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ELECTROMAGNETIC PROPERTIES OF ROTATING NUCLEI IN CRPA APPROACH

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In the last years, the cranked random phase approximation (CRPA) has been extensively used for the analysis of high spin states in even-even nuclei [1] . In this analysis the main accent is on the description of energy characteristics. However, the study of electromagnetic moments and transitions of collective rotational states can give equally important information. In most cases this information on elmg. moments and transitions is more detailed and deepened in comparison with the energy spectrum.

In the paper by Hamamoto and Sagawa [2] E1 and M1 transitions were analysed in the framework of the cranking model. In these papers, the matrix elements of transition operators in the yrast line states were determined with respect to quasiparticle vacuum. However, the question arises what is the role of quasiparticle correlations in the yrast line states in description of electromagnetic transitions, because the yrast line state is not in fact the quasiparticle vacuum but the phonon vacuum. The other problem is connected with linear boson image of a transition operator which is usually used in calculation of transition probabilities. The formulation of the CRPA based on conservation laws [3] showed the necessity of including the second order terms of boson expansion of single-particle operators in the cranking hamiltonian. Quantitative description of alignment of intrinsic angular momentum in octupole states in actinides, made in [4] , confirmed the importance of the boson second order term in expansion of the angular momentum operator J_x .

In this letter, the inclusion of boson second order terms of boson expansion of transition operators as well as inclusion of quasiparticle correlations in yrast line states is shown to be important for describing elmg. characteristics in even-even rotating nuclei in the framework of the CRPA approach.

The reduced transition probability connected with a spherical tensor $T_{\lambda\mu}$ is given by

$$B(T_\lambda ; I_1 \to I_2) = \sum_{\mu M_2} |\langle I_2 M_2 \alpha_2 | T_{\lambda\mu} | I_1 M_1 \alpha_1 \rangle|^2. \quad (1)$$

In the high spin limit $I_1 \approx I_2 \gg \lambda$ this probability can be rewritten as follows [3] :

$$B(T_\lambda ; I_1 \to I_2) \approx |\langle I_2 M_2 = I_2, \alpha_2 | T_{\lambda\mu = I_2 - I_1} | \alpha_1 I_1 M_1 = I_1 \rangle|^2. \quad (2)$$

where $|IM=I\alpha\rangle$ is the CRPA state in the coordinate system where the quantum axis coincides with the x-axis. For simplicity, further we will use a shortened assignment $|\alpha IM\rangle = |\alpha I\rangle$.

In the case of electromagnetic transitions from the one-phonon state $|\alpha_1 I\rangle = \mathcal{D}^+_{\alpha_1}|0I\rangle$ to the yrast line state, which is determined as the phonon vacuum $|\alpha_2 I\rangle = |0I\rangle$ $(\mathcal{D}_\alpha|0I\rangle = 0)$, one can write [3] *)

$$\langle\alpha_2 I_2|T_{\lambda\mu}|\alpha_1 I_1\rangle \simeq \langle 0I_1|\left[T_{\lambda\mu=I_2-I_1}, \mathcal{D}^+_{\alpha_1}\right]|0I_1\rangle . \qquad (3)$$

Electromagnetic transitions between the states $|\alpha_1 I_1\rangle = \mathcal{D}^+_{\alpha_1}|0I_1\rangle$ and $|\alpha_2 I_2\rangle = \mathcal{D}^+_{\alpha_2}|0I_2\rangle$ in the CRPA approach are characterised by the following matrix elements:

$$\langle\alpha_2 I_2|T_{\lambda\mu}|\alpha_1 I_1\rangle \approx \langle 0I_1|\left[\mathcal{D}_{\alpha_2},\left[T_{\lambda\mu},\mathcal{D}^+_{\alpha_1}\right]\right]|0I_1\rangle + \langle 0I_1|T_{\lambda\mu}|0I_1\rangle\delta_{\alpha_1\alpha_2} . \qquad (4)$$

