\lambda\mu\sigma}(\tau)$, $S^{\lambda'}_{nLK\sigma}(\tau)$ and others through the phonon operators. After simple transformations the QPNM Hamiltonian becomes

$$H=\sum_{q\sigma}\varepsilon_q\alpha^+_{q\sigma}\alpha_{q\sigma}+H_v+H_{vq}, \quad (8)$$

where the first two terms describe quasiparticles and phonons, and H_{vq} describes the quasiparticle-phonon interaction. They have the following form:

$$H_v=H_v^{00}+\sum_\lambda H_v^{\lambda 0}+\sum_K H_v^K, \quad (9)$$

$$H_v^K = -\sum_{i_1 i_2 \sigma} W^K_{i_1 i_2} Q^+_{K i_1 \sigma} Q_{K i_2 \sigma} , \qquad (10)$$

$$W^K_{i_1 i_2} = \sum_\lambda \left[W^{\lambda K}_{i_1 i_2} + W^{\lambda\lambda K}_{i_1 i_2} \right] + \sum_L \sum_{\lambda' = L \pm 1} W^{\lambda' L K}_{i_1 i_2} , \qquad (10^1)$$

$$W^{\lambda K}_{i_1 i_2} = \frac{1}{4} \sum_{n=1}^{n_{max}} \sum_\tau \Big\{ \sum_{\rho = \pm 1} (\varkappa_0^{\lambda K} + \rho \varkappa_1^{\lambda K}) D^{\lambda K i_1}_{n\tau} D^{\lambda K i_2}_{n\rho\tau} +$$

$$+ G^{\lambda K} \left[D^{\lambda K i_1}_{ng\tau} D^{\lambda K i_2}_{ng\tau} + D^{\lambda K i_1}_{nw\tau} D^{\lambda K i_2}_{nw\tau} \right] \Big\} ,$$

$$W^{\lambda' L K}_{i_1 i_2} = \frac{1}{4} \sum_{n=1}^{n_{max}} \sum_\tau \Big\{ \sum_{\rho = \pm 1} (\varkappa_0^{\lambda' L K} + \rho \varkappa_1^{\lambda' L K}) D^{\lambda' L K i_1}_{n\tau} D^{\lambda' L K i_2}_{n\rho\tau} +$$

$$+ G^{\lambda K} \left[D^{\lambda' L K i_1}_{ng\tau} D^{\lambda' L K i_2}_{ng\tau} + D^{\lambda' L K i_1}_{nw\tau} D^{\lambda' L K i_2}_{nw\tau} \right] \Big\} ,$$

$$H_{vq} = H^{00}_{vq} + \sum_\lambda H^{\lambda 0}_{vq} + \sum_K \Big\{ \sum_\lambda (H^{\lambda K}_{vq} +$$

$$+ H^{\lambda\lambda K}_{vq}) + \sum_L \sum_{\lambda' = L \pm 1} H^{\lambda' L K}_{vq} \Big\} , \qquad (11)$$

$$H^{\lambda K}_{vq} = -\frac{1}{4} \sum_{n i_2 \tau \sigma} \sum_{q q'}^{\tau} f_n^{\lambda K}(qq') V^{\lambda K i_2}_{n\tau}(qq') \Big[(Q^+_{K i_2 \sigma} + \qquad (12)$$

$$+ Q_{K i_2 -\sigma}) B(qq'; K -\sigma) + B(qq'; K\sigma) (Q^+_{K i_2 -\sigma} + Q_{K i_2 \sigma}) \Big] ,$$

$$H_{vq}^{\lambda' LK} = \frac{i}{4} \sum_{n i_2 \tau \sigma} \sum_{qq'}^{\tau} f_n^{\lambda' LK}(qq') V_{n\tau}^{\lambda' LK i_2}(qq') \left[(Q_{K i_2 \sigma}^{+} - Q_{K i_2 -\sigma}) \cdot \right.$$

$$\left. \cdot \mathcal{B}(qq'; K-\sigma) + \mathcal{B}(qq'; K\sigma)(Q_{K i_2 -\sigma}^{+} - Q_{K i_2 \sigma}) \right], \qquad (12^1)$$

$$V_{n\tau}^{\lambda K i_2}(qq') = \sum_{\rho = \pm 1} (\varkappa_0^{\lambda K} + \rho \varkappa_1^{\lambda K}) v_{qq'}^{(-)} D_{n\rho\tau}^{\lambda K i_2} - \qquad (13)$$

$$- G^{\lambda K} u_{qq'}^{(+)} D_{ng\tau}^{\lambda K i_2},$$

$$V_{n\tau}^{\lambda' LK i_2}(qq') = \sum_{\rho = \pm 1} (\varkappa_0^{\lambda' LK} + \rho \varkappa_1^{\lambda' LK}) v_{qq'}^{(+)} D_{n\rho\tau}^{\lambda' LK i_2}. \qquad (13^1)$$

Here ε_q is the quasiparticle energy with the monopole and quadrupole pairing; H_v^{00}, $H_v^{\lambda 0}$, H_{vq}^{00} and $H_{vq}^{\lambda 0}$ are given in [13],

$$D_{n\tau}^{\lambda K i_2} = \sum_{qq'}^{\tau} f^{\lambda K}(qq') f_n^{\lambda K}(qq') (u_{qq'}^{(+)})^2 g_{qq'}^{K i_2},$$

$$D_{ng\tau}^{\lambda K i_2} = \sum_{qq'}^{\tau} f^{\lambda K}(qq') f_n^{\lambda K}(qq') u_{qq'}^{(+)} v_{qq'}^{(-)} g_{qq'}^{K i_2},$$

$$D_{nw\tau}^{\lambda K i_2} = \sum_{qq'}^{\tau} f^{\lambda K}(qq') f_n^{\lambda K}(qq') u_{qq'}^{(+)} v_{qq'}^{(+)} w_{qq'}^{g i_2},$$

