объединенный
институт
ядерных
исследований
дубна

E4-89-882

A.I.Machavariani, A.G.Rusetsky

ON THE FIELD-THEORETICAL FORMULATION OF THE LOW-ENERGY PION-NUCLEON SCATTERING PROBLEM

Submitted to "Nuclear Physics A"

1989

INTRODUCTION

At present, interest in the investigation of low-energy pion-nucleon scattering is caused by the task of revealing the quark degrees of freedom in hadron-hadron interactions[1-6]. However, the models which do not take account of quarks can describe, often even more accurately, the same experimental data as various quark models. Moreover, the existing quark models are far more complicated and generally refer to similar assumptions as the conventional phenomenological models, e.g., the calculations of low-energy πN-phase shifts within bag models[1-4] are carried out on the basis of Lippmann-Schwinger and Low equations; for the qualitative description of low-energy πN-scattering the Skyrme model Lagrangian related to well-known phenomenological chiral Lagrangians is used, etc.

In our opinion, in the low and intermediate energy region the clear-cut separation of quark degrees of freedom cannot be carried out without the consistent field-theoretical formulation of the hadron-hadron interaction problem. One of the most general approaches to hadron-hadron interactions based on Bethe-Salpeter-type equations has been suggested in ref.[9]. However, at present this approach is very difficult to realize in practice owing to the difficulties caused by the absence of an adequate tool for handling with four-dimensional relativistic scattering equations. In the theory of connected πNN - NN [10] and πN - $\pi\pi N$ [11] systems the quasipotential reductions of Bethe-Salpeter-type equations have often been used. However, it is well known that the quasipotential equations cannot be obtained in a unique manner, and in different quasipotential formulations various meson-nucleon vertex functions should be chosen in a different way for describing the experimental data.

An alternative field-theoretical approach to the low-energy πN -scattering is based on Low-type equations[12-20]. The potential term of these equations is constructed out of the vertex functions with two particles on mass shell. Such vertex functions have often been used in the current algebra approach[8], and the calculation of these vertices within various quark models seems to be much less complicated than the ones with all particles being off--shell. However, the Low-type equations are nonlinear and, the driving term of these equations is nonhermitian due to the presence of intermediate particle propagators. In the present work the linearization procedure for Low-type equations is suggested. With the use of the above linearization procedure the Low-type equations are reduced to the Lippmann-Schwinger equation with a potential obtained within the framework of field theory. This energy-dependent potential is expressed via the same meson-nucleon vertex functions as the driving term of the Low equation and contains, in addition, a crossing term dependent on the πN -scattering t -matrix of interest. This nonlinear term in πN -interaction potential can be taken into account with the help of a special iteration procedure.

1. THE LOW-TYPE EQUATIONS

In this section we shall present the derivation of Low-type equations slightly different from the conventional ones. We follow the metric and conventions of ref.[21]). In particular, ps, qi and $p's', q'i'$ $(p'^2 = p^2 = M^2,\ q'^2 = q^2 = \mu^2)$ denote the four-momenta and the spin and isospin labels for incoming and outgoing nucleons and pions, respectively (fig. 1). According to the Lehmann-Symanzic-Zimmermann contraction rules[21]) the πN S-matrix $S_{fi} =$

$= \langle out, \vec{p}'s', \vec{q}'i' | \vec{p}s, \vec{q}i, in\rangle$ can be related to the scattering $\mathcal{T}$ -matrix in the following manner:

$$S_{fi} = \delta_{fi} + (2\pi)\,\delta(p'^0 + q'^0 - p^0 - q^0)\,\mathcal{T}_{fi} \tag{1.1}$$

$$\mathcal{T}_{fi} = \langle \vec{p}'s' |\, \mathring{a}_{\vec{q}'i'}(0) \,| \vec{p}s, \vec{q}i, in\rangle , \tag{1.2}$$

where $\mathring{a}_{\vec{q}'i'}(x^0) = \frac{d}{dx^0}\, a_{\vec{q}'i'}(x^0)$ is expressed via the pi-meson field operator $\vec{\varphi}_{i'}(x)$ and its derivative $\dot{\vec{\varphi}}_{i'}(x) = \frac{d}{dx^0}\vec{\varphi}_{i'}(x)$

$$a_{\vec{q}'i'}(x^0) = \frac{i}{\sqrt{Z}} \int d^3\vec{x}\, e^{iq'x} \left[\dot{\vec{\varphi}}_{i'}(x) - iq'^0 \vec{\varphi}_{i'}(x)\right] \tag{1.3a}$$

$$\mathring{a}_{\vec{q}'i'}(x^0) = \frac{i}{\sqrt{Z}} \int d^3\vec{x}\, e^{iq'x} (\Box_x + \mu^2)\vec{\varphi}_{i'}(x) \equiv \frac{i}{\sqrt{Z}} \int d^3\vec{x}\, e^{iq'x} j_{i'}(x) . \tag{1.3b}$$

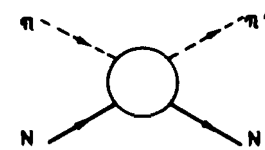

Fig. 1. Graphical representation for $\pi(qi) + N(ps) \to \pi'(q'i') + N'(p's')$ scattering t-matrix, where i denotes the π-meson isospin and s - the nucleon spin.

From the definition of πN -scattering $\mathcal{T}$ -matrix $\mathcal{T}_{fi}$ (1.2) and (1.3b) we obtain:

$$\mathcal{T}_{fi} = (2\pi)^3 i\, \delta^{(3)}(\vec{p}' + \vec{q}' - \vec{p} - \vec{q}) \langle \vec{p}'s' | j_{i'}(0) | \vec{p}s, \vec{q}i, in\rangle \equiv (2\pi)^3 i\, \delta^{(3)}(\vec{p}' + \vec{q}' - \vec{p} - \vec{q}) f . \tag{1.4}$$

On the basis of the contraction rules for $\mathcal{T}_{fi}$ we have

$$\mathcal{T}_{fi} = \langle \vec{p}'s' | \mathring{a}_{\vec{q}'i'}(0)\, a^+_{\vec{q}i}(in) | ps\rangle = -\int dx^0 \frac{d}{dx^0} \langle \vec{p}'s' | T(\mathring{a}_{\vec{q}'i'}(0)\, a^+_{\vec{q}i}(x^0)) | \vec{p}s\rangle =$$

$$= \langle \vec{p}'s' | [\mathring{a}_{\vec{q}'i'}(0), a^+_{\vec{q}i}(0)] | \vec{p}s\rangle - \int dx^0 \langle \vec{p}'s' | T(\mathring{a}_{\vec{q}'i'}(0)\, \mathring{a}_{\vec{q}i}(x^0)) | \vec{p}s\rangle . \tag{1.5}$$

The equal-time commutator in the relation (1.5) can be expressed in the following way:

$$Y = \langle \vec{p}'s' | [\mathring{a}_{\vec{q}'i'}(0), a^+_{\vec{q}i}(0)] | \vec{p}s\rangle = i(2\pi)^3 \delta^{(3)}(\vec{p}' + \vec{q}' - \vec{p} - \vec{q})\, \mathcal{Y}_{fi} \tag{1.6a}$$

$$\mathcal{Y}_{fi} = Z^{-1/2} \langle \vec{p}'s' | [j_{i'}(0), a^+_{\vec{q}i}(0)] | \vec{p}s\rangle \equiv \langle \vec{p}s', \vec{q}'i' | y | \vec{p}s, \vec{q}i\rangle . \tag{1.6b}$$

Further we insert the complete set of physical states $\sum_n |n,in\rangle\langle in,n| = 1$ between the operators $\dot{a}_{q'i'}$ and $\dot{a}^{+}_{qi}$ in the second term of the right-hand side of eq.(1.5) and make use of the translational invariance of the theory. The cancellation of δ -functions, corresponding to the conservation of the total 3-momentum of a πN system from the both sides of eq.(1.5) leads to the following expression:

$$\begin{aligned}\langle \vec{p}'s'|j_{i'}(0)|\vec{p}s,\vec{q}i,in\rangle &= \langle \vec{p}s',\vec{q}'i'|\mathcal{Y}|\vec{p}s,\vec{q}i\rangle - \\ &- (2\pi)^3\sum_n \langle \vec{p}'s'|j_{i'}(0)|n,in\rangle \frac{\delta^{(3)}(\vec{p}+\vec{q}-\vec{P}_n)}{p^0+q^0-P_n^0+i\varepsilon}\langle in,n|j_i(0)|\vec{p}s\rangle - \\ &- (2\pi)^3\sum_n \langle \vec{p}'s'|j_i(0)|n,in\rangle \frac{\delta^{(3)}(\vec{p}'-\vec{q}-\vec{P}_n)}{p'^0-q^0-P_n^0}\langle in,n|j_{i'}(0)|\vec{p}s\rangle .\end{aligned} \tag{1.7}$$

Here $P_n = (P_n^0, \vec{P}_n)$ stands for the total four-mementum of the intermediate state $|n,in\rangle$.

The well-known Low equation[12] can be derived from relation (1.7) if the contributions of all intermediate states except $n = N, \pi N$ are omitted in the right-hand side. Truncation of multiparticle states is necessary for obtaining a closed set of integral equations. This truncation is widely used in field-theoretical formulations of the equations for transition matrices in low- and intermediate energy region. The Chew-Low equation[13] is obtained from (1.7) if one neglects the equal-time commutator $\mathcal{Y}_{fi}$ and uses the static approximation for the nucleon. However, an important role of the nucleon recoil in the low-energy pi-nucleon scattering has been discussed in ref.[15,20]. In particular, it has been demonstrated[15] that in the case of a nonstatic nucleon the resonance in the P_{33} channel can be reproduced provided the contributions from the antinucleon states are taken into account. Following ref.[15] we pick out the disconnected parts of transition matrices in order to find this contribution. The resulting equation is written in the following form:

$$\begin{aligned}\langle \vec{p}'s'|j_{i'}(0)|\vec{p}s,\vec{q}i,in\rangle_c &= \langle \vec{p}s',\vec{q}'i'|\mathcal{Y}|\vec{p}s,\vec{q}i\rangle + \langle \vec{p}s',\vec{q}'i'|V|\vec{p}s,\vec{q}i\rangle + \\ &+ (2\pi)^3\sum_{n=\pi N} \langle \vec{p}'s'|j_{i'}(0)|n,in\rangle_c \frac{\delta^{(3)}(\vec{P}_n-\vec{p}-\vec{q})}{P_n^0-p^0-q^0-i\varepsilon}\langle in,n|j_i(0)|\vec{p}s\rangle_c + \\ &+ (2\pi)^3\sum_{n=\pi N} \langle \vec{p}'s'|j_i(0)|n,in\rangle_c \frac{\delta^{(3)}(\vec{P}_n-\vec{p}'+\vec{q})}{P_n^0-p'^0+q^0}\langle in,n|j_{i'}(0)|\vec{p}s\rangle_c ,\end{aligned} \tag{1.8}$$

where the subscript c denotes a connected matrix element, and V contains one-nucleon exchange in the s - and u -channels (fig. 2a,b) and one-antinucleon exchange in the $\bar{s}$ - and $\bar{u}$ -channels (fig. 2c,d), the latter being referred to as z -graphs.

$$\begin{aligned}\langle \vec{p}'s',\vec{q}'i'|V|\vec{p}s,\vec{q}i\rangle &= (2\pi)^3\sum_{\vec{P}_N S_N} \langle \vec{p}'s'|j_{i'}(0)|\vec{P}_N S_N\rangle \frac{\delta^{(3)}(\vec{P}_N-\vec{p}-\vec{q})}{P_N^0-p^0-q^0}\langle \vec{P}_N S_N|j_i(0)|\vec{p}s\rangle + \\ &+ (2\pi)^3\sum_{\vec{P}_N S_N} \langle \vec{p}'s'|j_i(0)|\vec{P}_N S_N\rangle \frac{\delta^{(3)}(\vec{P}_N-\vec{p}'+\vec{q})}{P_N^0-p'^0+q^0}\langle \vec{P}_N S_N|j_{i'}(0)|\vec{p}s\rangle - \\ &- (2\pi)^3\sum_{\vec{P}_{\bar{N}} S_{\bar{N}}} \langle 0|j_i(0)|\vec{P}_{\bar{N}} S_{\bar{N}},\vec{p}s\rangle \frac{\delta^{(3)}(\vec{P}_{\bar{N}}+\vec{p}+\vec{q})}{P_{\bar{N}}^0+p^0+q^0}\langle \vec{P}_{\bar{N}} S_{\bar{N}},\vec{p}'s'|j_{i'}(0)|0\rangle - \\ &- (2\pi)^3\sum_{\vec{P}_{\bar{N}} S_{\bar{N}}} \langle 0|j_{i'}(0)|\vec{P}_{\bar{N}} S_{\bar{N}},\vec{p}s\rangle \frac{\delta^{(3)}(\vec{P}_{\bar{N}}+\vec{p}'-\vec{q})}{P_{\bar{N}}^0+p'^0-q^0}\langle \vec{P}_{\bar{N}} S_{\bar{N}},\vec{p}'s'|j_i(0)|0\rangle .\end{aligned} \tag{1.9}$$

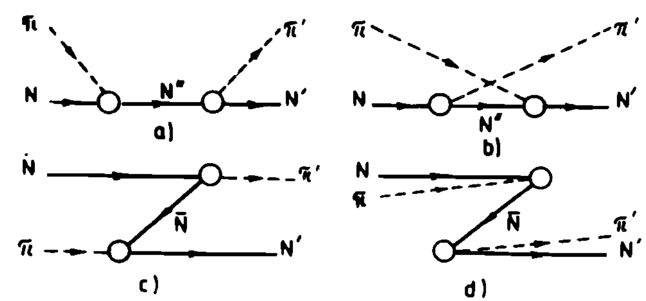

Fig. 2.

a) s -channel one-nucleon exchange graph; b) u -channel one-nucleon exchange graph; c) $\bar{u}$ -channel z -graph; d) $\bar{s}$ -channel z -graph.