In the CRPA approach each single-particle operator can be expressed in the form of the boson expansion (see, e.g. [3])

$$T = \langle\Omega|T|\Omega\rangle + T(1) + T(2), \qquad (5)$$

where $\langle\Omega|T|\Omega\rangle$ is the expectation value of the operator T with respect to vacuum $|\Omega\rangle$ of the cranking model quasiparticles, $T(1)$ and $T(2)$ are the linear boson term and quadratic boson term, respectively where bosons are understood as two-quasiparticle bosons $b^+_{ik} = \alpha^+_i\alpha^+_k$, $b^+_{\bar{i}\bar{k}} = \alpha^+_{\bar{i}}\alpha^+_{\bar{k}}$ or $b^+_{i\bar{k}} = \alpha^+_i\alpha^+_{\bar{k}}$ (see [3]). Concretely, for the positive signature ($\sigma = +$) operator $R_x(\pi)TR_x^{-1}(\pi) = +T$ we have

$$\langle\Omega|T(\sigma=+)|\Omega\rangle = \sum_{k\ell}\langle k|T|\ell\rangle\left(B^{\bar{i}}_k B^{\bar{i}}_\ell + \gamma_\tau h\, B^i_{\bar{k}} B^i_{\bar{\ell}}\right)$$
$$T^{(+)}(1) = \sum_{ij} t^{(+)}_{i\bar{j}}\left(b^+_{i\bar{j}} + h\tau\, b_{i\bar{j}}\right) \qquad (6)$$
$$T^{(+)}(2) = \sum_{ijm}\left\{t^{(+)}_{ij}\left(b^+_{im}b_{jm} + b^+_{i\bar{m}}b_{j\bar{m}}\right) + t^{(+)}_{\bar{i}\bar{j}}\left(b^+_{\bar{i}m}b_{\bar{j}m} + b^+_{\bar{i}\bar{m}}b_{\bar{j}\bar{m}}\right)\right\} .$$

where $B^i_k\,(B^{\bar{i}}_k)$ are the coefficients of the Bogolubov transformation (see [1]), and numbers $h = \pm 1$, $\tau = \pm 1$ and $\gamma_\tau = \pm 1$ characterise the properties of an operator T with respect to

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*) In the framework of the yrast line, states are interpreted as the phonon vacuum corresponding to a given value of rotational frequency or angular momentum I. Therefore, we denote these vacuum states as $|0I\rangle$. For simplicity, sometimes we will omit the index I; so the phonon vacuum is denoted by $|0\rangle$.

hermitian conjugation ($T^{+} = hT$), complex conjugation $\langle k|T|\ell\rangle^{*} = \tau\langle k|T|\ell\rangle$ and time-reversal ($\mathcal{J}T\mathcal{J}^{-1} = \gamma_{\tau}T$). One can obtain similar expression for the negative signature operator ($R_x(\pi)\,T\,R_x^{-1}(\pi) = -T$)

$$\begin{aligned} &\langle\Omega|T(\sigma=-)|\Omega\rangle = 0 \\ &T^{(-)}(1) = \sum_{\mu} t_{\mu}^{(-)}(b_{\mu}^{+} + b_{\mu}h\tau) \quad \mu = ik, \bar{i}\bar{k} \\ &T^{(-)}(2) = \sum_{ij\bar{m}}\{t_{i\bar{j}}^{(-)}(b_{im}^{+}b_{\bar{j}m} + b_{i\bar{m}}^{+}b_{\bar{j}\bar{m}}) + t_{\bar{i}j}^{(-)}(b_{\bar{i}m}^{+}b_{jm} + b_{\bar{i}\bar{m}}^{+}b_{j\bar{m}})\}. \end{aligned} \tag{7}$$

Corresponding negative signature and positive signature phonons have the following forms:

$$\begin{aligned} &\mathcal{D}_{\alpha}^{+}(\sigma=+) = \frac{1}{\sqrt{2}}(X_{\alpha} - iP_{\alpha})^{(+)} = \frac{1}{\sqrt{2}}\sum_{\mu}\{(X+P)_{\mu}^{(+)}b_{\mu}^{+} + (X-P)_{\mu}^{(+)}b_{\mu}\}, \quad \mu = i\bar{j}, \bar{i}j \\ &\mathcal{D}_{\alpha}^{(+)}(\sigma=-) = \frac{1}{\sqrt{2}}(X_{\alpha} - iP_{\alpha})^{(-)} = \frac{1}{\sqrt{2}}\sum_{\mu}\{(X+P)_{\mu}^{(-)}b_{\mu}^{+} + (X-P)_{\mu}^{(-)}b_{\mu}\}, \quad \mu = ij, \bar{i}\bar{j}. \end{aligned} \tag{8}$$