$$D_{n\tau}^{\lambda' LK i_2} = \sum_{qq'}^{\tau} f^{\lambda' LK}(qq') f_n^{\lambda' LK}(qq') (u_{qq'}^{(-)})^2 g_{qq'}^{K i_2}, \qquad (14)$$

$$D_{ng\tau}^{\lambda' LK i_2} = \sum_{qq'}^{\tau} f^{\lambda' LK}(qq') f_n^{\lambda' LK}(qq') u_{qq'}^{(-)} v_{qq'}^{(+)} g_{qq'}^{K i_2},$$

$$D_{nw\tau}^{\lambda' LK i_2} = \sum_{qq'}^{\tau} f^{\lambda' LK}(qq') f_n^{\lambda' LK}(qq') u_{qq'}^{(-)} v_{qq'}^{(-)} w_{qq'}^{K i_2},$$

$$g_{qq'}^{K i_2} = \psi_{qq'}^{K i_2} + \varphi_{qq'}^{K i_2}, \quad w_{qq'}^{K i_2} = \psi_{qq'}^{K i_2} - \varphi_{qq'}^{K i_2}.$$

One can easily verify that the Hamiltonian (8) and its parts (9), (10), (11), (12) and (12^1) are Hermitian.

3. The RPA equation

Let us derive the RPA equations for the energies ω_{Ki_0} and wave functions (5) of one-phonon states. The RPA equations for the $K^{\pi}=0^+$ states are given in [13]. To describe the states with $K^{\pi}\neq 0^+$ we use the following part of the Hamiltonian (9)

$$\sum_{q\sigma} \varepsilon_q \alpha^+_{q\sigma} \alpha_{q\sigma} + H^K_v . \tag{15}$$

Now, we find an average value (15) over the state (5) and using the variational principle

$$\delta \Big\{ \langle Q_{Ki_0\sigma} \Big[\sum_{q\sigma} \varepsilon_q \alpha^+_{q\sigma} \alpha_{q\sigma} + H^K_v \Big] Q^+_{Ki_0\sigma} \rangle -$$

$$- \frac{\omega_{Ki_0}}{2} \Big[\sum_{qq'} g^{Ki_0}_{qq'} w^{Ki_0}_{qq'} \gamma^K(qq') - 2 \Big] \Big\} = 0$$

we get the following equations:

$$\varepsilon_{qq'} \gamma^K(qq') g^{Ki_0}_{qq'} - \omega_{Ki_0} \gamma^K(qq') w^{Ki_0}_{qq'} -$$

$$- \sum_{n=1}^{n_{max}} \Big\{ \sum_{\rho=\pm 1} (\varkappa_0^{\lambda K} + \rho \varkappa_1^{\lambda K}) f^{\lambda K}(qq') f_n^{\lambda K}(qq') (u^{(+)}_{qq'})^2 D^{\lambda K i_0}_{n\rho\tau} +$$

$$+ G^{\lambda K} f^{\lambda K}(qq') f_n^{\lambda K}(qq') u^{(+)}_{qq'} v^{(-)}_{qq'} D^{\lambda K i_0}_{ng\tau} + \tag{16}$$

$$+ G^{\lambda\lambda K} f^{\lambda\lambda K}(qq') f_n^{\lambda\lambda K}(qq') u^{(-)}_{qq'} v^{(-)}_{qq'} D^{\lambda\lambda K i_0}_{ng\tau} +$$

$$+\sum_{\lambda'=L\pm1}\Big[\sum_{\rho=\pm1}(\varkappa_0^{\lambda' LK}+\rho\varkappa_1^{\lambda' LK})f^{\lambda' LK}(qq')f_n^{\lambda' LK}(qq')(u_{qq'}^{(-)})^2\cdot$$

$$\cdot D_{n\rho\tau}^{\lambda' LKi_0}+G^{\lambda' LK}f^{\lambda' LK}(qq')f_n^{\lambda' LK}(qq')\,u_{qq'}^{(-)}\,v_{qq'}^{(+)}\cdot$$

$$\cdot D_{ng\tau}^{\lambda' LKi_0}\Big]\Big\}=0,$$

$$\mathcal{E}_{qq'}\gamma^{K}(qq')\,w_{qq'}^{Ki_0}-\omega_{Ki_0}\gamma^{K}(qq')\,g_{qq'}^{Ki_0}-$$

$$-\sum_{n=1}^{n_{max}}\Big\{G^{\lambda K}f^{\lambda K}(qq')f_n^{\lambda K}(qq')\,u_{qq'}^{(+)}\,v_{qq'}^{(+)}\,D_{nw\tau}^{\lambda Ki_0}-$$

$$-\sum_{\rho=\pm1}(\varkappa_0^{\lambda\lambda K}+\rho\varkappa_1^{\lambda\lambda K})f^{\lambda\lambda K}(qq')f_n^{\lambda\lambda K}(qq')(u_{qq'}^{(-)})^2D_{n\rho\tau}^{\lambda\lambda Ki_0}+$$

$$+G^{\lambda\lambda K}f^{\lambda\lambda K}(qq')f_n^{\lambda\lambda K}(qq')\,u_{qq'}^{(-)}\,v_{qq'}^{(+)}\,D_{nw\tau}^{\lambda\lambda Ki_0}+$$

$$+\sum_{\lambda'=L\pm1}G^{\lambda' LK}f^{\lambda' LK}(qq')f_n^{\lambda' LK}(qq')\,u_{qq'}^{(-)}\,v_{qq'}^{(-)}\,D^{\lambda' LKi_0}\Big\}=0, \tag{17}$$

where $\mathcal{E}_{qq'}=\mathcal{E}_q+\mathcal{E}_{q'}$ and $\langle\dots\rangle$ means averaging over the phonon vacuum.