The connected contributions to the driving term (1.9) from $\pi\pi N$, $NN\bar{N}$ and other higher mass intermediate states are neglected. Equation (1.8) is the Low equation for the πN -scattering t -matrix for the first time obtained in ref.[15]). The nonlinear three-dimensional integral equation (1.8) contains the s-channel (fig. 3a) and u-channel (fig. 3b) terms, the latter being referred hereafter to as the crossing term and obeys the crossing-symmetry relations. It should be pointed out that within various chiral models the Weinberg-Tomozawa formulae for the s-wave πN-scattering lengths can be reproduced from eq.(1.8) in the tree approximation provided the equal-time commutator $\mathcal{Y}_{fi}$ and z-graph contribution are retained in the driving term (1.9) indicating the importance of these contributions being omitted in the Chew-Low theory.

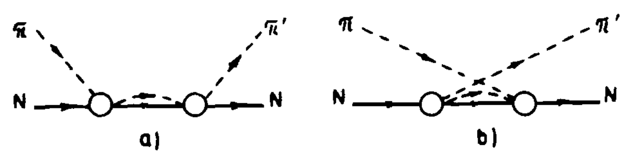

Fig. 3.

a) s-channel graph, corresponding to fG_0f^+ term in eq. (1.12).

b) u-channel graph, corresponding to (fG_0f^+) term in eq. (1.12).

In the formulation of Low-type equations considered above nucleons alwaya remain on-mass-shell in in- or out-states. In refs. [18,19]) the Low-type equations have been suggested where both pions and nucleons are allowed to go off-mass-shell. The driving term of these equations, in addition to s- and u-channel terms, includes t-channel exchange terms with σ, ρ, 2π **intermediate states (fig. 4)**. Equation (1.8) seems to be more simple and convenient for our purpose though the formalism given below can be generalized for the equations suggested in refs.[18,19]).

Note that the driving term V (1.9) is nonhermitian due to the presence of intermediate nucleon and antinucleon propagators. Moreover, if the πN- interaction Lagrangian depends on the derivative of the pion field, then the equal-time commutator

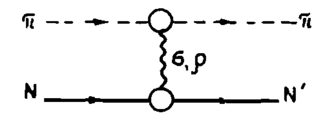

Fig. 4. σ- and ρ-exchange graphs in πN-scattering t-channel.

$\mathcal{Y}_{fi}$ (1.6b) is nonhermitian too. This is easy to observe if Y (1.6a) is written in the following manner:

$$Y = \frac{1}{2}\langle\vec{p}'s'|\{[\dot{a}_{\vec{q}'i'}(0), a^+_{\vec{q}i}(0)] - [a_{\vec{q}'i'}(0), \dot{a}^+_{\vec{q}i}(0)]\}|\vec{p}s\rangle +$$
$$+\frac{1}{2}\langle\vec{p}'s'|\frac{d}{dx^0}\{[a_{\vec{q}'i'}(x^0), a^+_{\vec{q}i}(x^0)]\}|_{x^0=0}|\vec{p}s\rangle . \qquad (1.10)$$

If the last term in (1.10) is omitted, then $\mathcal{Y}_{fi}$ is hermitian and can be written in the following manner

$$\tilde{\mathcal{Y}}_{fi} = \frac{1}{2} z^{-1/2}\langle\vec{p}'s'|[j_{i'}(0), a^+_{\vec{q}i}(0)] + [a_{\vec{q}'i'}(0), j_i(0)]|\vec{p}s\rangle . \qquad (1.11)$$

However, the last term in (1.10) vanishes provided the πN-interaction Lagrangian does not depend on $\dot{\phi}_i(x)$. In Appendix A a slightly different formulation of Low-type equations is presented where the hermitian equal-time commutators similar to (1.11) appear from the beginning in the case of derivative coupling too.

The Low-type equation (1.8) for the πN-scattering t-matrix f (1.4) can schematically be written in the following form:

$$f = W + fG_0f^+ \quad , \quad W = \mathcal{Y} + V + (fG_0f^+)_{cr} . \qquad (1.12)$$

Here $\mathcal{Y}$ and V are given by (1.6b) and (1.9), $G_0 = \frac{1}{p^0+q^0+i\varepsilon-p'^0-q'^0}$ is the Green function for a noninteracting πN-system, and $(\cdots)_{cr}$ denotes the crossing of initial and final pi-mesons in the expression in brackets, i.e. the change $q,i \to -(p+q-p'),i'$ and vice versa.

The solution of the nonlinear integral equations (1.12) can be found on the basis of the following iteration procedure

$$f^{(I+1)} = W^{(I)} + f^{(I+1)}G_0f^{(I+1)^+}, \quad W^{(I)} = \mathcal{Y} + V + (f^{(I)}G_0f^{(I)^+})_{cr}, \quad I = 0,1,2\ldots \quad . \qquad (1.13)$$

With some initial approximation $f^{(0)}$ and corresponding $W^{(0)}$, where the special choice of $f^{(0)}$ can provide the rapid convergence of iteration procedure (1.13). In ref.[17] $f^{(0)}=0$ and $W^{(0)}=Y+V$ have been assumed. It will be demonstrated below that for any given $W^{(N)}$ eq. (1.3) on half-energy-shell are equivalent to the Lippmann-Schwinger equation with some potential $U(E)$ dependent on the total energy E of a πN-system:

$$\mathcal{T}(E) = U(E) + U(E)G_0(E)\mathcal{T}(E). \qquad (1.14)$$

Here $\mathcal{T}(E)$ coincides with the πN-scattering t-matrix on energy shell.

2. LINEARIZATION OF LOW-TYPE EQUATIONS

Let us consider eq.(1.12) in the center-of-mass frame where $\vec{p}'=-\vec{q}'$ and $\vec{p}=-\vec{q}$. The driving term W in this equation can be written in the following form:

$$\langle \vec{p}'s', -\vec{p}'i' | W | \vec{p}s, -\vec{p}i\rangle \equiv W_{a'a}(\vec{p}',\vec{p}) = A_{a'a}(\vec{p}',\vec{p}) + E_{\vec{p}'} B_{a'a}(\vec{p}',\vec{p}). \qquad (2.1a)$$

Here a denotes the set of quantum numbers of the πN- system. $a=(s,i)$, $E_{\vec{k}} \equiv \omega_N(\vec{k})+\omega_\pi(\vec{k}) = \sqrt{\vec{k}^2+M^2}+\sqrt{\vec{k}^2+\mu^2}$ is the total energy of the πN system, and A and B are hermitian matrices. According to (1.6), (1.9), (1.10), (1.11) and (1.12) we obtain the following explicit expression for A and B:

$$B_{a'a}(\vec{p}',\vec{p}) = \langle \vec{p}'s', -\vec{p}'i' | Y' | \vec{p}s, -\vec{p}i\rangle + \sum_{j=1}^{4} B^{j}_{a'a}(\vec{p}',\vec{p}) + \int \frac{d^3\vec{q}''\, d^3\vec{p}''\, M}{(2\pi)^3\, 2\omega_\pi(\vec{q}'')\,\omega_N(\vec{p}'')} B^{5}_{a'a}(\vec{p}',\vec{p};\vec{q}'',\vec{p}'') \qquad (2.1b)$$

$$A_{a'a}(\vec{p}',\vec{p}) = \langle \vec{p}'s', -\vec{p}'i' | \tilde{Y}_{ji} - \frac{1}{2}(E_{\vec{p}'}+E_{\vec{p}})Y' | \vec{p}s, -\vec{p}i\rangle -$$
$$- M B^{1}_{a'a}(\vec{p}',\vec{p}) + M B^{2}_{a'a}(\vec{p}',\vec{p}) - (p^0+p'^0-\ell^0) B^{3}_{a'a}(\vec{p}',\vec{p}) - (q^0+q'^0-\ell^0) B^{4}_{a'a}(\vec{p}',\vec{p}) -$$
$$- \int \frac{d^3\vec{q}''\, d^3\vec{p}''\, M}{(2\pi)^3\, 2\omega_\pi(\vec{q}'')\,\omega_N(\vec{p}'')} (p^0+p'^0-\omega_\pi(\vec{q}'')-\omega_N(\vec{p}''))\, B^{5}_{a'a}(\vec{p}',\vec{p};\vec{q}'',\vec{p}''). \qquad (2.1c)$$

where $\ell^0=\sqrt{(\vec{p}'+\vec{p})^2+M^2}$, $\tilde{Y}_{ji}$ is defined by (1.11) and:

$$\langle \vec{p}'s', -\vec{p}'i' | Y' | \vec{p}s, -\vec{p}i\rangle =$$
$$= \int d^3\xi\, e^{i(\vec{p}+\vec{p}')\frac{\vec{\xi}}{2}} \langle \vec{p}'s' | [(\dot{\phi}_{i'}(0,\tfrac{\vec{\xi}}{2}) - iq'^0\phi_{i'}(0,\tfrac{\vec{\xi}}{2})), (\dot{\phi}_{i}(0,-\tfrac{\vec{\xi}}{2}) + iq^0\phi_{i}(0,-\tfrac{\vec{\xi}}{2}))] | \vec{p}s\rangle \qquad (2.2a)$$

$$B^{1}_{a'a}(\vec{p}',\vec{p}) = -\sum_{s_N} \frac{\langle \vec{p}'s' | j_{i'}(0) | 0\, s_N\rangle}{E_{\vec{p}'}-M}\, \frac{\langle 0 s_N | j_i(0) | \vec{p}s\rangle}{E_{\vec{p}}-M} \qquad (2.2b)$$

$$B^{2}_{a'a}(\vec{p}',\vec{p}) = -\sum_{s_{\bar{N}}} \frac{\langle 0 | j_i(0) | \vec{p}s, 0 s_{\bar{N}}\rangle}{E_{\vec{p}}+M}\, \frac{\langle 0 s_{\bar{N}}, \vec{p}'s' | j_{i'}(0) | 0\rangle}{E_{\vec{p}'}+M} \qquad (2.2c)$$

$$B^{3}_{a'a}(\vec{p}',\vec{p}) = \sum_{s_N} \frac{M}{\ell^0}\, \frac{\langle \vec{p}'s' | j_i(0) | (\vec{p}+\vec{p}') s_N\rangle}{p^0-q'^0-\ell^0}\, \frac{\langle (\vec{p}+\vec{p}') s_N | j_{i'}(0) | \vec{p}s\rangle}{p'^0-q^0-\ell^0} \qquad (2.2d)$$

$$B^{4}_{a'a}(\vec{p}',\vec{p}) = -\sum_{s_{\bar{N}}} \frac{M}{\ell^0}\, \frac{\langle 0 | j_{i'}(0) | \vec{p}s, -(\vec{p}+\vec{p}') s_{\bar{N}}\rangle}{q^0-p'^0-\ell^0}\, \frac{\langle \vec{p}'s', -(\vec{p}+\vec{p}') s_{\bar{N}} | j_i(0) | 0\rangle}{q'^0-p^0-\ell^0} \qquad (2.2e)$$

$$B^{5}_{a'a}(\vec{p}',\vec{p};\vec{q}'',\vec{p}'') = \sum_{s''i''} \frac{\langle \vec{p}'s' | j_i(0) | \vec{p}''s'', \vec{q}''i''\rangle}{p'^0-q^0-\omega_N(\vec{p}'')-\omega_\pi(\vec{q}'')}\, \delta^{(3)}(\vec{p}'+\vec{p}-\vec{p}''-\vec{q}'')\, \frac{\langle \vec{p}''s'', \vec{q}''i'' | j_{i'}(0) | \vec{p}s\rangle}{p^0-q'^0-\omega_N(\vec{p}'')-\omega_\pi(\vec{q}'')} \qquad (2.2f)$$

$B^5_{a'a}$ and $A^5_{a'a}$ contain the πN-scattering t-matrix of interest, therefore the solution of Low equation (1.12) is given by the iteration series (1.13).

