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IS IT POSSIBLE TO OBSERVE THE CUSP PHENOMENA IN PION-NUCLEUS REACTIONS AT LOW-ENERGIES?

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1. Introduction

It is well known that the study of the energy dependence of one reaction near the threshold of another reaction can provide us with additional information on the dynamics of interaction of particles [1 - 4]. One can think that intense monochromatic low-energy pion beams are a good tool for searching such phenomena in pion-nucleus interactions.

The motivation of the present paper is that recent data on the pion produotion [5,6] and absorption [7] on nuclei at low energies indicate some anomalies in the energy dependence of the cross sections. There are several qualitative explanations of this phenomenon such as the effect of dibaryons [8,9], excitation of Δ bounded in the nucleus [10] or ΔΔ-states in nuclei [11].

Here, we consider a possibility to explain the observed anomalies in the energy behaviour as a threshold effect in pion-nucleus reactions. To simplify the analysis, we consider the three-channel model for pion induced reactions in the low-energy range (below say 50 MeV), i. e.

$$(i):\ \pi + A \to (f): \begin{cases} (1)\ \ N + N + X\,, \\ (2)\ \ \pi + A\,, \\ (3)\ \ \pi' + A'\,, \end{cases} \qquad (1.1)$$

where (1) is the pion absorption ohannel, the seqond is the elastic scattering, and the third is the inelastic reaction in which the nucleus in an excited state appears.

The pion absortion followed by the emission of two nucleons leaving the rest nucleus X in an excited state (three-body final state) seems to be the dominant channel for light nuclei [12]. The approximation of the excitation spectra of the nucleus by one excited state (channel (3)), the model of two energy levels of nucleus, is justified by the analysis of the elastic scattering of low energy pions by light nuclei in the framework of the UST-approach [13-15]. The mean excitation energy (Δ) of the p-shell nuclei which provided the best description of the scattering data is about 15 - 25 MeV.

2. General formalism

To describe the influence of the inelastic reaction (3) on the elastic scattering (2) .and the absorption (1) cross sections, it is necessary to calculate the energy dependence of the elements $S_{\alpha\beta}$ (α,β=1,2,3) of the S-matrix in the vicinity of the threshold. It has been shown in [1-4] (see especially [4]) that using the unitarity condition for the S-matrix and the relations $S_{\alpha\beta} = S_{\beta\alpha}$ following from time reversal invariance one can obtain

$$S_{\alpha 3} = m_\alpha \, k_\Delta^{1/2} \quad \text{for } \alpha = 1,2, \qquad (2.1)$$

$$S_{\alpha\beta} = S_{\alpha\beta}^{(o)} - \frac{1}{2} m_\alpha m_\beta k_\Delta \quad \text{for} \quad \alpha,\beta \neq 3,$$

where $S_{\alpha\beta}^{(o)}$ are calculated at the energy of the threshold and the momentum $k_\Delta = \sqrt{2M_3(E-\Delta)}$: E is the energy of relative motion of particles (π and A) in the initial state, M_3 is the reduced mass of particles in the final state (π' and A') and Δ is the energy of the excited nucleus A' counted from the ground state; below the threshold $k_\Delta = i|k_\Delta|$. The constants m_α (α = 1,2) are proportional to the to the square roots of the reaction cross sectins σ_{31} and σ_{32}, respectively.

It should be stressed that relations (2.1) are valid if the particles π' and A' (or one of them) are neutral and are created in the S-state. For the reactions with pions this case is realized if π' is the neutral pion.

Using (2.1) one can calculate [4] the energy dependence of the differential and total cross sections for the elastic scattering and the pion absorption.

3. Elastic and inelastic scattering

We consider for simplicity the interaction of a pion with a nucleus having the positive parity (P) and both zero spin (S) and isospin (T), i.e. $A(S^P,T) = (0^+,0)$. The reaction of the pion absorption by carbon which has been studied in [7], is just this case. The quantum numbers (α) of the partial reaction channel are defined as (J,L,S,P,T), where J_α and L_α are the total and orbital angular momenta; S_α, P_α and T_α are the total spin, parity and isospin, respectively.

Let us consider first the reaction (2) → (3), i.e.

$$\pi + A(0^+,0) \rightarrow \pi' + A'(S'^{P'},T'). \qquad (3.1)$$

The total isospin of the reaction is determined by the initial state $T=t_\pi= 1$. Taking into account that the particles π' and A' are created in the S-state ($L_{\pi' A'} = 0$) and the spin of the pion is zero, we can determine the total angular momentum $J = S'$. Finally, from the parity conservation we obtain the selection rule

$$(-1)^{L_{\pi A}} = P', \qquad (3.2)$$

where P' is the parity of the excited nucleus state.