From eqs. (16) and (17) we get the functions $g_{qq'}^{Ki_0}$ and $w_{qq'}^{Ki_0}$ and substitute them into formulae for $D_{n\tau}^{\lambda Ki_0}$, $D_{ng\tau}^{\lambda Ki_0}$, $D_{nw\tau}^{\lambda Ki_0}$, $D_{n\tau}^{\lambda\lambda Ki_0}$, $D_{ng\tau}^{\lambda\lambda Ki_0}$, $D_{nw\tau}^{\lambda\lambda Ki_0}$, $D_{n\tau}^{L\pm1LKi_0}$, $D_{ng\tau}^{L\pm1LKi_0}$ and $D_{nw\tau}^{L\pm1LKi_0}$. With the allowance made for $\tau=\rho, n$ and $n=1,2,\dots n_{max}$ the secular equation for the energies ω_{Ki_0} has the form of the determinant of the rank $24\cdot n_{max}$, i.e.

$$\det \| 24 \cdot n_{max} \| = 0. \tag{18}$$

If the spin-multipole interactions of the electric type $\lambda\lambda K$ are disregarded, the rank of the determinant is $18 n_{max}$. If only ph interactions are taken into account, the rank of the determinant is $8 n_{max}$. The most interesting case is when ph and pp multipole and ph $L\text{-}1LK$ spin-multipole interactions are taken into consideration; then, the rank of the determinant is $8 n_{max}$. It is to be noted that a particular case of eqs. (16) and (17) for $n_{max} = 1$ and ph, pp multipole interactions is given in [14]. The tensor forces being added in the Hamiltonian, as in [12], won't change the rank of the determinant (18).

To illustrate the RPA solutions we shall consider two particular cases. The first case is the inclusion of ph multipole λK and spin-multipole $L\text{-}1LK$ interactions with $n_{max} = 1$. We denote $\varkappa^{L-1LK} = \varkappa^{LK}$ and $f^{L-1LK}(qq') \equiv f^{LK}(qq')$. The Hamiltonian is taken in the form

$$H_0^K = \sum_{q\sigma} \varepsilon_q \alpha^+_{q\sigma} \alpha_{q\sigma} - \frac{1}{4} \sum_{\rho=\pm 1 \tau} \sum_{i_1 i_2 \sigma} \Big\{ (\varkappa_0^{\lambda K} + \rho \varkappa_1^{\lambda K}) D_\tau^{\lambda K i_1} D_{\rho\tau}^{\lambda K i_2} + \tag{19}$$

$$+ (\varkappa_0^{LK} + \rho \varkappa_1^{LK}) D_\tau^{LK i_1} D_{\rho\tau}^{LK i_2} \Big\} Q^+_{K i_1 \sigma} Q_{K i_2 \sigma}.$$

Using the variational principle we get instead of (16) and (17) the following equations:

$$(\varepsilon^2_{qq'} - \omega_{K i_2}) \gamma^K(qq') g^{K i_2}_{qq'} - (f^{\lambda K}(qq') u^{(+)}_{qq'})^2 \varepsilon_{qq'} \sum_{\rho=\pm 1} (\varkappa_0^{\lambda K} +$$

$$+ \rho \varkappa_1^{\lambda K}) D^{\lambda K i_2}_{\rho\tau} - (f^{LK}(qq') u^{(-)}_{qq'})^2 \varepsilon_{qq'} \sum_{\rho=\pm 1} (\varkappa_0^{LK} + \rho \varkappa_1^{LK}) D^{LK i_2}_{\rho\tau} = 0,$$

$$w^{K i_2}_{qq'} = \frac{\omega_{K i_2}}{\varepsilon_{qq'}} g^{K i_2}_{qq'}.$$

Then, the normalisation condition of the wave function (5) is

$$\frac{1}{2}\sum_{qq'}\frac{\omega_{Ki_2}}{\varepsilon_{qq'}}\left(g_{qq'}^{Ki_2}\right)^2\gamma^K(qq')=1\,,$$

and the secular RPA equation becomes

$$\begin{vmatrix}
(\varkappa_0^{\lambda K}+\varkappa_1^{\lambda K})Z_p^{\lambda K i_2}-1 & (\varkappa_0^{\lambda K}-\varkappa_1^{\lambda K})Z_p^{\lambda K i_2} & (\varkappa_0^{LK}+\varkappa_1^{LK})Z_p^{\lambda LK i_2} & (\varkappa_0^{LK}-\varkappa_1^{LK})Z_p^{\lambda LK i_2}\\
(\varkappa_0^{\lambda K}-\varkappa_1^{\lambda K})Z_n^{\lambda K i_2} & (\varkappa_0^{\lambda K}+\varkappa_2^{\lambda K})Z_n^{\lambda K i_2}-1 & (\varkappa_0^{LK}-\varkappa_1^{LK})Z_n^{\lambda LK i_2} & (\varkappa_0^{LK}+\varkappa_1^{LK})Z_n^{\lambda LK i_2}\\
(\varkappa_0^{\lambda K}+\varkappa_1^{\lambda K})Z_p^{\lambda LK i_2} & (\varkappa_0^{\lambda K}-\varkappa_1^{\lambda K})Z_p^{\lambda LK i_2} & (\varkappa_0^{LK}+\varkappa_1^{LK})Z_p^{LK i_2}-1 & (\varkappa_0^{LK}-\varkappa_1^{LK})Z_p^{LK i_2}\\
(\varkappa_0^{\lambda K}-\varkappa_1^{\lambda K})Z_n^{\lambda LK i_2} & (\varkappa_0^{\lambda K}+\varkappa_1^{\lambda K})Z_n^{\lambda LK i_2} & (\varkappa_0^{LK}-\varkappa_1^{LK})Z_n^{LK i_2} & (\varkappa_0^{LK}+\varkappa_1^{LK})Z_n^{LK i_2}-1
\end{vmatrix}=0, \tag{20}$$