In the center-of-mass system the Low equation (1.12) is written in the following form:

$$f_{a'a}(\vec{p}',\vec{p}) = W_{a'a}(\vec{p}',\vec{p}) + \sum_{a''} \int d\tilde{k}\, \frac{f_{a'a''}(\vec{p}',\vec{k})\, f^{*}_{aa''}(\vec{p},\vec{k})}{E_{\vec{k}} - E^{+}_{\vec{p}}}, \qquad (2.3)$$

where $E^{\pm}_{\vec{p}} \equiv E_{\vec{p}} \pm i\varepsilon$, and $d\tilde{k} \equiv d^3\vec{k}\, M/((2\pi)^3\, 2\omega_\pi(\vec{k})\omega_N(\vec{k}))$.

Further the iteration series for eq.(2.3) are considered:

$$f_{a'a}(\vec{p}',\vec{p}) = \sum_{N=1}^{\infty} f^{(N)}_{a'a}(\vec{p}',\vec{p}) \qquad (2.4a)$$

$$f^{(N)}_{a'a}(\vec{p}',\vec{p}) = \sum_{n=1}^{N-1} \sum_{a''} \int d\tilde{k}\, \frac{f^{(n)}_{a'a''}(\vec{p}',\vec{k})\, f^{(N-n)*}_{aa''}(\vec{p},\vec{k})}{E_{\vec{k}} - E^{+}_{\vec{p}}}, \quad N \geq 2. \qquad (2.4b)$$

In particular, for $N=1,2,3$ we obtain:

$$f^{(1)}_{a'a}(\vec{p}',\vec{p}) = W_{a'a}(\vec{p}',\vec{p}) \qquad (2.5a)$$

$$f^{(2)}_{a'a}(\vec{p}',\vec{p}) = \sum_{\ell}\int d\tilde{k}\,\frac{W_{a'\ell}(\vec{p}',\vec{k})\,W^{*}_{a\ell}(\vec{p},\vec{k})}{E_{\vec{k}} - E^{+}_{\vec{p}}} = \sum_{\ell}\int d\tilde{k}\,\frac{W_{a'\ell}(\vec{p}',\vec{k})\,U_{\ell a}(\vec{k},\vec{p};E_{\vec{p}})}{E_{\vec{k}} - E^{+}_{\vec{p}}} \tag{2.5b}$$

$$f^{(3)}_{a'a}(\vec{p}',\vec{p}) = \sum_{\ell_1\ell_2}\int d\tilde{k}_1 d\tilde{k}_2\,\frac{W_{a'\ell_1}(\vec{p}',\vec{k}_1)\,W^{*}_{a\ell_2}(\vec{p},\vec{k}_2)}{E_{\vec{k}_1} - E^{+}_{\vec{k}_2}}\left(\frac{W^{*}_{\ell_2\ell_1}(\vec{k}_2,\vec{k}_1)}{E_{\vec{k}_2} - E^{+}_{\vec{p}}} - \frac{W_{\ell_1\ell_2}(\vec{k}_1,\vec{k}_2)}{E_{\vec{k}_1} - E^{+}_{\vec{p}}}\right) =$$

$$= \sum_{\ell_1\ell_2}\int d\tilde{k}_1 d\tilde{k}_2\, W_{a'\ell_1}(\vec{p}',\vec{k}_1)\,\frac{U_{\ell_1\ell_2}(\vec{k}_1,\vec{k}_2;E_{\vec{p}})\,U_{\ell_2 a}(\vec{k}_2,\vec{p},E_{\vec{p}})}{(E_{\vec{k}_1} - E^{+}_{\vec{p}})(E_{\vec{k}_2} - E^{+}_{\vec{p}})}\,. \tag{2.5c}$$

Here U is a hermitian matrix for any real parameter E

$$U_{\ell_1\ell_2}(\vec{k}_1,\vec{k}_2;E) = A_{\ell_1\ell_2}(\vec{k}_1,\vec{k}_2) + E\,B_{\ell_1\ell_2}(\vec{k}_1,\vec{k}_2)\,. \tag{2.6}$$

From the definition of $U(E)$ (2.6) we obtain:

$$W^{*}_{\ell_2\ell_1}(\vec{k}_2,\vec{k}_1) = U_{\ell_1\ell_2}(\vec{k}_1,\vec{k}_2;E_{\vec{k}_2}) \tag{2.7a}$$

$$W_{\ell_1\ell_2}(\vec{k}_1,\vec{k}_2) = U_{\ell_1\ell_2}(\vec{k}_1,\vec{k}_2;E_{\vec{k}_1}) \tag{2.7b}$$

(2.5a)-(2.5c) can be generalized to higher-order terms of itera-tion series (2.4b) in the following way:

$$f^{(n)}_{a\ell_n}(\vec{p}',\vec{k}_n) = \sum_{\ell_1\cdots\ell_{n-1}}\int W_{a'\ell_1}(\vec{p}',\vec{k}_1)\prod_{j=1}^{n-1}\left(\frac{d\tilde{k}_j}{E_{\vec{k}_j} - E^{+}_{\vec{k}_n}}\,U_{\ell_j\ell_{j+1}}(\vec{k}_j,\vec{k}_{j+1};E_{\vec{k}_n})\right),\ n\geqslant 3. \tag{2.8}$$

Equation (2.8) can be proved for any n by mathematical induction We assume that (2.8) holds for a finite $N>3$. Then, using (2.4b), $f^{(N+1)}_{a'a}$ is written in the following manner:

$$f^{(N+1)}_{a'\ell_{N+1}}(\vec{p}',\vec{k}_{N+1}) = \sum_{n=1}^{N}\left\{\sum_{\ell_n}\int\!\!\left[\sum_{\ell_1\cdots\ell_{n-1}}\int W_{a'\ell_1}(\vec{p}',\vec{k}_1)\prod_{j=1}^{n-1}\left(\frac{d\tilde{k}_j}{E_{\vec{k}_j} - E^{+}_{\vec{k}_n}}\,U_{\ell_j\ell_{j+1}}(\vec{k}_j,\vec{k}_{j+1};E_{\vec{k}_n})\right)\right]\times\right.$$

$$\left.\times\,\frac{d\tilde{k}_n}{E_{\vec{k}_n} - E^{+}_{\vec{k}_{N+1}}}\left[\sum_{\ell_{n+1}\cdots\ell_N}\int W_{\ell_{N+1}\ell_N}(\vec{k}_{N+1},\vec{k}_N)\prod_{i=n+1}^{N}\left(\frac{d\tilde{k}_i}{E_{\vec{k}_i} - E^{+}_{\vec{k}_n}}\,U_{\ell_i\ell_{i-1}}(\vec{k}_i,\vec{k}_{i-1};E_{\vec{k}_n})\right)\right]^{*}\right\} \tag{2.9}$$

where $\prod_{j=L}^{M}(\cdots)=1$ and $\sum_{\ell_L\cdots\ell_M}(\cdots)=1$, when $M<L$ is assumed.

With the use of (2.7a)-(2.7b) from (2.9) we obtain:

$$f^{(N+1)}_{a'\ell_{N+1}}(\vec{p}',\vec{k}_{N+1}) = \sum_{n=1}^{N}\left\{\sum_{\ell_1\cdots\ell_N}\int W_{a'\ell_1}(\vec{p}',\vec{k}_1)\prod_{j=1}^{N-1}\left(d\tilde{k}_j\,U_{\ell_j\ell_{j+1}}(\vec{k}_j,\vec{k}_{j+1};E_{\vec{k}_n})\right)\times\right.$$

$$\left.\times\,\frac{1}{(E_{\vec{k}_n} - E^{+}_{\vec{k}_{N+1}})\prod_{i=1}^{n-1}(E_{\vec{k}_i} - E^{+}_{\vec{k}_n})\prod_{\ell=n+1}^{N}(E_{\vec{k}_\ell} - E^{-}_{\vec{k}_n})}\,d\tilde{k}_N\,U_{\ell_N\ell_{N+1}}(\vec{k}_N,\vec{k}_{N+1};E_{\vec{k}_{N+1}})\right\}. \tag{2.10}$$

In Appendix B the identity:

$$\sum_{n=1}^{N}\frac{E_n^m}{(E_n - E^{\pm})\prod_{i=1}^{n-1}(E_i - E_n^{+})\prod_{\ell=n+1}^{N}(E_\ell - E_n^{-})} = \frac{E^m}{\prod_{\ell=1}^{N}(E_\ell - E^{\pm})} \tag{2.11}$$

for any N and $m=0,1\ldots N-1$ is proved.

With the use of explicit expression for $U(E)$ (2.6) as well as (2.10) and (2.11) one makes sure that $f^{(N+1)}_{a'a}$ is given by an expression similar in form to (2.8) with $n=N+1$. Thus, the proof of eq.(2.8) is completed.

The iteration series where the n -th term is given by eq. (2.8) can be reproduced from the following linear integral equation

$$\langle\vec{k}'|\,\mathcal{T}_{a'a}(E_{\vec{k}})|\vec{k}\rangle = U_{a'a}(\vec{k}',\vec{k};E_{\vec{k}}) + \sum_{a''}\int U_{a'a''}(\vec{k}',\vec{q};E_{\vec{k}})\,d\tilde{q}\,\frac{\langle\vec{q}|\,\mathcal{T}_{a''a}(E_{k})|\vec{k}\rangle}{E_{\vec{q}} - E^{+}_{\vec{k}}}\,. \tag{2.12}$$

From the comparison of each term of the iteration series for eq. (2.12) with the corresponding expression (2.8) one obtains:

$$f^{(N+1)}_{a'a}(\vec{p}',\vec{p}) = \sum_{\ell}\int d\tilde{q}\,W_{a'\ell}(\vec{p}',\vec{q})\,\frac{1}{E_{\vec{q}} - E^{+}_{\vec{p}}}\,\langle\vec{q}|\,\mathcal{T}^{(N)}_{\ell a}|\vec{p}\rangle \tag{2.13}$$

$$f^{(N)}_{a'a}(\vec{p}',\vec{p}) = \langle\vec{p}'|\,\mathcal{T}^{(N)}_{a'a}(E_{\vec{p}})|\vec{p}\rangle\,.$$

With the use of (2.5a) we finally arrive at the following result:

$$f_{a'a}(\vec{p}',\vec{p}) = \sum_{\ell}\int d\tilde{q}\,W_{a'\ell}(\vec{p}',\vec{q})\left[\delta_{a\ell}\,\delta^{(3)}(\vec{p}-\vec{q}) + \frac{1}{E_{\vec{q}} - E^{+}_{\vec{p}}}\,\langle\vec{q}|\,\mathcal{T}_{\ell a}(E_{\vec{p}})|\vec{p}\rangle\right] \tag{2.14}$$

$$f_{a'a}(\vec{p}',\vec{p}) = \langle\vec{p}'|\,\mathcal{T}_{a'a}(E_{\vec{p}})|\vec{p}\rangle\,.$$

Thus, for any given $W_{a'a}$ (2.1a) each term in the iteration series of eq.(2.12) with potential $U(E)$ (2.6) coincides with the corres-ponding term in the iteration series for the matrix $f_{a'a}$ on energy shell. Hence the solution of eq.(2.12) reproduces the solution of nonlinear equation (2.3) which is given via the iteration series (2.4a)-(2.4b).It should be pointed out that the series (2.4a)-(2.4b) may diverge but the solution $f_{a'a}$ can still exist; e.g. in refs.[15,16]) Pade approximants have been used to sum up such divergent series. Recalling the explicit expressions for W (1.12) and cor-responding $U(E)$ one can easily observe that in fact eq.(2.12) is nonlinear due to the term $(f G_0 f^+)_{cr}$ in W (1.12). Such nonlinear equations can be treated by the iteration procedure (1.13). Similar nonlinearity due to the symmetry of the amplitude of interest with respect to the pi-meson crossing is inherent of Low-type equations for $\pi\pi$- scattering[19]. However, in the case of NN - scattering the nonlinear terms in the potential of the resulting Lippmann-Schwinger equation do not appear.

It should be pointed out that if the nonhermitian term $B_{\alpha'\alpha}$ in W (2.1) is set equal to zero, the equivalence of the Low equation and Lippmann-Schwinger equation can be rigorously proved in the scattering theory [22] provided the potential $A_{\alpha'\alpha}$ does not create bound states. On the other hand, if all terms in W except the term proportional to $(E_\beta - M)^{-1}$ are omitted, then eq. (2.12) will be similar to the linear equation suggested in ref.[17]. Hence, the linearization procedure suggested in the present work generalizes, on the one hand, the linearization scheme known in the potential scattering theory and, on the other hand, the result obtained in ref.[17] to the case of any given driving term.