From the above analysis it follows that if the parity of A'

is positive ($P'=+1$) than the relative angular momentum in the initial state must be even, i. e., $L_{\pi A} = 0, 2$, etc., and the spin of A' is $S' = L_{\pi A}$. On the other hand, if the parity of A' is negative than $L_{\pi A} = S' = 1, 3$, etc.. For example, the latter case means that the P-wave pion can create near the threshold the nucleus $A'(1^-,1)$ which has the quantum numbers of the nuclear giant dipole resonance (GDR).

4. Pion absorption channel

Considering the pion absorption by nucleus we suppose (see (1.1)) the dominance of the quasi-deuteron mechanism [12]. As it has been shown by Brack and Riska [16] in their study of pionic disintegration of the deuteron, the partial singlet $^1D_2$ final state of the nucleons dominates in the cross section. For the initial π-deuteron state it means the dominance of the P-wave.

It is natural to suppose that in pionic disintegration of the nucleus the P-wave mechanism producing nucleons in the $^1D_2$ is also dominant. Taking into account the results of the partial wave analysis presented in the preceding section we conclude that the most interesting reaction channel, from the point of view of the cusp phenomena has the following quantum numbers:

$$J=T=1,\ P=+1. \qquad (4.1)$$

Hence, the initial state in (1.1) (and the elastic channel) is specified by

$$L_{\pi A}= 1,\ S = 0\ ,\ A(0^+,0), \qquad (4.2)$$

and the final state in the inelastic channel

$$L_{\pi' A'}= 0,\ \ S = 1,\ A'(1^-,T'). \qquad (4.3)$$

where the isospin $T'= 1,2$. The value $T'=0$ is excluded by the

requirement for the pion to be neutral in the final state.

The final state in the absorpion channel is determined by the total orbital momentum $L = L_{NN} (+) L_X$, the total spin $S = S_X$ (nucleons are in the singlet state), and the total isospin $T = T_{NN} (+) T_X = 1 (+) T_X$. Supposing that the pion is absorbed by two closed correlated nucleons (the pion absorption operator is of short range) and neglecting the final state interaction in the NNX system, one can show that $L_X \approx 0$. Taking into account that the NN state is dominated by $^1D_2$ we obtain that $L = 2$. Finally, in conformity with the quasi-deuteron mechanism it is natural to suppose that the rest nucleus X has the quantum numbers of the deuteron, i. e. $S_X = 1$ and $T_X = 0$. Therefore, one can specify the final state of the absorption channel as

$$L = 2,\ \ S = 1\ \ ,\ \ NN(^1D_2,\ T_{NN} = 1)\ \text{and}\ X(1^+,0) \qquad (4.4)$$

It is worthwhile to stress that the pion absorption in the state (4.3) takes place in the S-wave of the π-nucleus system. It provides a strong coupling of the inelastic channel with the pion absorption channel due to the well known 1/v - law behaviour of the total absorption cross section at the threshold.

5. Total cross sections

Taking into account the quantum numbers (4.1) - (4.4) of the most important (from the point of view of the threshold anomalies) S-matrix elements (J=T=1) and their energy dependence near the threshold (2.1), we obtain the following expressions for the total elastic and absorption cross sections

$$\sigma_{el}(E) = \sigma_{el}(E = \Delta) - \frac{3\pi}{k^2} \mathrm{Re}\Big[\Big(S^{11}_{el}(E = \Delta) - 1\Big)^* m_2^2\, k_\Delta \Big], \qquad (5.1)$$

$$\sigma_{abs}(e) = \sigma_{abs}(E = \Delta) - \frac{3\pi}{2k^2} \mathrm{Re}\left[S^{11*}_{abs}(E = \Delta)\, m_1 m_2 k_\Delta \right]. \qquad (5.2)$$

Here, $S^{JT}_{\beta(L',S');\alpha(L,S)}$:

$$S^{11}_{el} \equiv S^{11}_{2(1,0);2(1,0)}, \quad S^{11}_{abs} = S^{11}_{1(2,1);2(1,0)}, \qquad (5.3)$$

Δ is the energy of the threshold of the inelastic channel ($\alpha = 3$), $k_\Delta = \sqrt{2M_3(E - \Delta)}$ above the threshold and $k_\Delta = i|k_\Delta|$ below the threshold. The factor 1/2 in the total absorption cross section (5.2) takes into account the indisguishability of the final nucleons.

From the unitarity condition one can obtain the absolute value of S^{11}_{abs} at $E = \Delta$

$$|S^{11}_{abs}| = (1 - |S^{11}_{el}|^2)^{1/2}. \qquad (5.4)$$

Estimations. The constants m_1 and m_2 are determined by the partial (J=T=1) reaction cross sections σ_{13} and σ_{32} in the following way:

$$|m_1| = (2\sigma_{13} k_\Delta /\pi)^{1/2}, \qquad (5.5)$$

$$|m_2| = (\sigma_{32} k^2/3\pi k_\Delta)^{1/2}, \qquad (5.6)$$

as $k_\Delta \to 0$. Here $k \approx \sqrt{2M_2 E}$ is the pion momentum and M_2 is the reduced mass of the π-nucleus system. Formula (5.5) is derived by using the detailed balance relation $\sigma_{31} = (3k^2_\Delta/2M\mu)\sigma_{13}$; M and μ are the masses of the nucleon and pion, respectively.