where

$$Z_\tau^{\lambda K i_2}=\sum_{qq'}^{\tau}\frac{\left(f^{\lambda K}(qq')\,u_{qq'}^{(+)}\right)^4\varepsilon_{qq'}}{\gamma^K(qq')(\varepsilon_{qq'}^2-\omega_{Ki_2}^2)}\,,$$

$$Z_\tau^{LK i_2}=\sum_{qq'}^{\tau}\frac{\left(f^{LK}(qq')\,u_{qq'}^{(-)}\right)^4\varepsilon_{qq'}}{\gamma^K(qq')(\varepsilon_{qq'}^2-\omega_{Ki_2}^2)}\,,$$

$$Z_\tau^{\lambda LK i_2}=\sum_{qq'}^{\tau}\frac{\left(f^{\lambda K}(qq')\,u_{qq'}^{(+)}f^{LK}(qq')\,u_{qq'}^{(-)}\right)^2\varepsilon_{qq'}}{\gamma^K(qq')(\varepsilon_{qq'}^2-\omega_{Ki_2}^2)}\,.$$

Owing to the fact that $Z_{\tau}^{\lambda LKi_2}$ differs from zero, the secular equation (20) does not disintegrates into two equations for the electric and magnetic parts.

The second particular case is the same interactions as in the Hamiltonian (19) but for one (neutron or proton) system. In this case we get explicit expressions for the functions $\psi_{qq'}^{Ki_2}$ and $\varphi_{qq'}^{Ki_2}$. The secular eq.(20) takes the form

$$(\varkappa^{\lambda K} Z^{\lambda K i_2} - 1)(\varkappa^{LK} Z^{LKi_2} - 1) = \varkappa^{\lambda K} \varkappa^{LK} (Z^{\lambda LKi_2})^2, \quad (21)$$

and

$$D^{LKi_2} = y^{Ki_2} D^{\lambda K i_2}, \quad y^{Ki_2} = \frac{1 - \varkappa^{\lambda K} Z^{\lambda K i_2}}{\varkappa^{LK} Z^{\lambda LKi_2}} .$$

Then

$$\psi_{qq'}^{Ki_2} = \frac{\varepsilon_{qq'} + \omega_{Ki_2}}{2\varepsilon_{qq'}} g_{qq'}^{Ki_2}, \quad \varphi_{qq'}^{Ki_2} = \frac{\varepsilon_{qq'} - \omega_{Ki_2}}{2\varepsilon_{qq'}} g_{qq'}^{Ki_2},$$

$$g_{qq'}^{Ki_2} = \sqrt{\frac{2}{\mathcal{Y}^{Ki_2}}} \frac{\varepsilon_{qq'}}{\gamma^K(qq')(\varepsilon_{qq'}^2 - \omega_{Ki_2}^2)} \Big\{ \varkappa^{\lambda K} (f^{\lambda K}(qq') u_{qq'}^{(+)})^2 +$$

$$+ y^{Ki_2} \varkappa^{LK} (f^{LK}(qq') u_{qq'}^{(-)})^2 \Big\} .$$

It is seen that the function $g_{qq'}^{Ki_2}$ consists of the terms of the electric and magnetic type. Here

$$\mathcal{Y}^{Ki_2} = (\varkappa^{\lambda K})^2 Y^{\lambda K i_2} + (y^{Ki_2})^2 Y^{LKi_2} (\varkappa^{LK})^2 + 2 \varkappa^{\lambda K} \varkappa^{LK} y^{Ki_2} Y^{\lambda LKi_2},$$

$$Y^{\lambda K i_2} = \sum_{qq'} \frac{(f^{\lambda K}(qq') u_{qq'}^{(+)})^4 \varepsilon_{qq'} \omega_{Ki_2}}{\gamma^K(qq')(\varepsilon_{qq'}^2 - \omega_{Ki_2}^2)^2},$$

$$Y^{\mu K i_2} = \sum_{qq'} \frac{\left(f^{\mu K}(qq') u^{(-)}_{qq'}\right)^4 \varepsilon_{qq'} \omega_{Ki_2}}{\gamma^K(qq')\left(\varepsilon^2_{qq'} - \omega^2_{Ki_2}\right)^2},$$

$$Y^{\lambda \mu K i_2} = \sum_{qq'} \frac{\left(f^{\lambda K}(qq') u^{(+)}_{qq'} f^{\mu K}(qq') u^{(-)}_{qq'}\right)^2 \varepsilon_{qq'} \omega_{Ki_2}}{\gamma^K(qq')\left(\varepsilon^2_{qq'} - \omega^2_{Ki_2}\right)^2}.$$

One can easily show that when the energy $\omega_{K_0 i_0}$ tends to the pole $\varepsilon_{q_1 0 q_2 0}$ the wave function of the one-phonon state (5) tends to

$$Q^+_{K_0 i_0 \sigma_0} \Psi_0 \Big|_{\varepsilon_{q_1 0 q_2 0} - \omega_{K_0 i_0} \to 0} = \delta_{\sigma(K_1^0 - K_2^0), \sigma_0 K_0} \alpha^+_{q_1^0 \sigma} \alpha^+_{q_2^0 - \sigma} \Psi_0 ,$$

i.e. to the wave function of the two-quasiparticle state. Moreover, for the solutions of the secular eq. (21) the following condition holds:

$$\langle \Big\{ \sum_{q\sigma} \varepsilon_q \alpha^+_{q\sigma} \alpha_{q\sigma} - \frac{1}{8} \sum_{i_2 \sigma} \left[\varkappa^{\lambda K} (D^{\lambda K i_2})^2 + \varkappa^{\mu K} (D^{\mu K i_2})^2 \right] \cdot$$

$$\cdot Q_{K i_2 \sigma} Q_{K i_2 - \sigma} \Big\} Q^+_{K i_2 \sigma} Q^+_{K i_2 - \sigma} \rangle = 0 ,$$

i.e. "dangerous" diagrams are compensated. These facts indicate additionally that the choice of the phonon creation operator in the form (4) is correct.