3. OTHER FORMULATIONS FOR THE PION-NUCLEON SCATTERING PROBLEM

In this section we demonstrate that the nonlinearities arising in the Low equation (2.3) and corresponding Lippmann-Schwinger equation (2.12) are inherent of other formulations of pi-nucleon scattering too. In particular, below eqs. (2.3) and (2.12) will be compared with the corresponding multichannel Lippmann-Schwinger equatinns obtained within the framework of potential scattering theory as well as with the Bethe-Salpeter or quasipotential equations for πN -scattering.

3.1. Multichannel Scattering Theory Equations

Below we consider the multichannel scattering theory for relativistic particles with channel states $\alpha = N$, πN , $\pi\pi N$,

$N\bar{N}$, $\pi\pi N N\bar{N} = 1 \cdots 6$. Our choice is motivated by the fact that the same intermediate states are present in the Low equation (1.8). In particular, N and πN intermediate states appear in s - channel diagrams (fig. 2a and 3a); $\pi\pi N$ and $\pi\pi\pi N$ states, in u -channel diagrams (fig. 2b and 3b), and $NN\bar{N}$ and $\pi\pi NN\bar{N}$ states, in $\bar{s}$- and $\bar{u}$ -channel diagrams (fig. 2c,d). The transition matrix $T_{\beta\alpha}(E)$ is a solution of the Lippmann-Schwinger integral equation:

$$T_{\alpha\beta}(E) = V_{\alpha\beta} + \sum_{\sigma=1}^{6} V_{\alpha\sigma} G_0(E, E_\sigma) T_{\sigma\beta}(E), \tag{3.1}$$

where E is the total energy of the πN system, E_σ is the energy of the asymptotic state σ , and $G_0(E,E_\sigma) = \langle\sigma|(E-H_0)^{-1}|\sigma\rangle$ is the free Green function in the σ -channel. Further the set of scattering states $|\psi_\alpha^{(+)}\rangle$ of the total Hamiltonian $H_{\alpha\beta} = (H_0 + V)_{\alpha\beta}$ is introduced. The potential $V_{\alpha\beta}$ is assumed to be hermitian. Then, if $V_{\alpha\beta}$ does not create bound states, one obtains the following representation for the total Green function $G(E)$

$$[G(E)]_{\alpha\beta} = [G_0^{-1}(E) - V]_{\alpha\beta}^{-1} = \sum_{\sigma=1}^{6} \int \frac{\langle\alpha|\psi_\sigma^{(+)}\rangle\langle\psi_\sigma^{(+)}|\beta\rangle}{E + i\varepsilon - E_\sigma} . \tag{3.2}$$

With the use of the relation $T_{\alpha\beta}(E_\beta) = \sum_{\sigma=1}^{6} V_{\alpha\sigma}\langle\sigma|\psi_\beta^{(+)}\rangle$ one arrives at the nonlinear singular integral Low-type equation for the multichannel scattering t -matrix:

$$T_{\alpha\beta}(E) = V_{\alpha\beta} + \sum_{\sigma,\sigma'=1}^{6} V_{\alpha\sigma'}[G(E)]_{\sigma'\sigma} V_{\sigma\beta} = V_{\alpha\beta} + \sum_{\sigma=1}^{6} \int \frac{T_{\alpha\sigma}(E_\sigma) T_{\sigma\beta}^{+}(E_\sigma)}{E + i\varepsilon - E_\sigma} . \tag{3.3}$$

Below it is demonstrated that the special choice for $V_{\alpha\beta}$ will allow us to obtain the Low equation for the πN -scattering t - matrix $T_{2'2}(E)$ almost similar in form to (1.8). For this purpose $V_{2'2}$ is chosen to be:

$$V_{2'2} = -(2\pi)^3 \delta^{(3)}(\vec{p}\,' + \vec{q}\,' - \vec{p} - \vec{q}) \langle \vec{p}\,'s', \vec{q}\,'i' | \tilde{y} | \vec{p}s, \vec{q}i \rangle \cdot \tag{3.4}$$

Here one has to work with the formulation of Low-type equation with the hermitian Y -term (1.11) presented in Appendix A in order to provide the required hermiticity of potential $V_{\alpha\beta}$. Moreover, for $\alpha = 3 \cdots 6$, by analogy with the derivation of (1.8), only the disconnected terms in transition matrices $T_{\alpha 2}(E)$, describing interactions in πNN vertices will be retained. In particular, we assume that

$$T_{21'}(E_{1'}) = -(2\pi)^3 \delta^{(3)}(\vec{p}\,' - \vec{p} - \vec{q}) \langle \vec{p} s | \tilde{j}_{\vec{q} i}(0) | \vec{p}\,' s' \rangle \qquad (3.5a)$$

$$T_{23'}(E_{3'}) = -\frac{1}{\sqrt{2}} (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{q}\,'_{\pi_1} - \vec{p}\,') \delta^{(3)}(\vec{q}\,'_{\pi_2} - \vec{q}) \delta_{i'_{\pi_2} i} \langle \vec{p} s | \tilde{j}_{\vec{q}\,'_{\pi_1} i'_{\pi_1}}(0) | \vec{p}\,' s' \rangle + \{\pi_1', \pi_2'\} \qquad (3.5b)$$

$$T_{24'}(E_{4'}) = -\frac{1}{\sqrt{6}} (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{q}\,'_{\pi_1} - \vec{q}\,'_{\pi_2} - \vec{p}\,') \delta^{(3)}(\vec{q}\,'_{\pi_3} - \vec{q}) \delta_{i'_{\pi_3} i} \langle \vec{p} s | \tilde{j}_{\vec{q}\,'_{\pi_1} i'_{\pi_1}}(0) | \vec{p}\,' s', \vec{q}\,'_{\pi_2} i'_{\pi_2} \rangle + \{\pi_1', \pi_2', \pi_3'\} \qquad (3.5c)$$

$$T_{25'}(E_{5'}) = -\frac{1}{\sqrt{2}} (2\pi)^3 \delta^{(3)}(\vec{q} - \vec{p}\,'_{N_1} - \vec{p}_{\bar{N}}) \delta^{(3)}(\vec{p}\,'_{N_2} - \vec{p}) \delta_{s'_{N_2} s} \langle 0 | \tilde{j}_{\vec{q} i}(0) | \vec{p}\,'_{N_1} s'_{N_1}, \vec{p}_{\bar{N}} s_{\bar{N}} \rangle - \{N_1', N_2'\} \qquad (3.5d)$$

$$T_{26'}(E_{6'}) = -\frac{1}{2} (2\pi)^3 \delta^{(3)}(\vec{q}\,'_{\pi_1} + \vec{p}\,'_{N_1} + \vec{p}_{\bar{N}}) \delta^{(3)}(\vec{p}\,'_{N_2} - \vec{p}) \delta_{s'_{N_2} s} \delta^{(3)}(\vec{q}\,'_{\pi_2} - \vec{q}) \delta_{i'_{\pi_2} i} \times$$
$$\times \langle 0 | \tilde{j}_{\vec{q}\,'_{\pi_1} i'_{\pi_1}}(0) | \vec{p}\,'_{N_1} s'_{N_1}, \vec{p}_{\bar{N}} s_{\bar{N}} \rangle - \{\pi_1' N_1', \pi_2' N_2'\}. \qquad (3.5e)$$

Here $\{a, b\}$ denotes the permutation of particles a and b . An explicit expression for the pion source operator $\tilde{j}_{\vec{q} i}(0)$ is given in Appendix A (formula (A.5)). The disconnected amplitudes (3.5a)-(3.5e) are depicted in fig. 5.

In transition amplitudes $T_{\alpha\beta}(E)$ and potential $V_{\alpha\beta}$ the δ-functions corresponding to conservation of the total three-momentum are singled out

$$T_{\alpha\beta}(E) = -(2\pi)^3 \delta^{(3)}(\vec{P}_\alpha - \vec{P}_\beta) t_{\alpha\beta}(E)$$
$$V_{\alpha\beta} = -(2\pi)^3 \delta^{(3)}(\vec{P}_\alpha - \vec{P}_\beta) v_{\alpha\beta} \,. \qquad (3.6)$$

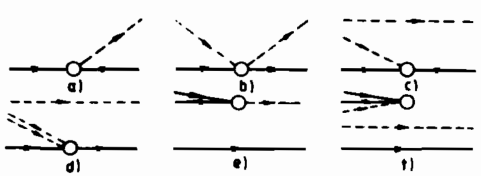

Fig. 5. Graphical representation for multichannel scattering matrix $T_{\alpha' 2}$ for $\alpha' = 1 \cdots 6 \equiv a, b, c, d, e, f = N', \pi' N', \pi'\pi' N', \pi'\pi'\pi' N', N' N' \bar{N}', \pi'\pi' N' N' \bar{N}'$.

With the use of (3.4) and (3.5a)-(3.5e) from (3.3) the Low equation for the πN- scattering t-matrix $t_{2'2}(E)$ is obtained:

$$t_{2'2}(E) = \omega_{2'2}(E) - (2\pi)^3 \sum_{2''} \int \frac{t_{2'2''}(E_{2''}) \, t^{+}_{2''2}(E_{2''})}{E + i\varepsilon - E_{2''}} \qquad (3.7a)$$

$$\omega_{2'2}(E) = v_{2'2} - (2\pi)^3 \sum_{\alpha \neq 2} \int \frac{t_{2'\alpha}(E_\alpha) \, t^{+}_{\alpha 2}(E_\alpha)}{E + i\varepsilon - E_\alpha} . \qquad (3.7b)$$

The effective pion-nucleon potential $\omega_{2'2}(E)$ (3.7b) contains the connected $\omega^{c}_{2'2}(E)$ and disconnected $\omega^{d}_{2'2}(E)$ (fig. 6) parts. Using (3.5a)-(3.5e) one obtains the following expression for the connected part

$$\langle \vec{p}\,' s', \vec{q}\,' i' | \omega^{c}_{2'2}(E) | \vec{p} s, \vec{q} i \rangle = \langle \vec{p}\,' s', \vec{q}\,' i' | \tilde{y} | \vec{p} s, \vec{q} i \rangle -$$
$$- \sum_{s_N} \int \frac{d^3 \vec{p}_N M}{\omega_N(\vec{p}_N)} \langle \vec{p}\,' s' | \tilde{j}_{\vec{q}\,' i'}(0) | \vec{p}_N s_N \rangle \frac{\delta^{(3)}(\vec{p}_N - \vec{p} - \vec{q})}{E - \omega_N(\vec{p}_N) + i\varepsilon} \langle \vec{p}_N s_N | \tilde{j}^{+}_{\vec{q} i}(0) | \vec{p} s \rangle -$$
$$- \sum_{s_N} \int \frac{d^3 \vec{p}_N M}{\omega_N(\vec{p}_N)} \langle \vec{p}\,' s' | \tilde{j}^{+}_{\vec{q} i}(0) | \vec{p}_N s_N \rangle \frac{\delta^{(3)}(\vec{p}_N - \vec{p}\,' + \vec{q})}{E - \omega_N(\vec{p}_N) - q^0 - q'^0 + i\varepsilon} \langle \vec{p}_N s_N | \tilde{j}_{\vec{q}\,' i'}(0) | \vec{p} s \rangle + \qquad (3.8)$$
$$+ \sum_{s_{\bar{N}}} \int \frac{d^3 \vec{p}_{\bar{N}} M}{\omega_N(\vec{p}_{\bar{N}})} \langle 0 | \tilde{j}_{\vec{q}\,' i'}(0) | \vec{p} s, \vec{p}_{\bar{N}} s_{\bar{N}} \rangle \frac{\delta^{(3)}(\vec{q} - \vec{p}\,' - \vec{p}_{\bar{N}})}{E - \omega_N(\vec{p}_{\bar{N}}) - p^0 - p'^0 + i\varepsilon} \langle \vec{p}\,' s', \vec{p}_{\bar{N}} s_{\bar{N}} | \tilde{j}^{+}_{\vec{q} i}(0) | 0 \rangle +$$
$$+ \sum_{s_{\bar{N}}} \int \frac{d^3 \vec{p}_{\bar{N}} M}{\omega_N(\vec{p}_{\bar{N}})} \langle 0 | \tilde{j}^{+}_{\vec{q} i}(0) | p s, p_{\bar{N}} s_{\bar{N}} \rangle \frac{\delta^{(3)}(\vec{q} + \vec{p} + \vec{p}_{\bar{N}})}{E - \omega_N(\vec{p}_{\bar{N}}) - p^0 - p'^0 - q^0 - q'^0 + i\varepsilon} \langle \vec{p}\,' s', \vec{p}_{\bar{N}} s_{\bar{N}} | j_{\vec{q}\,' i'}(0) | 0 \rangle -$$
$$- \sum_{s'' i''} \int \frac{d^3 \vec{p}\,'' d^3 \vec{q}\,'' M}{(2\pi)^3 \, 2\omega_\pi(\vec{q}\,'') \omega_N(\vec{p}\,'')} \cdot \frac{\delta^{(3)}(\vec{p}\,' - \vec{q} - \vec{p}\,'' - \vec{q}\,'')}{E - \omega_N(\vec{p}\,'') - \omega_\pi(\vec{q}\,'') - q^0 - q'^0 + i\varepsilon} \times$$
$$\times \langle \vec{p}\,' s' | \tilde{j}^{+}_{\vec{q} i}(0) | \vec{p}\,'' s'', \vec{q}\,'' i'' \rangle \langle \vec{p}\,'' s'', \vec{q}\,'' i'' | \tilde{j}_{\vec{q}\,' i'}(0) | \vec{p} s \rangle .$$

Here the (-) sign in the fourth and fifth terms in eq.(3.8) appears due to the nucleon permutation.