Eq.(8) can be understood as a transformation from the space of two-quasiparticle bosons to the space of phonons. The inverse transformations are

$$\begin{aligned} &b_{i\bar{j}}^{+} = \sqrt{2}\sum_{\alpha}\{(X+P)_{i\bar{j}}^{(+)}\mathcal{D}_{\alpha}^{+}(\sigma=+) - (X-P)_{i\bar{j}}^{(+)}\mathcal{D}_{\alpha}(\sigma=+)\} \\ &b_{\mu}^{+} = \sqrt{2}\sum_{\alpha}\{(X+P)_{\mu}^{(-)}\mathcal{D}_{\alpha}^{+}(\sigma=-) - (X-P)_{\mu}^{(-)}\mathcal{D}_{\alpha}(\sigma=-)\} \qquad \mu = ij, \bar{i}\bar{j}. \end{aligned} \tag{9}$$

Further, we will analyse the following situations.

1. Transitions along the yrast line and the moments in the yrast line states.

Since the yrast line contains only states with positive signature (even values of angular momentum), transitions along the yrast line (ΔI = even) as well as moments in the yrast line states are connected with the positive signature transition operator T (in the case of R - symmetric nucleus this operator does not change parity as well). The corresponding transition matrix element is

$$\langle 0I_2|T_{\lambda\mu}(\sigma=+)|0I_1\rangle \approx \langle\Omega|T_{\lambda\mu=I_2-I_1}|\Omega\rangle + \langle 0I_2|T_{\lambda\mu}(2)|0I_1\rangle, \tag{10}$$

where the expectation value in the quasiparticle vacuum $|\Omega\rangle$ is determined by the cranking model in terms of the cranking condition $\langle\Omega|\mathcal{J}_x|\Omega\rangle = \sqrt{I(I+1)}$. Using (5), (6) and (9) one can obtain

$$\langle 0I_2 | T_{\lambda\mu=I_2-I_1}(\sigma=+) | 0I_1 \rangle = \langle \Omega | T_{\lambda\mu} | \Omega \rangle +$$

$$+ 2 \sum_{ijm} \Big\{ t^{(+)}_{ij} \big[(X-P)^{(-)}_{im} (X-P)^{(-)}_{jm} + (X-P)^{(+)}_{i\bar{m}} (X-P)^{(+)}_{j\bar{m}} \big] + t^{(+)}_{\bar{i}\bar{j}} \big[(X-P)^{(+)}_{\bar{i}m} (X-P)^{(+)}_{\bar{j}m} + (X-P)^{(-)}_{\bar{i}\bar{m}} (X-P)^{(-)}_{\bar{j}\bar{m}} \big] \Big\} . \tag{11}$$

It can be seen from eq. (11) that quasiparticle correlations in the phonon vacuum (which is involved in the second term in eq. (10)) leads to an additional nontrivial term in (11) for transition matrix elements ($\Delta I = I_2 - I_1 \neq 0$) as well as for an expectation value of the transition operator moment ($\Delta I = I_2 - I_1 = 0$). It must be noted that this additional term depends on two-quasiparticle amplitudes of all phonons of both signatures and parities. The difference between $\langle 0I_2 | T_{\lambda\mu}(\sigma=+) | 0I_1 \rangle$ and $\langle \Omega | T_{\lambda\mu} | \Omega \rangle$ allows one to estimate the influence of quasiparticle correlations in the phonon vaccum.

2. Electromagnetic transitions from one-phonon states to the yrast line states.

In this case, we have to analyze transitions with and without the signature change. If we assume that the lowest excited states in an even-ever nucleus are really one-phonon states (without any anharmonicity), the quadratic boson part $T_{\lambda\mu}(2)$ does not give any contribution to the matrix element of the transition operator. With use of (3), (6) and (9) we can write for this matrix element the following relations:

for the transition from the negative signature one-phonon state

$$\langle 0I_2 | T_{\lambda\mu} \mathcal{O}^+_\alpha(\sigma=-) | I_1 0 \rangle = \langle 0I_2 | [T_{\lambda\mu}, \mathcal{O}^+_\alpha(\sigma=-)] | I_1 0 \rangle \simeq$$

$$= \sqrt{2} \sum_\nu \big\{ (X+P)^{(-)}_\nu t_\nu h\tau - (X-P)^{(-)}_\nu t_\nu \big\} \qquad \nu = ik, \bar{i}\bar{k} \tag{12}$$

and for the transition from the positive signature one-phonon state

$$\langle 0I_2 | T_{\lambda\mu} \mathcal{O}^+_\alpha(\sigma=+) | I_1 0 \rangle = \langle 0I_2 | [T_{\lambda\mu}, \mathcal{O}^+_\alpha(\sigma=+)] | I_1 0 \rangle \simeq$$

$$= \sqrt{2} \sum_{i\bar{j}} \big\{ (X+P)^{(+)}_{i\bar{j}} t_{i\bar{j}} h\tau - (X-P)^{(+)}_{i\bar{j}} t_{i\bar{j}} \big\} . \tag{13}$$