4. The QPNM equations for doubly even deformed nuclei

Here we give formulae for describing nonrotational states with $K^{\pi} \neq 0^{+}$ in the QPNM with the new phonons $Q^{+}_{Ki_2\sigma}$ and $Q_{Ki_2\sigma}$. The wave function (as in [14]) can be written in the form

$$\Psi_{\nu}(K_0^{\pi_0}\sigma_0) = \Big\{ \sum_{i_0} R^{\nu}_{i_0} Q^{+}_{K_0 i_0 \sigma_0} + \sum_{\substack{K_1 i_1 \sigma_1 \\ K_2 i_2 \sigma_2}} \frac{1}{2}(1+\delta_{K_1 i_1, K_2 i_2})^{1/2} \cdot$$

$$\cdot \delta_{\sigma_1 K_1 + \sigma_2 K_2, \sigma_0 K_0} P^{\nu}_{K_1 i_1, K_2 i_2} Q^{+}_{K_1 i_1 \sigma_1} Q^{+}_{K_2 i_2 \sigma_2} \Big\} \Psi_0 , \tag{22}$$

where $\nu = 1,2,3,...$ is the number of the state with $K_0^{\pi_0}$. To take the Pauli principle into account in two-phonon terms of the wave function (22) we introduce the function

$$\mathcal{K}^{K_0}(K_2 i_2, K_1 i' \,|\, K_1 i_1, K_2 i_2) = (1+\delta_{K_1 i_1, K_2 i_2})^{-1} \cdot$$

$$\cdot \sum_{\sigma_1 \sigma_2} \delta_{\sigma_1 K_1 + \sigma_2 K_2, \sigma_0 K_0} \langle Q_{K_2 i_2 \sigma_2} \big[[Q_{K_1 i' \sigma_1}, Q^{+}_{K_1 i_1 \sigma_1}] Q^{+}_{K_2 i_2 \sigma_2} \big] \rangle , \tag{23}$$

$$\mathcal{K}^{K_0}(K_1 i_1, K_2 i_2) \equiv \mathcal{K}^{K_0}(K_2 i_2, K_1 i_1 \,|\, K_1 i_1, K_2 i_2).$$

Its explicit form is given in [5,15].

The normalisation condition of the wave function (22) in the diagonal in $\mathcal{K}^{K_0}$ approximation has the form

$$\sum_{i_0}(R^{\nu}_{i_0})^2+\sum_{K_1 i_1\leq K_2 i_2}(P^{\nu}_{K_1 i_1,K_2 i_2})^2\left[1+\mathcal{K}^{K_0}(K_1 i_1,K_2 i_2)\right]=1. \tag{24}$$

Now, let us find an average value of the Hamiltonian (8) over the state (22) and using the variational principle derive the following equations for the energies η_ν and wave function (22)

$$(\omega_{K_0 i_0}-\eta_\nu)R^{\nu}_{i_0}-\sum_{K_1 i_1\leq K_2 i_2}(1+\delta_{K_1 i_1,K_2 i_2})^{-1/2}P^{\nu}_{K_1 i_1,K_2 i_2}\cdot$$

$$\cdot U^{K_0 i_0}_{K_1 i_1,K_2 i_2}\left[1+\mathcal{K}^{K_0}(K_1 i_1,K_2 i_2)\right]=0,$$

$$\left[\omega_{K_1 i_1}+\omega_{K_2 i_2}+\Delta\omega(K_1 i_1,K_2 i_2)-\eta_\nu\right]P^{\nu}_{K_1 i_1,K_2 i_2}-$$

$$-\sum_{i_0}(1+\delta_{K_1 i_1,K_2 i_2})^{-1/2}R^{\nu}_{i_0}U^{K_0 i_0}_{K_1 i_1,K_2 i_2}=0. \tag{25}$$

Hence, we get the secular equation

$$\det\left\|(\omega_{K_0 i_0}-\eta_\nu)\delta_{i_0 i_0'}-\sum_{K_1 i_1\leq K_2 i_2}(1+\delta_{K_1 i_1,K_2 i_2})^{-1}\cdot\right. \tag{26}$$

$$\left.\cdot\frac{U^{K_0 i_0}_{K_1 i_1,K_2 i_2}U^{K_0 i_0'}_{K_1 i_1,K_2 i_2}\left[1+\mathcal{K}^{K_0}(K_1 i_1,K_2 i_2)\right]}{\omega_{K_1 i_1}+\omega_{K_2 i_2}+\Delta\omega(K_1 i_1,K_2 i_2)-\eta_\nu}\right\|=0.$$

From (24) and (25) we find $R^{\nu}_{i_0}$ and $R^{\nu}_{K_1 i_1,K_2 i_2}$ for each value of η_ν. The rank of the determinant (26) equals the number of one-phonon terms in the wave function (22).

It is important that eqs. (25) and (26) coincide in form with the equations given in [4,5,15] in which only ph multipole interactions are taken into account, with the equations in [5,7] in which ph multipole interactions $\lambda\mu=22$ and 42 are considered and with the equations in [13,14] in which ph and pp multipole interactions $\lambda\mu$ at $n_{max}=1$ are taken into account. Thus, the form of equations (25) and (26) and the rank of the determinant (26) are independent of what multipole and spin-multipole interactions are taken into account and are independent of the rank n_{max} of separable interactions. This means that calculations in the QPNM can be made with any complex interactions in the separable form. The QPNM was formulated so that all complications caused by the form of interactions were concentrated in the RPA equations. It is not difficult to solve the RPA equations with complex interactions.

The inclusion of ph and pp separable $n_{max}>1$ interactions of the electric and magnetic types complicates the formulae for the two-phonon pole shift $\Delta\omega(K_1 i_1, K_2 i_2)$ and the function $U^{K_0 i_0}_{K_1 i_1, K_2 i_2}$.