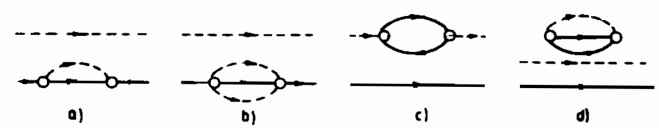

Fig. 6.

Disconnected diagrams in $\omega_{2'2}(E)$ (3.7b), which stem from the disconnectedness of $T_{\alpha' 2}$, when $\alpha' \geq 3$ (fig. 5c-5f).

The disconnected part of potential $\omega_{2'2}(E)$ depicted in fig. 6 contains the nucleon (fig. 6a,b) and pion (fig. 6c) self-energy graphs and the vacuum graph (fig. 6d). Often such disconnected diagrams are included into the pion- and nucleon-mass renormalization in the free Hamiltonian H_0 [23]). In ref.[24]) it is demonstrated that if one works with the renormalized propagators from the beginning, then the disconnected part can be set equal to zero. According to the result of ref.[24]), in the present work we omit the disconnected part $\omega^d_{2'2}(E)$. Once $\omega^d_{2'2}(E)$ is omitted in the potential, one can readily obtain the Low equation for the connected πN -scattering matrix $t^c_{2'2}$. The corresponding equation is completely similar in form to (3.7a), except $t^c_{2'2}$ and $\omega^c_{2'2}$ are substituted for $t_{2'2}$ and $\omega_{2'2}$ with $\omega^c_{2'2}$ defined by (3.8).

On the next step the equation for $t^c_{2'2}$ obtained above is compared with the corresponding Low equation (1.8) on half energy shell, where $E=E_2=p^0+q^0$. One makes sure that by analogy with (1.8) the nonlinear term $(f G_0 f^+)_{cr}$ is present in $\omega^c_{2'2}(E)$. However the propagators in u and $\bar{u}$ -channels in these two equations differ off energy shell, i.e. when $q'^0=p^0+q^0-p'^0$. Notwithstanding this difference the Low equation for $t^c_{2'2}$ can be reduced to the Lippmann-Schwinger equation along similar lines as in section 2 The resulting equation is written in the following way:

$$\tilde{t}_{2'2}(E_2)=U_{2'2}(E_2)+\sum_{2''}U_{2'2''}(E_2)G_0(E_2,E_{2''})\,\tilde{t}_{2''2}(E_2), \tag{3.9}$$

where $\tilde{t}_{2'2}(E_2)$ and $t^c_{2'2}(E)$ coincide on energy shell, and $U_{2'2}(E)$ is expressed via $\omega^c_{2'2}(E_2)$ according to (2.6).

It is of interest to compare the solution of eq.(3.9) with the solution of the following linear equation:

$$t_{2'2}(E_2)=K_{2'2}(E_2)-(2\pi)^3\sum_{2''}K_{2'2''}(E_2)G_0(E_2,E_{2''})\,t_{2''2}(E_2) \tag{3.10a}$$

$$K_{2'2}(E_2)=V_{2'2}-(2\pi)^3\sum_{\alpha\neq2}\sum_{\beta\neq2}V_{2'\alpha}\,(G(E_2))_{\alpha\beta}\,V_{\beta2} \tag{3.10b}$$

which is obtained directly from (3.1). Here $K_{2'2}(E)$, by analogy with $\omega_{2'2}(E)$, contains both disconnected and connected parts. In the kernel of integral equation (3.10a) $K_{2'2}(E_2)$ (3.10b) the logarithmic singularities due to the presence of intermediate multiparticle propagators appear whereas no such singularities in the kernels of eqs. (2.12) and (3.9) are present. However, the solution of eq. (3.10a) obeys the two- and three-particle unitarity whereas the solutions of eqs. (3.7) and (1.8) obey the two-particle unitarity only on energy shell. Hence from the comparison of solutions of eqs. (3.9) and (3.10a) the contribution of multiparticle connected diagrams being neglected in the derivation of (3.9) can be evaluated. Further, the advantage of the approach based on eq. (2.12) as compared to the one based on (3.10a) consists in the fact that the former allows the unambiguous field-theoretical prescription for constructing πN -potential. Moreover, within the framework of the former approach (2.6) defines the fully off-shell πN -potential while in (1.8) the driving term is defined only half-off-shell.

3.2. Bethe-Salpeter-Type Equations for Pion-Nucleon Scattering

The derivation of Bethe-Salpeter-type equations for pion--nucleon scattering carried out recently in refs.[11,23,25,26]) is based on the last cut lemma[37]). According to this method, in the infinite sum of Feynman graphs the sums of n -particle irreducible graphs, i.e. graphs which contain more than n -particles in intermediate states are singled out. These sums define n -particle irreducible transition matrices $M^{(n)}_{\beta\alpha}$. The system of four--dimensional singular integral equations for $M^{(n)}_{\beta\alpha}$ is derived. In particular, according to ref.[26]), for the πN -scattering matrix $M_{22}(\pi N)$ one obtains:

$$M_{22}(\pi N)=M^{(1)}_{22}(\pi N)+M^{(1)}_{21}(\pi N)d_N M^{(1)}_{12}(\pi N), \tag{3.11}$$

$$M_{22}^{(1)}(\pi N) = M_{22}^{(2)}(\pi N) + M_{22}^{(2)}(\pi N)\, d_N d_\pi\, M_{22}^{(1)}(\pi N), \tag{3.12}$$

where d_N and d_π denote nucleon and π -meson propagators and one-particle irreducible πN -scattering matrix $M_{22}^{(1)}$ and πNN vertex function $M_{21}^{(1)}$ do not contain one-nucleon intermediate states. $M_{22}^{(2)}$ and $M_{21}^{(2)}$ do not contain two-particle intermediate states, etc. For $M_{21}^{(1)}$ the following equation is obtained:

$$M_{21}^{(1)}(\pi N) = M_{21}^{(2)}(\pi N) + M_{22}^{(2)}(\pi N)\, d_N d_\pi\, M_{21}^{(1)}(\pi N). \tag{3.13}$$

A graphical representation of eqs. (3.11)-(3.13) is given in fig. 7. Further, in close analogy with the derivation of Low-type equations (1.8) or (3.7a) the disconnected contribution from multiparticle intermediate states is taken into account in $M_{22}^{(2)}$, whereas the contribution of connected amplitudes for transition from the πN state to $\pi\pi N$, $\rho\pi N$, $\pi\pi\pi N$, $\pi\pi\rho N$ ... states has been neglected. Then on the basis of the last cut lemma one obtains:

$$M_{22}^{(2)}(\pi N) = M_{22}^{(3)}(\pi N) + \sum_{i\neq j} \gamma_-^{(1)}(i,N)\, d_{\pi_1} d_{\pi_2} d_N\, \gamma_+^{(2)}(j,N) + \sum_{m=\sigma,\rho}\sum_{i\neq j} \gamma_-^{(1)}(i,m)\, d_m d_N d_\pi\, \gamma_+^{(2)}(j,m) + \cdots \tag{3.14}$$

Here $\gamma_\pm^{(n)}(i,m)$ denote the disconnected n -particle irreducible amplitudes:

$$\begin{aligned} \gamma_+^{(n)}(i,N) &= M_{21}^{(n)}(\pi_j,N)\, d_{\pi_i}^{-1} \; ; \quad \gamma_-^{(n)}(j,N) = M_{12}^{(n)}(\pi_i,N)\, d_{\pi_j}^{-1} \\ \gamma_+^{(n)}(i,\sigma) &= M_{21}^{(n)}(\pi_j,\sigma)\, d_{\pi_i}^{-1} \; ; \quad \gamma_-^{(n)}(j,\sigma) = M_{12}^{(n)}(N_i,\sigma)\, d_{N_j}^{-1}. \end{aligned} \tag{3.15}$$

Using again the last cut lemma, for $M_{22}^{(3)}(\pi N)$ one obtains:

$$M_{22}^{(3)}(\pi N) = \sum_{i\neq j} \Gamma_+^{(3)}(i)\, d_{\pi_1} d_{\pi_2} d_{\pi_3} d_N\, \Gamma_-^{(2)}(j) + \cdots \tag{3.16}$$

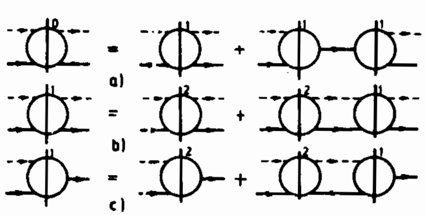

Fig. 7. Graphical representation of the equation for: a) M_{22} (3.11), b) $M_{22}^{(1)}$ (3.12), c) $M_{21}^{(1)}$ (3.13).

where $\Gamma_\pm^{(n)}$ denote the disconnected n -particle irreducible amplitudes for the transition $\pi N \to \pi\pi N$

$$\Gamma_+^{(n)}(i) = M_{13}^{(n)}(\pi_j,\pi_k,N)\, d_{\pi_i}^{-1} \; ; \; \Gamma_-^{(n)}(j) = M_{31}^{(n)}(\pi_k,\pi_i,N)\, d_{\pi_j}^{-1} \; ; \; ijk = 123, 231, 312. \tag{3.17}$$

The graphical representation of eqs. (3.14)-(3.16) which define the pion-nucleon potential $M_{22}^{(2)}(\pi N)$ is given in fig. 8. It is easy to observe that under the crossing transformation (3.16) transforms into the s -channel term of eq. (3.12) with the πN intermediate state (fig. 9):

$$\Big(\sum_{i\neq j} \Gamma_+^{(3)}(i)\, d_{\pi_1} d_{\pi_2} d_{\pi_3} d_N\, \Gamma_-^{(2)}(j)\Big)_{cr} = M_{22}^{(2)}(\pi N)\, d_\pi d_N\, M_{22}^{(1)}(\pi N). \tag{3.18}$$

Hence, the potential of the Bethe-Salpeter equation, $M_{22}^{(2)}(\pi N)$ contains the nonlinear crossing term as well as the potential of eqs. (2.12) or (3.9)-(3.10a). However, unlike (2.12), (3.9) and (3.10a), eq. (3.12) is a covariant four-dimensional equation. Consequently, in this equation the nucleon and antinucleon degrees of freedom in intermediate states cannot be separated from each other. Moreover in the Bethe-Salpeter equations considered above the nucleon pole term in $M_{22}(\pi N)$ has been singled out (3.11). The quasipotential equations obtained on the basis of the Bethe--Salpeter equation are similar in form to (3.12), except all the integrals are taken over three-momentum space.

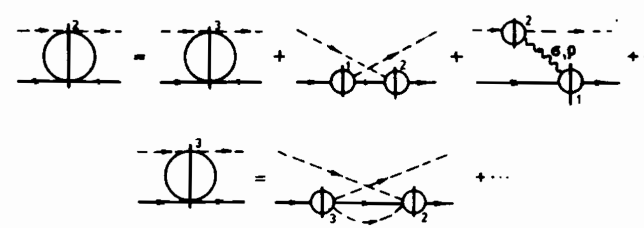

Fig. 8.

Graphical representation for the two-particle irreducible potential $M_{22}^{(2)}(\pi N)$ ((3.14) and (3.16)).

The advantage of the approach based on Low equation (1.8) as compared to the one considered in this section consists in the fact that the πN -potential in the former is built up via the vertex functions with nucleons and antinucleons on mass shell. Moreover, all intermediate state propagators in this approach are linear and contain on-mass-shell particles.