In this case expressions (12) and (13) coincide with analogous ones giver in [3] , and one can see that if excited states are described by pure one-phonon states neither quadratic boson part $T_{\lambda\mu}(2)$ of the transition operator nor quasiparticle correlations give contribution to (12) or (13).

3. Electromagnetic transitions between one-phonon states and moments in them.

In the general case, one-phonon states are characterized by even as well as odd values of angular momentum, and therefore, by both values of signature. According to the selection rule with respect to signature, the transitions with spin change ΔI = even values (ΔI =0,2,4) are described by the transition operator of positive signature while the transitions with ΔI = odd values (ΔI = 1,3,5) are connected with the transition operator of negative signature.

Transition matrix elements with odd spin change (signature change) can be expressed as follows:

$$\langle I_2\alpha_2(\sigma=\mp)|T_{\lambda\mu}(\sigma=-)|\alpha_1(\sigma=\pm)I_1\rangle = \langle 0I_2|\left[\mathcal{D}_{\alpha_2}(\sigma=\mp),\left[T^{\sigma=-}_{\lambda\mu}(2),\mathcal{D}^{+}_{\alpha_1}(\sigma=\pm)\right]\right]|0I_1\rangle \quad (14)$$

Using (7), (8) and (9) it is possible to rewrite this expression in the following form:

$$\langle\alpha_2(\sigma=-)I_2|T_{\lambda\mu}(\sigma=-)|\alpha_1(\sigma=+)I_1\rangle \approx 2\sum_{ijm}\Big\{t^{(-)}_{i\bar{j}}(X+P)^{(+)}_{\bar{j}m}(X+P)^{(-)}_{im} +$$

$$+ t^{(-)}_{\bar{i}j}(X+P)^{(+)}_{j\bar{m}}(X+P)^{(-)}_{\bar{i}\bar{m}} + t^{(-)}_{i\bar{j}}(X-P)^{(+)}_{i\bar{m}}(X-P)^{(-)}_{\bar{j}\bar{m}} + t^{(-)}_{\bar{i}j}(X-P)^{(+)}_{\bar{i}m}(X-P)^{(-)}_{jm}\Big\} \quad (15a)$$

$$\langle\alpha_2(\sigma=+)|T_{\lambda\mu}(\sigma=-)|\alpha_1(\sigma=-)\rangle \approx 2\sum_{ijm}\Big\{t^{(-)}_{i\bar{j}}(X+P)^{(-)}_{\bar{j}\bar{m}}(X+P)^{(+)}_{i\bar{m}} +$$

$$+ t^{(-)}_{\bar{i}j}(X+P)^{(+)}_{\bar{i}m}(X+P)^{(-)}_{jm} + t^{(-)}_{i\bar{j}}(X-P)^{(-)}_{im}(X-P)^{(+)}_{\bar{j}m} + t^{(-)}_{\bar{i}j}(X-P)^{(-)}_{\bar{i}\bar{m}}(X-P)^{(+)}_{j\bar{m}}\Big\}. \quad (15b)$$

From (14) one can see that only inclusion of the linear boson term in the transition operator means zero probability of transitions between one-phonon states with signature change (with ΔI = 1, 3, 5,...). For description of transition with signature change in even-even nuclei one has to involve the quadratic boson term in the transition operator. It should be pointed out that transitions of that type are experimentally observed (see e.g. quadrupole transitions with ΔI = 1 between the states in the lowest rotational bands in ^{168}Er [5]).