Indeed,

$$\Delta\omega(K_1 i_1, K_2 i_2) = -\sum_{i'} \{ \mathcal{K}^{K_0}(K_2 i_2, K_1 i' | K_1 i_1, K_2 i_2) W^{K_0}_{i_1 i'} + \tag{27}$$

$$+ \mathcal{K}^{K_0}(K_2 i', K_1 i_1 | K_1 i_1, K_2 i_2) W^{K_0}_{i_2 i'} \},$$

where $W^{K}_{i_1 i_2}$ is given by formula (10′). The function $\widetilde{W}^{20}_{i_1 i_2}$ for the case when a phonon with $K^{\pi}=0^+$ enters into the two-phonon part of the wave function (22) is given in [13]. Then,

$$U^{K_0 i_0}_{K_1 i_1, K_2 i_2}[1+\mathcal{K}^{K_0}(K_1 i_1, K_2 i_2)] = -\frac{1}{2}\sum_{\sigma_1\sigma_2} \delta_{\sigma_1 K_1+\sigma_2 K_2, \sigma_0 K_0} \cdot \tag{28}$$

$$\cdot \{ \langle Q_{K_0 i_0 \sigma_0} H_{vq} Q^+_{K_1 i_1 \sigma_1} Q^+_{K_2 i_2 \sigma_2} \rangle + \langle Q_{K_2 i_2 \sigma_2} Q_{K_1 i_1 \sigma_1} H_{vq} Q_{K_0 i_0 \sigma_0} \rangle \},$$

where H_{vq} is determined by formula (11). Now we use the commutation relations

$$[B(qq';K\sigma)_\tau, Q^+_{K_0 i_0 \sigma_0}]\rangle = \sum_{K_3 i_3 \sigma_3} T_\tau \begin{pmatrix} K_0 i_0 \sigma_0, K_3 i_3 \sigma_3 \\ qq' \; K\sigma \end{pmatrix} Q^+_{K_3 i_3 \sigma_3}\rangle, \quad (29)$$

$$[\mathcal{B}(qq';K\sigma)_\tau, Q^+_{K_0 i_0 \sigma_0}]\rangle = -i \sum_{K_3 i_3 \sigma_3} \mathcal{T}_\tau \begin{pmatrix} K_0 i_0 \sigma_0, K_3 i_3 \sigma_3 \\ qq'; K_\sigma \end{pmatrix} Q^+_{K_3 i_3 \sigma_3}\rangle, \quad (30)$$

with the functions T_τ and $\mathcal{T}_\tau$ being real, and get

$$U^{K_0 i_0}_{K_1 i_1, K_2 i_2} = 2 \sum_{\sigma_1 \sigma_2 \tau} \delta_{\sigma_1 K_1 + \sigma_2 K_2, \sigma_0 K_0} \sum_{qq'}^{\tau} \{ V_\tau^{\lambda_1 K_1 i_1}(qq') f^{\lambda_1 K_1}(qq') \cdot$$

$$\cdot T_\tau \begin{pmatrix} K_0 i_0 \sigma_0, K_2 i_2 \sigma_2 \\ qq'; K_1 - \sigma_1 \end{pmatrix} + V_\tau^{\lambda_2 K_2 i_2}(qq') f^{\lambda_2 K_2}(qq') \cdot$$

$$\cdot T_\tau \begin{pmatrix} K_0 i_0 \sigma_0, K_1 i_1 \sigma_1 \\ qq', K_2 - \sigma_2 \end{pmatrix} + V_\tau^{L_1 K_1 i_1}(qq') f^{L_1 K_1}(qq') \cdot$$

$$\cdot \mathcal{T}_\tau \begin{pmatrix} K_0 i_0 \sigma_0, K_2 i_2 \sigma_2 \\ qq'; K_1 - \sigma_1 \end{pmatrix} + V_\tau^{L_2 K_2 i_2}(qq') f^{L_2 K_2}(qq') \cdot \quad (31)$$

$$\cdot \mathcal{T}_\tau \begin{pmatrix} K_0 i_0 \sigma_0, K_1 i_1 \sigma_1 \\ qq'; K_2 - \sigma \end{pmatrix} \},$$

where $V_\tau^{\lambda K i_2}(qq')$ and $V_\tau^{LK i_2}(qq')$ are given by (13) and (13').

Let us obtain the matrix elements of $E\lambda$ and $M\lambda$ transitions. Using phonons (4) we can write the corresponding operators in the form

$$\mathfrak{M}(E\lambda\mu) = \sum_{\tau\sigma} \sum_{qq'}^{\tau} \Gamma_\tau(E\lambda\mu; qq') \{ v^{(-)}_{qq'} B(qq';\mu\sigma) +$$

$$+ \frac{1}{2} f^{\lambda\mu}(qq')(u^{(+)}_{qq'})^2 \sum_{i'} g^{\mu i'}_{qq'} (Q^+_{\mu i' \sigma} + Q_{\lambda\mu' - \sigma}) \}, \quad (32)$$

$$\mathfrak{M}(M\lambda\mu)=\sum_{\sigma\tau}\sum_{qq'}^{\tau}\Gamma_\tau(M\lambda\mu;qq')\Big\{v_{qq'}^{(+)}\mathcal{B}(qq';\mu\sigma)-$$

$$-\frac{1}{2}f^{L\mu}(qq')(u_{qq'}^{(-)})^2\sum_{i'}g_{qq'}^{\mu i'}(Q_{\mu i'\sigma}^{+}-Q_{\mu i'-\sigma}), \qquad (33)$$

where

$$\Gamma_\tau(E\lambda\mu;qq')=\langle q|e_{eff}^{\lambda}(\tau)r^\lambda Y_{\lambda\mu}(\theta\varphi)|q'\rangle,$$

$$\Gamma_\tau(M\lambda\mu;qq')=\langle q\,|\,\frac{M_0}{2}[\lambda(2\lambda+1)]^{1/2}\Big\{g_s^{eff}(\tau)(\vec{\sigma}\vec{Y}_{\lambda-1})_{\lambda\mu}+$$

$$+g_l^{eff}\frac{4}{\lambda+1}(\vec{l}\,Y_{\lambda-1})_{\lambda\mu}\,|\,q'\rangle.$$