4. THE DYNAMICAL INPUT AND CALCULATIONS OF πN-SCATTERING PHASE SHIFTS

In this section the results of numerical calculations of P - wave πN -scattering phase shifts performed on the basis of equivalent linear equation (2.12) are presented. First we discuss the parametrization of various meson-nucleon vertex functions needed for the construction of effective pion-nucleon potential $U(E)$ (2.6)

4.1. πNN - Vertex

In the expression for s - and u -channel terms of the driving term V (1.9) the vertex functions $\langle\vec{p}s'|j_i(0)|\vec{p}s\rangle$ appear, while in the expression for $\bar{s}$ - and $\bar{u}$ -channel terms the vertices $\langle 0|j_i(0)|\vec{p}s,\vec{p}_{\bar{N}}s_{\bar{N}}\rangle$ are present. From the Lorentz-covariance and isotopic invariance one obtains

$$\langle\vec{p}'s'|j_i(0)|\vec{p}s\rangle = ig_\pi((p'-p)^2)\,\bar{u}(\vec{p}'s')\gamma^5\tau_i\,u(\vec{p}s) \tag{4.1a}$$

$$\langle 0|\,j_i(0)|\vec{p}s,\vec{p}_{\bar{N}}s_{\bar{N}},out\rangle = ig_\pi((p+p_{\bar{N}})^2)\,\bar{U}(\vec{p}_{\bar{N}}s_{\bar{N}})\gamma^5\tau_i\,u(\vec{p}s)\,. \tag{4.1b}$$

For the function $g_\pi(t)$ in the region $t\leqslant 0$ the parametrization from ref.[16]) has been used:

$$g_\pi(t) = g_\pi(0)\left[1+\frac{t(t-4M^2)}{4M^2\mu_N^2}\right]^{-1},\quad t\leqslant 0\,, \tag{4.2a}$$

where μ_N is a cutoff and $g_\pi(0)$ is determined from the Goldberger-Treiman relation, $g_\pi(0)=12.79$; $g_\pi(t)$ when $t\geqslant 4M^2$ is a complex function. According to ref.[15]) the completeness condition in the form $1=\frac{1}{2}\left(\sum_n|n,in\rangle\langle in,n|+|n,out\rangle\langle out,n|\right)$ has been used in $\bar{s}$ - and $\bar{u}$ -channel terms of the driving term V . Making use of this trick one makes sure that in these terms $Re(g_\pi(t_1)g_\pi^*(t_2))$ appears instead of the products $g_\pi(t_1)g_\pi^*(t_2)$. Then, following ref.[15]) for further simplification of the problem the product of two real functions, called $\bar{g}_\pi(t)$, $\bar{g}_\pi(t_1)\bar{g}_\pi(t_2)$ is substituted for the expression $Re(g_\pi(t_1)g_\pi^*(t_2))$, and the following parametrization for $\bar{g}_\pi(t)$ similar to (4.2a) is chosen

$$\bar{g}_\pi(t) = \bar{g}_\pi\left[1+\frac{t(t-4M^2)}{4M^2\mu_{\bar{z}}^2}\right]^{-1},\quad t\geqslant 4M^2. \tag{4.2b}$$

Here $\bar{g}_\pi\equiv|g_\pi(4M^2)|$ and $\mu_{\bar{z}}$ are adjustable parameters.

4.2. Equal-Time Commutator

To calculate the explicit expression of the equal-time commutator y_{ji} (1.6b), the πN -system interaction Lagrangian is to be known. In the present work, a simple effective Lagrangian which describes the interaction between π,ρ,σ mesons and nucleons (see, e.g. ref.[27])) has been used for this purpose:

$$\mathcal{L}_I = ig_{\pi NN}\bar{\psi}\gamma^5\vec{\tau}\vec{\phi}\psi + g_{\sigma NN}\bar{\psi}\psi\sigma + g_{\sigma\pi\pi}\sigma\vec{\phi}\vec{\phi} - g_{\rho\pi\pi}(\vec{\rho}_\mu\cdot[\vec{\phi}\times\partial^\mu\vec{\phi}]) + \tag{4.3}$$

$$+ g_{\rho NN}\bar{\psi}\gamma^\mu\frac{1}{2}\vec{\tau}\vec{\rho}_\mu\psi + \frac{f_{\rho NN}}{4M}\bar{\psi}\sigma^{\mu\nu}\frac{1}{2}\vec{\tau}\psi(\partial_\mu\vec{\rho}_\nu-\partial_\nu\vec{\rho}_\mu)\,,$$

where σ, $\vec{\rho}_\mu$, $\vec{\phi}$ and ψ denote σ,ρ,π -meson and nucleon field operators, respectively. We restrict ourselves to the tree approximation in calculating the equal-time commutator. In this approximation for $z^{-1/2}[j_{i'}(0),a^+_{\vec{q}i}(0)]$ one obtains the following expression

$$z^{-1/2}[j_{i'}(0),a^+_{\vec{q}i}(0)] = 2g_{\sigma\pi\pi}\delta_{i'i}\sigma(0)+2i\epsilon_{i'ij}g_{\rho\pi\pi}q^\mu\vec{\rho}^{\,j}_\mu(0)\,. \tag{4.4}$$

Note that the expression (4.4) can be obtained in the linear σ -model with vector and axial-vector mesons[8,40]) in tree approximation, if A_1 -meson field is omitted in the corresponding Lagrangian. However, in order to describe experimental data the coupling constants must be somewhat shifted from their theoretical values. The matrix element of (4.4) in the tree approximation coincides with the sum of σ - and ρ -exchange Feynman graphs (fig. 4). The second term in (4.4) violates the hermiticity of y_{ji} . As it has been demonstrated above, if one neglects the term y' (2.2a) which in the case considered here equals to $i(E_{\vec p}-E_{\vec p'})g_{\rho\pi\pi}\epsilon_{i'ij}\langle\vec p's'|\vec\rho^{\,j}_0(0)|\vec ps\rangle$, then the hermitian y -term can be written in the following way:

$$\langle\vec p's',\vec q'i'|\tilde y_{ji}|\vec ps,\vec qi\rangle = 2g_{\sigma\pi\pi}\delta_{i'i}\langle\vec p's'|\sigma(0)|\vec ps\rangle + ig_{\rho\pi\pi}\epsilon_{i'ij}(q'^{\mu}+q^{\mu})\langle\vec p's'|\vec\rho^{\,j}_{\mu}(0)|\vec ps\rangle . \quad (4.5)$$

Below the calculations of P -wave phase shifts are carried out with y_{ji} (4.4) and the hermitian $\tilde y_{ji}$ (4.5). With the use of the condition $\partial^{\mu}\vec\rho^{\,j}_{\mu}=0$ (4.4) can be written in the following form:

$$\langle\vec p's',\vec q'i'|y_{ji}|\vec ps,\vec qi\rangle = 2g_{\sigma\pi\pi}\delta_{i'i}\langle\vec p's'|\sigma(0)|\vec ps\rangle + ig_{\rho\pi\pi}\epsilon_{i'ij}\,Q^{\mu}_{(1)}\langle\vec p's'|\vec\rho^{\,j}_{\mu}(0)|\vec ps\rangle + \quad (4.6a)$$
$$+\, i\epsilon_{i'ij}\,g_{\rho\pi\pi}Q^{\mu}_{(2)}\langle\vec p's'|\vec\rho^{\,j}_{\mu}(0)|\vec ps\rangle ,$$

where

$$Q^{\mu}_{(1)} = (2E_{\vec p'}+2E_{\vec p}-p'^0-p^0, -(\vec p'+\vec p))$$
$$Q^{\mu}_{(2)} = (-2E_{\vec p'},0). \quad (4.6b)$$

The first two terms in (4.6a) are included into $A_{\alpha'\alpha}$ whereas the last term is included into $B_{\alpha'\alpha}$ (see eq. (2.1a)). For σNN and ρNN vertices the following parametrization is chosen:

$$\langle\vec p's'|\sigma(0)|\vec ps\rangle = g_{\sigma NN}\,\Gamma_\sigma(t)\,\bar u(\vec p's')u(\vec ps)\frac{1}{m_\sigma^2-t} \quad (4.7a)$$

$$\langle\vec p's'|\vec\rho^{\,j}_{\mu}(0)|\vec ps\rangle = -g_{\rho NN}\,\Gamma_\rho(t)\,\bar u(\vec p's')\tfrac{1}{2}\vec\tau^{\,j}[\gamma_\mu(1+f_v)-\frac{(p+p')_\mu}{2M}]u(\vec ps)\frac{1}{m_\rho^2-t}, \quad (4.7b)$$

where $t=(p-p')^2$ and $\Gamma_\sigma(t)$ and $\Gamma_\rho(t)$ are the phenomenological form factors. The parameters of these form factors are determined by fitting experimental data for P -wave πN-phase shifts. The constant $g_{\sigma NN}$ is taken to be approximately equal to the πNN coupling constant due to the chiral symmetry constraints. The constant $g_{\rho NN}$ is determined with the use of the ρ -universality condition and Kawarabayashi-Suzuki-Riazuddin-Fayazuddin relation

$g_{\rho NN}=-\frac{m_\rho}{\sqrt2 f_\pi}$ [8]). For the constant f_v the value 3.7 is taken which is obtained from the analysis of nucleon electromagnetic form factors.[28]). The constants $g_{\sigma\pi\pi}$ and $g_{\rho\pi\pi}$ are expressed via the σ,ρ meson masses and decay widths $\Gamma_{\sigma\to2\pi}$ and $\Gamma_{\rho\to2\pi}$:

$$g_{\sigma\pi\pi} = 2m_\sigma\,(2\pi\,\Gamma_{\sigma\to2\pi}/(3(m_\sigma^2-4\mu^2)^{1/2}))^{1/2} \quad (4.8)$$
$$g_{\rho\pi\pi} = 2m_\rho\,(3\pi\,\Gamma_{\rho\to2\pi}/(m_\rho^2-4\mu^2)^{3/2})^{1/2}.$$

For the ρ -meson mass and width the experimental values $m_\rho=770$ MeV and $\Gamma_{\rho\to2\pi}=153$ MeV are taken, whereas m_σ and $\Gamma_{\sigma\to2\pi}$ are regarded as adjustable parameters and are determined from fitting pion-nucleon scattering P -wave phase shifts.

4.3. The Crossing Term

The numerical treatment of the crossing term $(f G_0 f^+)_{cr}$ turns out to be very cumbersome in practice. The calculations carried out in refs.[16,29]) have demonstrated that the contribution of the crossing-term to the pion-nucleon scattering P -wave amplitude is dominated by the Δ -resonance contribution to the crossing term, i.e. $(fG_0f^+)_{cr}\approx(f_{33}G_0f^+_{33})_{cr}$. Preliminary calculations of the crossing-term have motivated our choice for the initial approximation $f^{(0)}$ in the iteration series (1.13)

$$f^{(0)}=g(|p'|)g(|p|)\Big[\lambda^{-1}-\frac{M}{4\pi^2}\int\frac{k^2d|k|}{\omega_N(\vec k)\omega_\pi(\vec k)}\frac{g^2(|k|)}{E_{\vec k}-E_{\vec p}}\Big]^{-1}|P_{33}\rangle\langle P_{33}| . \quad (4.9)$$

Here $g(|k|)=\frac{|k|}{k^2+\mu_c^2}$, $\lambda^{-1}=\frac{M}{4\pi^2}P\int\frac{k^2d|k|}{\omega_N(\vec k)\omega_\pi(\vec k)}\frac{g^2(|k|)}{E_{\vec k}-E_R}$,

where $E_R=1.236$ GeV is the resonance energy; $|P_{33}\rangle\langle P_{33}|$ is the

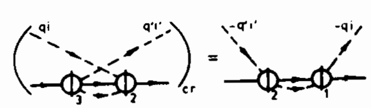

Fig. 9. Graphical representation for the crossing transformation, corresponding to (3.18), with $q'=p+q-p'$.