Transition matrix elements with even spin change (without signature change) are given by

$$\langle \alpha_2(\sigma=\pm)I_2|T_{\lambda\mu}(\sigma=+)|\alpha_1(\sigma=\pm)I_1\rangle = \langle 0I_2|T_{\lambda\mu}(\sigma=+)|0I_1\rangle\delta_{\alpha_1\alpha_2} + \langle 0I_2|\left[\mathfrak{Q}_{\alpha_2}(\sigma=\pm),\left[T^{\sigma=+}_{\lambda\mu}(2); \mathfrak{Q}^{+}_{\alpha_1}(\sigma=\pm)\right]\right]|0I_1\rangle \tag{16}$$

where $\langle 0I_2|T_{\lambda\mu}(\sigma=+)|0I_1\rangle$ is given by (11) and the second term in (16) can be rewritten by means of (6), (8) and (9); so eventually we obtain

$$\begin{aligned}\langle \alpha_2(\sigma=+)I_2|T_{\lambda\mu}(\sigma=+)|\alpha_1(\sigma=+)I_1\rangle &= \langle 0I_2|T_{\lambda\mu}(\sigma=+)|0I_1\rangle\delta_{\alpha_1\alpha_2} + \\ &+2\sum_{ijm}\Big\{ t^{(+)}_{ij}(X+P)^{(\alpha_2+)}_{i\bar m}(X+P)^{(\alpha_1+)}_{j\bar m} + t^{(+)}_{\bar i\bar j}(X+P)^{(\alpha_2+)}_{\bar i m}(X+P)^{(\alpha_1+)}_{\bar j m} + \\ &+ t^{(+)}_{ij}(X-P)^{(\alpha_1+)}_{j\bar m}(X-P)^{(\alpha_2+)}_{i\bar m} + t^{(+)}_{\bar i\bar j}(X-P)^{(\alpha_1+)}_{\bar j m}(X-P)^{(\alpha_2+)}_{\bar i m}\Big\}\end{aligned} \tag{17a}$$

$$\begin{aligned}\langle \alpha_2(\sigma=-)I_2|T_{\lambda\mu}(\sigma=+)|\alpha_1(\sigma=-)I_1\rangle &= \langle 0I_2|T_{\lambda\mu}(\sigma=+)|0I_1\rangle\delta_{\alpha_1\alpha_2} + \\ &+2\sum_{ijm}\Big\{ t^{(+)}_{ij}(X+P)^{(\alpha_1-)}_{jm}(X+P)^{(\alpha_2-)}_{im} + t^{(+)}_{\bar i\bar j}(X+P)^{(\alpha_1-)}_{\bar j\bar m}(X+P)^{(\alpha_2-)}_{\bar i\bar m} + \\ &+ t^{(+)}_{ij}(X-P)^{(\alpha_1-)}_{jm}(X-P)^{(\alpha_2-)}_{im} + t^{(+)}_{\bar i\bar j}(X-P)^{(\alpha_1-)}_{\bar j\bar m}(X-P)^{(\alpha_2-)}_{\bar i\bar m}\Big\}.\end{aligned} \tag{17b}$$

The comparison of (17) with (15) shows the different character of transitions with and without the signature change. This comparison supports the well-known fact that with increasing rotational frequency a given rotational band splits into two bands, each being characterized by a good signature number $\sigma=+$ and $\sigma=-$. Intraband transitions in each of these bands have another character than interband transitions: $\sigma=\pm \rightarrow \sigma=\mp$. The similar behaviour of transitions with $\Delta I=1$ with respect to the signature change was shown in paper by Hamamoto and Mottelson [6] in the framework of the particle-rotor model for odd nonaxial nuclei. In [7], the contribution to E2 and M1 transitions with $\Delta I=1$ in odd nuclei caused by polarisation of the core via rotational Goldstone mode is discussed. This Goldstone mode gives also contribution to the second term in right-hand side of (11)*).

It can be seen from eqs. (10),(11),(15) and (17) that the inclusion of quasiparticle correlations in the yrast line of even-even nuclei leads to the appearance of the second term in (11) and therefore, to the redetermination of electromagnetic moments in the yrast

*) Notice that the importance of the quadratic boson term for the description of the interband electric transitions between the excited nonrotational states had been shown also in the quasiparticle-phonon model for deformed even-even nuclei [8].

line states as well as in the excited states above the yrast line. The signature dependence of electromagnetic transitions between the states near the yrast line can be explained in the framework of the CRPA by both the inclusion of the quasiparticle correlations in the RPA vacuum and the inclusion of the quadratic boson term in expansion (5) of the transition operator. These inclusions are valid for both axial and nonaxial nuclei because the assumption of axiality was not used anywhere in this paper.

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