In calculating the matrix elements of $E\lambda$ and $M\lambda$ transitions from the ground states of doubly even nuclei to the states with the dominating one-phonon components in the wave function (22), we use only the phonon parts of the operators $\mathfrak{M}(E\lambda\mu)$ and $\mathfrak{M}(M\lambda\mu)$. As a result we get

$$(\Psi_\nu^*(K_0^{\pi}\sigma_0)\,\mathfrak{M}(E\lambda\mu)\,\Psi_0)=$$

$$=\frac{1}{2}\sum_{i_0\tau}R_{i_0}^{\nu}\sum_{qq'}^{\tau}\Gamma_\tau(E\lambda K_0;qq')f^{\lambda K_0}(qq')(u_{qq'}^{(+)})^2g_{qq'}^{K_0 i_0}, \qquad (34)$$

$$(\Psi_\nu^*(K_0^{\pi_0}\sigma_0)\,\mathfrak{M}(M\lambda\mu)\,\Psi_0)=$$

$$=-\frac{i}{2}\sum_{i_0\tau}R_{i_0}^{\nu}\sum_{qq'}^{\tau}\Gamma_\tau(M\lambda K_0;qq')f^{LK_0}(qq')(u_{qq'}^{(-)})^2g_{qq'}^{K_0 i_0}. \qquad (35)$$

These matrix elements differ from the formulae used earlier, for instance in [14], by that the functions $g_{qq'}^{K_0 i_0}$ belong to both parts of the phonon operator (4), the electric and magnetic parts. If, for example, in the normalisation (6) the magnetic part appears to be much smaller than the electric one, then this will result in the hindrance of the $M\lambda$ transition. It can be expected that with the use of the phonon (4) the $E\lambda$ transition probabilities from the ground states will not differ considerably from the calculated ones [14,16].

Reliable experimental data and numerous calculations are available on M1 transitions from the ground states of doubly even deformed nuclei to the 1^+ states. The treatment of the 1^+ states as mixed symmetry states is undoubtedly a success of the IBM-2. The energies and $B(M1)$ values are well described in the RPA with the quadrupole ph interactions with excluding a spurious state [11]. Since many calculations were performed, we will not calculate the energies of the 1^+ states and $B(M1)$ values.

The experimental data on M2 to M3 transitions from the ground states are rather scarce. Thus, states with the mixed symmetry with $I^{\pi} = 3^+$ were searched for in [17]. They measured the $B(M3)\uparrow$ -value for excitation of the $I^{\pi}K_{\nu} = 3^+2_1$ state and did not observe M3 transitions to the states with an energy higher than 1 MeV. There are only few calculations of the M2 and M3 transition probabilities [18-20].

It is expedient to calculate the M2 and M3 transition probabilities within the QPNM using formulae (34) and (35), which may stimulate new experiments.

In experimental investigations on the Coulomb excitation, ($n\gamma$) and other reactions (see, for instance, [21,22]) a large number of M1 values and M1 + E2 mixtures were observed for transitions between excited states of doubly even deformed nuclei. Calculations of the M1, E2, M3 and other transition probabilities between excited states can be made within the formalism expounded in this paper.

Let us find the matrix elements of $E\lambda$ and $M\lambda$ transitions between excited states with the dominating one-phonon components of their wave functions (22). In these calculations we use the quasiparticle parts of the operators (32) and (33) and the commutators (29) and (30); as a result, we get

$$
\left(\Psi^{*}_{\nu_4}(K_4^{\pi_4}\sigma_4)\mathfrak{M}(E\lambda\mu)\Psi_{\nu_0}(K_0^{\pi_0}\sigma_0)\right) =
$$

$$
= \sum_{i_0 i_4} R^{\nu_0}_{i_0} R^{\nu_4}_{i_4} \delta_{\sigma_0 K_0+\sigma\mu,\sigma_4 K_4} \sum_{\tau} \sum_{q_1 q_2 q_3}^{\tau} \Gamma_\tau(E\lambda\mu; q_1 q_2)\, v^{(-)}_{q_1 q_2} \cdot
\tag{36}
$$

$$
\cdot\left(\psi^{K_0 i_0}_{q_2 q_3}\psi^{K_4 i_4}_{q_3 q_1} + \varphi^{K_0 i_0}_{q_2 q_3}\varphi^{K_4 i_4}_{q_3 q_1}\right)\cdot
$$

$$
\cdot\left(f^{\lambda K_0}(q_2 q_3)u^{(+)}_{q_2 q_3} f^{\lambda K_4}(q_3 q_1)u^{(+)}_{q_3 q_1} + f^{\lambda K_0}(q_2 q_3)u^{(-)}_{q_2 q_3} f^{\lambda K_4}(q_3 q_1)u^{(-)}_{q_3 q_1}\right).
$$

$$
\left(\Psi^{*}_{\nu_4}(K_4^{\pi_4}\sigma_4)\mathfrak{M}(M\lambda\mu)\Psi_{\nu_0}(K_0^{\pi_0}\sigma_0)\right) =
$$

$$
= i\sum_{i_0 i_4} R^{\nu_0}_{i_0} R^{\nu_4}_{i_4} \delta_{\sigma_0 K_0+\sigma\mu,\sigma_4 K_4} \sum_{\tau} \sum_{q_1 q_2 q_3}^{\tau} \Gamma_\tau(M\lambda\mu; q_1 q_2)\, v^{(+)}_{q_1 q_2} \cdot
$$

$$
\cdot\left(\psi^{K_0 i_0}_{q_2 q_3}\psi^{K_4 i_4}_{q_3 q_1} + \varphi^{K_0 i_0}_{q_2 q_3}\varphi^{K_4 i_4}_{q_3 q_1}\right)\left(f^{\lambda K_0}(q_2 q_3)u^{(+)}_{q_2 q_3} f^{\lambda K_4}(q_3 q_1)u^{(-)}_{q_3 q_1} + \right.
\tag{37}
$$

$$
\left. + f^{\lambda K_0}(q_2 q_3)u^{(-)}_{q_2 q_3} f^{\lambda K_4}(q_3 q_1)u^{(+)}_{q_3 q_1}\right).
$$