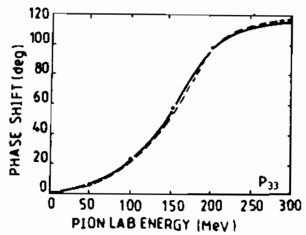

Fig. 10. Input πN P_{33} phase shifts, calculated according to (4.9). Solid line corresponds to $\mu_c = 9\mu$ and dashed line to $\mu_c = 10\mu$.

projection operator on the P_{33} channel; the separable approximation is used for the P_{33} -partial-wave pion-nucleon scattering amplitude; μ_c is an adjustable parameter. In the present work μ_c is set equal to $9\mu = 1.25$ GeV. However, it should be pointed out that the contribution of the crossing term does not change significantly when the parameter μ_c is somewhat shifted from the given value. This fact is illustrated in figs. 10 and 11. In fig. 10 the comparison of the input amplitude for $\mu_c=9\mu$ and $\mu_c=10\mu$ with experiment is given (the experimental data are taken from

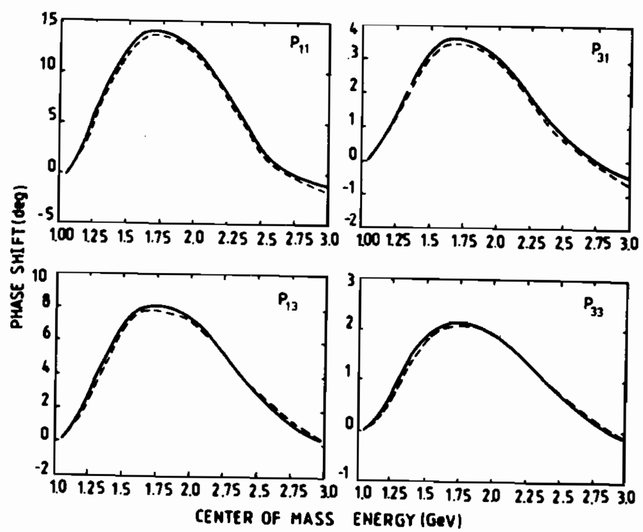

Fig. 11.

Crossing contribution to P -wave driving term. P -wave phase shifts are plotted for πN- scattering R-matrix taken equal to $(fG_0f^+)_{cr}$ on energy shell.

ref.[30])). In fig. 11 the crossing contribution to $W^{(0)}$ on energy shell is depicted for the same values of the parameter μ_c . As can be easily observed, the crossing terms calculated for $\mu_c = 9\mu$ and $\mu_c = 10\mu$ do not differ significantly.

The numerical calculations have demonstrated that the inclusion of the crossing term in P_{33} channel leads to the 10-15% enhancement of the P_{33} phase shift. The solution in the P_{33} channel approximately equals the input amplitude on energy shell. Consequently, one can expect that subsequent iterations of the crossing term will give rise to small corrections to P -wave partial amplitudes which can be compensated by a slight change of adjustable parameters.

4.4. The Results of Numerical Calculations

The Lippmann-Schwinger equation (2.12) after the partial-wave decomposition is reduced to the one-dimensional singular integral equation . The methods of numerical solution of such equation are well known. In particular, in the present work the matrix inversion method suggested in ref.[31]) was used. To investigate the role of ρ -meson exchange in πN- interactions within the approach presented above the P -wave πN -scattering phase shifts have been calculated with the full expression for $\mathcal{Y}_{fi}$ (4.4) as well as without the ρ -meson exchange term. Moreover, the P-wave phase shifts have been calculated with and without the crossing term. The calculations of P -wave phase shifts with the hermitian $\tilde{\mathcal{Y}}_{fi}$ (4.5) have also been performed to evaluate the contribution from nonhermitian $\mathcal{Y}'$ (2.2a) to the effective pion-nucleon potential.

In fig 12 the P -wave πN -phase shifts without the ρ -meson exchange contribution are depicted (a dashed line). For the most meson-nucleon vertex functions a parametrization similar to the one from ref.[16]) has been used; e.g. according to ref.[16]) the cut-

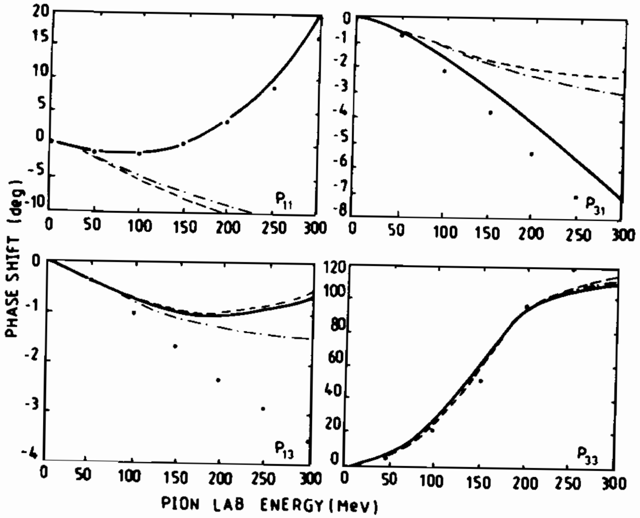

Fig. 12.

πN-scattering P-wave phase shifts with (solid line) and without (dashed and dot-dashed lines) the ρ-meson exchange contribution. The dashed line corresponds to the σNN vertex parametrization used in ref.[16]).

off mass μ_N in (4.2a) is set to be equal to 1.195 GeV. The σNN vertex function is taken to be $\Gamma_\sigma(t) = \frac{1 - t/m_\sigma^2}{(1 - t/2\mu_\sigma^2)^2}$ with $\mu_\sigma = 1.67$ GeV, and the product $2g_{\sigma\pi\pi}\, g_{\sigma NN}$ is set equal to $0.95\, \bar{g}_\pi^2 m_\sigma^2/M$. However, for a better description of experimental data we have used the parameterization for the πNN vertex function in the region $t \geq 4M^2$ (4.2b), different from the one used in ref.[16]). Here $\bar{g}_\pi$ was set equal to 9.1, and $\mu_2 = 1.67$ GeV. As one can observe from fig. 12, the qualitative description of all partial-wave phase shifts, except P_{11}, is obtained. In our opinion, the change of parametrization of vertex functions for describing the experimental data is necessary because, on the one hand, in ref.[16]) the authors have restricted themselves to the Pade [1,1] approximant on energy shell instead of summing up the Pade series for

the Low equation (1.8), and on the other hand, with the different treatment of the crossing term in ref.[16]) and in the present work.

The important role of the ρ-meson exchange in describing the experimentally observed change of sign of the P_{11} phase shift was indicated in the early sixties, in ref.[14]). In refs.[3,4]) the πN-scattering phase shifts were calculated in the Cloudy Bag Model, however, the nucleon recoil and crossing-symmetry in the πN-scattering amplitude have not been taken into account. As discussed in ref.[33]), these effects may turn out to be essential for understanding the pion-nucleon dynamics in P_{11} and P_{33} channels. In refs.[34,35]) the experimental data for low-energy pion-nucleon scattering has been described on the basis of the dispersion approach. In ref.[32]) πN-phase shifts were calculated on the basis of quasipotential equations in the separable pair interaction model, however, the crossing-symmetry has not been taken into consideration. The consideration of crossing-symmetry and nucleon recoil has been made in calculations of πN-phase shifts on the basis of Low-type equations[15,16,20,29]). In particular, in ref.[16]) the contribution of inelastic channels has been phenomenologically included into the driving term in order to describe the experimental data for the P_{11} phase shift. In ref.[29]) on the basis of a direct numerical solution of the Low equation it has been demonstrated that the consideration of the ρ-meson exchange contribution in the driving term allows one to obtain the desirable behaviour of the P_{11} phase shift. A similar result is obtained in the present work after the numerical solution of equivalent linear equation (2.12).

In numerical calculations in the case when the ρ-meson exchange term is present in the potential $U(E)$, the σNN form factor was taken to be $\Gamma_\sigma(t) = \frac{\mu_\sigma^2 - m_\sigma^2}{\mu_\sigma^2 - t}$ with $\mu_\sigma = 1$ GeV. According to ref.[36]) the parameters characterizing the σ-meson were set as follows: $m_\sigma = 760$ MeV, $\Gamma_{\sigma\to2\pi} = 200$ MeV; for σNN coupling the value $g_{\sigma NN}^2/4\pi = 10$ was taken; the dipole form factor $\Gamma_\rho(t) = \left[\frac{\mu_\rho^2 - m_\rho^2}{\mu_\rho^2 - t}\right]^2$ with $\mu_\rho = 1.359$ GeV was used in the ρNN vertex; other parameters were: $\mu_N = 1.195$ GeV, $\mu_z = 2.1$ GeV, $\bar{g}_\pi = 9.75$. As it can be easily observed from fig. 12 (a solid line), the description of all partial waves can be regarded satisfactory. It should be pointed out that "small" partial waves P_{13} and P_{31} reveal strong dependence on adjustable parameters, so we have not tried to fit these partial waves with high accuracy. Furthermore, it turns out that if one neglects the ρ-meson exchange, and the monopole form factor in the σNN vertex is used, results obtained after a slight readjustment of model parameters, are similar to those found with the use of the parametrization from ref.[16]) (a dot-dashed line). Note that the experimentally observed change of

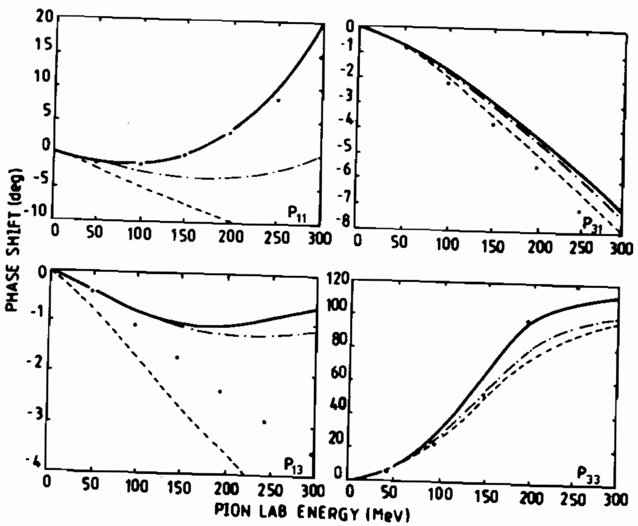

Fig. 13.

The same as in fig. 12 with (solid line) and without (dashed line) the crossing term. The dot-dashed line corresponds to the results obtained with the hermitian Y-term (4.5).

sign of the P_{11} phase shift in the approach described in this paper cannot be obtained for an arbitrary set of physically accepted model parameters if the ρ-meson exchange contribution is not taken into account.

In fig. 13 the P-wave πN-scattering phase shifts with the ρ-meson exchange (a solid line), without the crossing-term (a dashed line) and with the hermitian Y-term (4.5) (a dot-dashed line) are depicted. One can observe from fig. 13 that though the crossing term and the nonhermitian part y' (2.2a) are important for the description of P-wave scattering, the qualitative behaviour of phase shifts does not alter when these contributions are neglected. It should be pointed out that the effects caused by either of these contributions may be fully compensated by a slight readjustment of model parameters.

5. CONCLUSION

In the present work, the Lippmann-Schwinger-type linear integral equation (2.12) with the energy-dependent potential equivalent to the Low equation for πN-scattering (1.8) has been suggested. The effective pion-nucleon potential which appears in the equivalent Lippmann-Schwinger equation turns out to be a functional of the πN-scattering t-matrix. The existence of this nonlinear term in the potential stems from symmetry of the t-matrix with respect to the pion crossing. It is demonstrated that nonlinearity of that kind due to pion crossing is inherent of the πN-scattering problem and arises in the formulations of the latter within the framework of the multichannel scattering theory or the Bethe-Salpeter (or quasipotential) approach. However, for the case of NN-scattering such nonlinear terms are not present. It should be pointed out that eqs. (2.12) and (2.6) define fully-off-shell effective πN-potential which can be further employed in calculations of pion-nucleus interactions.

The major uncertainty arising in the construction of the effective pion-nucleon potential comes from arbitrariness in the choice of the phenomenological parametrization for various meson-nucleon form factors. From the structure of eqs. (1.8) and (2.12) it is clear that these vertices depend only on three-momenta of external particles. On the basis of current algebra and the dispersion approach the coupling constants which appear in each vertex can be evaluated. In our opinion, further progress in the investigation of low-energy πN-scattering within this approach is concerned with the construction of the above-mentioned form factors in realistic quark models.

The advantage of eq. (2.12) suggested in the present work as compared to the Bethe-Salpeter equation consists in that in the vertex functions used for construction of the potential $U(E)$ all nucleons are on mass shell, and all intermediate state propagators in $U(E)$ also contain the particles on mass shell. For this reason, a considerable simplification of the problem may emerge when the approach which explicitly takes into account the quark degrees of freedom is used to construct the pion-nucleon-potential or scattering t-matrix. In future we plan to carry out the calculations of πN-scattering phase shifts on the basis of eq. (2.12) suggested in the present paper, with the use of various meson--nucleon vertex functions, calculated in Quark Confinement Model [38,39].

The authors would like to thank T.I.Kopaleishvili for the current interest in the work and helpful remarks, and to A.D.Chelidze, G.V.Efimov, M.A. Ivanov, M.Kh.Khankhasayev and V.E.Lyubovitsky for the discussion of the results.

APPENDIX A

The approach based on Low-type equations can be formulated in such a form that the hermitian equal-time commutators will

appear from the beginning. For this purpose we start with the following asymptotic conditions for the pion field operator $\phi_i(x)$ and its canonical conjugate $\pi_i(x)$

$$\lim_{x_0\to\pm\infty} \langle\alpha|\vec{\phi}_i(x)|\beta\rangle = \sqrt{Z}\,\langle\alpha|\vec{\phi}_i^{\,in}_{\,out}(x)|\beta\rangle \tag{A.1}$$

$$\lim_{x_0\to\pm\infty} \langle\alpha|\vec{\pi}_i(x)|\beta\rangle = \sqrt{Z}\,\langle\alpha|\vec{\pi}_i^{\,in}_{\,out}(x)|\beta\rangle . \tag{A.2}$$

Here α and β are arbitrary states, Z is the renormalization constant, $\vec{\pi}_i^{\,in}_{\,out}(x) = \dot{\vec{\phi}}_i^{\,in}_{\,out}(x)$; $\vec{\pi}_i(x) = \frac{\partial\mathcal{L}}{\partial\dot{\vec{\phi}}_i(x)} = \dot{\vec{\phi}}_i(x) + \frac{\partial\mathcal{L}_I}{\partial\dot{\vec{\phi}}_i(x)}$, where $\mathcal{L}$ and $\mathcal{L}_I$ are the total and interaction Lagrangians of the system, respectively. Note that in quantum field theory usually instead of the asymptotic condition (A.2), a similar condition with $\dot{\phi}_i(x)$ substituted for $\pi_i(x)$ is used. Due to this the operator $a_{\vec{q}i}(x^0)$ (1.3a) appears in contraction rules. In the case considered here applying the contraction rule to the S-matrix sandwiched between πN-states one obtains:

$$\begin{aligned}\langle out,\vec{p}'s',\vec{q}'i'|\vec{p}s,\vec{q}i,in\rangle &= \langle\vec{p}'s'|a_{\vec{q}'i'}(in)|\vec{p}s,\vec{q}i,in\rangle + \\ &+ iZ^{-1/2}\Big(\lim_{x^0\to+\infty} - \lim_{x_0\to-\infty}\Big)\int d^3x\, e^{iq'x}\langle\vec{p}'s'|(\vec{\pi}_{i'}(x) - iq'^0\vec{\phi}_{i'}(x))|\vec{p}s,\vec{q}i,in\rangle = \\ &= \delta_{fi} + \int dx^0 \langle\vec{p}'s'|\,\mathring{\tilde{a}}_{\vec{q}'i'}(x^0)|\vec{p}s,\vec{q}i,in\rangle ,\end{aligned} \tag{A.3}$$

where

$$\tilde{a}_{\vec{q}'i'}(x^0) = iZ^{-1/2}\int d^3x\, e^{iq'x}[\vec{\pi}_{i'}(x) - iq_0'\vec{\phi}_{i'}(x)] = a_{\vec{q}'i'}(x^0) + iZ^{-1/2}\int d^3\vec{x}\, e^{iq'x}\frac{\partial\mathcal{L}_I}{\partial\dot{\vec{\phi}}_{i'}(x)} \tag{A.4}$$

$$\mathring{\tilde{a}}_{\vec{q}'i'}(x^0) = iZ^{-1/2}\int d^3x\Big[e^{iq'x}j_{i'}(x) + \frac{d}{dx^0}\Big(e^{iq'x}\frac{\partial\mathcal{L}_I}{\partial\dot{\vec{\phi}}_{i'}(x)}\Big)\Big] \equiv iZ^{-1/2}\int d^3\vec{x}\, e^{iq'x}\tilde{j}_{\vec{q}'i'}(x) . \tag{A.5}$$

From the definition of the operator $\tilde{a}_{\vec{q}'i'}(x^0)$ it is easy to make sure that $\tilde{a}_{\vec{q}'i'}(x^0)$ and $\tilde{a}^{+}_{\vec{q}i}(x^0)$ obey canonical commutation relations. Consequently, the third nonhermitian term in (1.10) vanishes identically in this case. Further, on the basis of (A.3) the Low equation for the matrix element $\langle\vec{p}'s'|\tilde{j}_{q'i'}(0)|\vec{p}s,\vec{q}i,in\rangle_c$ can be derived. This equation will be similar in form to (1.8), except that $\tilde{\mathcal{Y}}_{fi}$ (1.11) and $\tilde{j}_{\vec{q}i}(x) = j_i(x) + iq^0\frac{\partial\mathcal{L}_I}{\partial\dot{\phi}_i(x)} + \frac{\partial}{\partial x^0}\frac{\partial\mathcal{L}_I}{\partial\dot{\vec{\phi}}_i(x)}$

will be substituted for $\mathcal{Y}_{i}$ (1.6b) and $j_i(x)$. It is clear that the matrix elements $\langle\alpha|\tilde{j}_{\tilde{q}_i}(0)|\beta\rangle$ and $\langle\alpha|j_i(0)|\beta\rangle$ coincide on energy shell.

APPENDIX B

In this Appendix the derivation of eq. (2.11) is considered. For $N=2$ it reduces to the well-known identities.

$$\frac{1}{E_1-E^{\pm}}\,\frac{1}{E_2-E_1^-}+\frac{1}{E_2-E^{\pm}}\,\frac{1}{E_1-E_2^+}=\frac{1}{(E_1-E^{\pm})(E_2-E^{\pm})}$$
$$\frac{E_1}{E_1-E^{\pm}}\,\frac{1}{E_2-E_1^-}+\frac{E_2}{E_2-E^{\pm}}\,\frac{1}{E_1-E_2^+}=\frac{E}{(E_1-E^{\pm})(E_2-E^{\pm})} \tag{B.1}$$

We assume that (2.11) is valid for some $N=N_0>2$ and consider the case $N=N_0+1$. For the first time we take $m<N_0$

$$\sum_{n=1}^{N_0+1}\frac{E_n^m}{(E_n-E^{\pm})\prod\limits_{i=1}^{n-1}(E_i-E_n^+)\prod\limits_{i=n+1}^{N_0+1}(E_i-E_n^-)}=\frac{E_{N_0+1}^m}{(E_{N_0+1}-E^{\pm})\prod\limits_{i=1}^{N_0}(E_i-E_{N_0+1}^+)}+$$
$$+\sum_{n=1}^{N_0}\frac{E_n^m}{(E_{N_0+1}-E_n^-)(E_n-E^{\pm})\prod\limits_{i=1}^{n-1}(E_i-E_n^+)\prod\limits_{\ell=n+1}^{N_0}(E_\ell-E_n^-)}\,. \tag{B.2}$$

Further we transform (B.2) with the use of the following relation:

$$\frac{1}{(E_{N_0+1}-E_n^-)(E_n-E^{\pm})}=\frac{1}{(E_{N_0+1}-E^{\pm})}\left[\frac{1}{E_{N_0+1}-E_n^-}+\frac{1}{E_n-E^{\pm}}\right]. \tag{B.3}$$

We obtain:

$$\frac{E_{N_0+1}^m}{(E_{N_0+1}-E^{\pm})\prod\limits_{i=1}^{N_0}(E_i-E_{N_0+1}^+)}+\frac{1}{(E_{N_0+1}-E^{\pm})}\sum_{n=1}^{N_0}\frac{E_n^m}{(E_n-E^{\pm})\prod\limits_{i=1}^{n-1}(E_i-E_n^+)\prod\limits_{\ell=n+1}^{N_0}(E_\ell-E_n^-)}-$$
$$-\frac{1}{(E_{N_0+1}-E^{\pm})}\sum_{n=1}^{N_0}\frac{E_n^m}{(E_n-E_{N_0+1}^+)\prod\limits_{i=1}^{n-1}(E_i-E_n^+)\prod\limits_{\ell=n+1}^{N_0}(E_\ell-E_n^-)}\,. \tag{B.4}$$

If in (B.4) (2.11) for $N=N_0$ is used, the first and third terms cancel out, and we obtain the desired relation for $N=N_0+1$.

Equation (2.11) in the case when $m=N_0$ can be derived along similar lines, except that the relation:

$$\frac{E_n}{(E_{N_0+1}-E_n^-)(E_n-E^{\pm})}=\frac{1}{(E_{N_0+1}-E^{\pm})}\left[\frac{E_{N_0+1}}{E_{N_0+1}-E_n^-}+\frac{E}{E_n-E^{\pm}}\right] \tag{B.5}$$

is substituted, instead of (B.3) in (B.2) and again (2.11) is used for $N=N_0$.

REFERENCES

1. A.W.Thomas. Adv.Nucl.Phys. 13 (1984), 1.
2. S.Theberge, A.W.Thomas and G.A.Miller. Phys.Rev. D22 (1980), 2856.
3. O.G.Ernst and R.G.Meleod. Phys.Rev. C29 (1984), 906.
4. B.G.Pearce and I.R.Afnan. Phys.Rev. C34 (1986), 991.
5. E.A.Veit, B.K. Jennings and A.W.Thomas. Phys.Rev. D33 (1986), 1859.
6. B.Schwesinger, H.Weigel, G.Holzwarth and A.Hayashi. Phys.Rep. 173 (1989), 173.
7. T.H.R.Skyrme. Proc. Roy.Soc. London. Ser A260 (1961), 127.
8. V.De Alfaro, S.Fubini, G.Furlan and C.Rosetti. Currents in Hadron Physics. (North Holland Publ.Company, Amsterdam-London--New-York 1973).
9. K.Huang and R.Weldon.Phys.Rev. D11 (1975) 257.
10. H.Garcilazo and T.Mizutani. Acta Phys.Austriaca. Few-Body Syst. 5 (1988), 127.
11. I.R.Afnan. Phys.Rev. C38 (1988), 1792.
12. F.Low. Phys.Rev. 97 (1955), 1392.
13. G.F.Chew and F.Low. Phys.Rev. 101 (1956), 1570.
14. G.F.Chew. Phys.Lett. 9 (1962), 233.

15. G.B.Cammarata and M.K.Banerjee. Phys.Rev. C17 (1978),1125.
16. Nieh-Chih Wei and M.K.Banerjee. Phys.Rev. C22 (1980) 2065.
17. G.Miller. Phys.Rev. C14 (1976), 2230.
18. T.I.Kopaleishvili and A.I.Machavariani. Ann.Phys. 174 (1987), 1
19. A.I.Machavariani, JINR Preprint E4-88-610, Dubna (1988).
20. D.J.Ernst and M.B.Johnson. Phys.Rev. C17 (1978) 247.
21. C.Itzukson and J.B.Zuber. Quantum Field Theory. (McGraw Hill, 1980, New - York, 1980).
22. M.Goldberger and K.Watson. Collision Theory (John Willey and Sons, New-York-London-Sydnay, 1964).
23. Y.Avishai and T.Mizutani. Nucl.Phys. A326 (1979), 352, A338 (1980), 337.
24. A.I.Machavariani. Nucl.Phys. A403 (1983), 480.
25. I.R.Afnan and B.C.Pearce. Phys.Rev. C35 (1987), 737.
26. Y.Avishai and T.Mizutani. Phys.Rev. C27 (1983); 312.
27. B.Dutta-Roy, J.R.Lapidus and M.J.Tausner. Phys.Rev. 181 (1969), 2091.
28. F.Iachello, A.D.Jackson and A.Lande. Phys.Lett. 43B (1973), 191
29. T.I.Kopaleishvili, A.I.Machavariani and A.G.Rusetsky. Bulletin of Acad. Sciences of Georgian SSR 134 (1989), 524.
30. B.A.Arndt, J.M.Ford and L.D.Roper. Phys.Rev. D32 (1985), 1085.
31. M.I.Haftel and F.Tabakin. Nucl.Phys. A158 (1970), 1.
32. T.Mizutani, C.Fayard, G.H.Lamot and S.Nahabetian. Phys.Rev. C24 (1981), 2633.
33. M.G.Fuda.Phys.Rev. C30 (1984), 666; C31 (1985), 1365; C32 (1985) 2024;
34. M.G.Reiner. Ann.Phys. 154 (1984), 24.
35. R.Koch, Nucl.Phys. A448 (1986) 707.
36. J.L. Petersen and J.Risut. Nucl.Phys. B38 (1972), 207.

37. T.D.Taylor. Nuovo Cim. Suppl. 1 (1963), 857, Phys.Rev. 150 (1966), 1321.
38. G.V.Efimov, M.A.Ivanov and V.E.Lyubovitsky, JINR Preprint E2-88-915, Dubna, (1988).
39. G.V.Efimov and M.A.Ivanov. Inter. J.Mod.Phys. A4, (1989), 2031. G.V.Efimov, M.A.Ivanov and V.E.Luybovitsky. Few-Body Syst. 6, (1989), 17.
40. S.Gasiorowicz and D.A.Geffen. Rev.Mod.Phys. 41 (1969), 531.

Received by Publishing Department
on December 29, 1989.