These formulae can be useful for further calculations of the M1, M2 and E2 transition probabilities between excited states. Similar calculations of the quantities $\delta(E2:M1)$, as in [23], can be made for transitions between quadrupole states of deformed nuclei.

5. Conclusion

The axial symmetry of well deformed nuclei complicates the description of their vibrational states in comparison with spherical nuclei. If the projection onto the symmetry axis of K is assumed to be a good quantum number, then the vibrational state with a fixed K^{π} can be described by multipole and spin-multipole interactions of the electric type and spin-multipole interactions of the magnetic type. Thus, the states with $K^{\pi}=2^{+}$ can be described by interactions of the electric type $\lambda K = 22+42+\ldots$, $\lambda\lambda K=222+442+\ldots$ and magnetic type $\lambda' L K=232+432+\ldots$, and the states with $K^{\pi}=3^{+}$ by $\lambda K=43+63+\ldots$, $\lambda\lambda K=443+663+\ldots$, $\lambda' L K=233+433+\ldots$. To avoid nonphysical multiplicativity of a number of the calculated vibrational states, we have introduced a new phonon operator. It consists of the electric part taking account of the λK and $\lambda\lambda K$ interactions and the magnetic part taking account of the $L\pm 1 L K$ interaction. This new RPA phonon should be used for describing doubly even, doubly odd and odd-A deformed nuclei and first of all for describing $M\lambda$ and $E\lambda$ transitions between excited states.

In the present paper we have formulated the most general version of the QPNM. We have constructed the Hamiltonian and derived equations for ph and pp isoscalar and isovector multipole and spin-multipole finite rank separable interactions between quasiparticles. Introduction of the finite rank $n_{max}>1$ separable interactions leads to complication of the RPA equations, which is nonessential in computer calculations. All difficulties connected with the electric and magnetic types of interactions and with the $n_{max}>1$ separable interactions are concentrated in the RPA equations. It is important that they do not lead a noticeable complication of the QPNM equations for calculating the fragmentation of vibrational states including giant resonances. Additional difficulties caused by $n_{max} > 1$ do not arise if three-phonon terms are added to the wave function (22). They also do not arise in calculating the fragmentation of one-quasiparticle states in odd deformed nuclei.

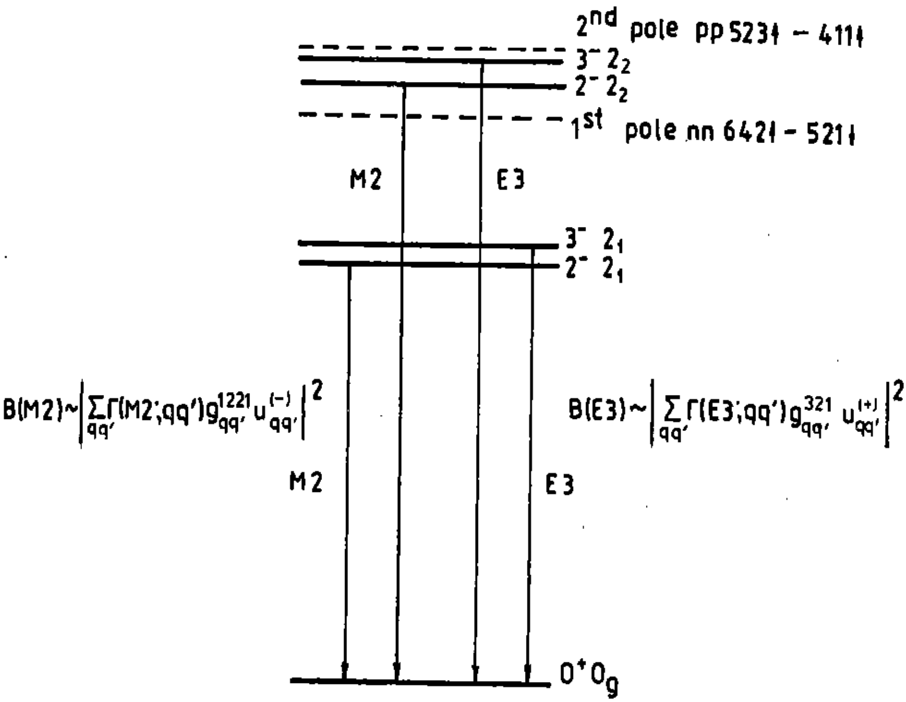

Fig. 1. The first two states with $K^{\pi} = 2^-$ described either as quadrupole magnetic states with enhancement of M2 transitions or as octupole electric states with enhancement of E3 transitions and the first two two-quasiparticle states.

I should like to emphasize that in solving such a complicated problem as the many-body nuclear problem one should aim at exposing the most important parts of effective interactions to be used in concrete calculations rather than at solving the problem in the most general form.

The mathematical apparatus of the QPNM constructed in this paper for deformed nuclei can serve as a basis for calculations of many characteristics of low-lying and high-lying states. We hope that the QPNM calculations will stimulate further experimental study of the structure of deformed nuclei at a new generation of accelerators and detectors.

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