The absorption cross section σ_{13} is determined near the threshold by the S-wave interaction of the pion with the nucleus A'. Taking into account the 1/v - dependence of the total cross section as $k_\Delta \to 0$: $\sigma_{13} = (4\pi/k_\Delta)\mathrm{Im}\,a_{\pi A'}$, we obtain

$$|m_1| = 2\sqrt{2\, \mathrm{Im}\,a_{\pi A'}}, \qquad (5.7)$$

where $a_{\pi A'}$ is the π-A' scattering length.

To estimate m_1 one can suppose that the interaction of the pion with the nucleus A' (in the channel 3 → 3) is the same as in the elastic scattering channel 2 → 2, i. e. $a_{\pi A'} \approx a_{\pi A}$. The value for $a_{\pi A}$ can be obtained from the pionic atom data.

The partial (J=T=1) reaction cross section σ_{32} reads as

$$\sigma_{32} = \frac{3\pi}{k^2} |m_2|^2 k_\Delta . \qquad (5.8)$$

On the other hand, using the unitarity condition one can obtain

$$\sigma_{32} = \frac{3\pi}{k^2} \left[1 - |S^{11}_{abs}|^2 - |S^{11}_{el}|^2 \right], \qquad (5.9)$$

where the S-matrix elements are defined in (5.3). Taking into account that below the threshold $E \le \Delta$

$$1 - |S^{11}_{abs}|^2 = |S^{11}_{el}(E=\Delta)|^2,$$

we obtain

$$\sigma_{32} = \frac{3\pi}{k^2} \left[|S^{11}_{el}(E=\Delta)|^2 - |S^{11}_{el}(E)|^2 \right]. \qquad (5.10)$$

Using (5.8) and (5.10) we can express $|m_2|$ in terms of S^{11}_{el} which describes the P-wave π-nucleus scattering

$$S^{11}_{el}(k) = \exp[2i\delta_{\pi A}(k)], \qquad (5.11)$$

which can be calculated or obtained from the phase shift analysis.

The model. Here we use the UST-approach (unitary scattering theory) [13-15] which provides good description of the low-energy scattering data. The main equations of this approach are formulated for the direct calculation of the π-nucleus phase shifts. The general formula reads as

$$\delta_{\pi A}(k) = \delta^{pot}_{\pi A}(k) + \delta^{abs}_{\pi A}(k), \qquad (5.12)$$

where δ^{pot} is the potential part calculated in terms of thr πN phase shifts, nuclear form factors and correlation functions, and δ^{abs} is the absorption correction.

In calculating $Im\delta^{pot}$ the approximation of completeness is used which brings about the parameter Δ, a mean excitation energy of the nucleus, i. e. the model of two energy levels of a nucleus is used. The parameter Δ determines the threshold behaviour of the inelasticity parameters

$$Im\delta^{pot} \quad k_\Delta, \quad k_\Delta \approx \sqrt{2M_2(E-\Delta)}, \qquad (5.13)$$

where M_2 is the reduced mass of the πA system.

Using (5.12) we can write

$$|S^{11}_{el}| = \eta^{pot}\eta^{abs}, \qquad (5.14)$$

where the inelasticity parameters $\eta^{pot} \equiv \exp(-2\ Im\delta^{pot})$ and $\eta^{abs} \equiv \exp(-2\ Im\delta^{abs})$.

Below the threshold $\eta^{pot} = 1$ and using (5.8). (5.10) and (5.14), we obtain

$$|m_2| = 2\ \eta^{abs}(E=\Delta)\ [(Im\delta^{pot}(E)/k_\Delta)]^{1/2}_{E\to\Delta}. \qquad (5.15)$$

Now, we are able to estimate absolute values of the threshold anomalies in the total cross sections (5.1) and (5.2)

$$\delta\sigma_{el}(E) = \frac{3\pi}{k^2}\ |S^{11}_{el}(E=\Delta) - 1|\ |m_2|^2\ |k_\Delta|, \qquad (5.16)$$

$$\delta\sigma_{abs}(E) = \frac{3\pi}{2k^2}\ [1 - (\eta^{abs}_p(E=\Delta))^2]^{1/2}\ |m_1|\ |m_2|\ |k_\Delta|. \qquad (5.17)$$

π-^{12}C. We present the results of our calculations of (5.16) and (5.17) for the interaction of a pion with ^{12}C. The value for the threshold energy Δ is taken to be 25 MeV. This value provides good description of the π-^{12}C scattering data at low energies [14,15]. The imaginary part of the π-^{12}C scattering length is $Im a = -0.132$ fm [17]

The P-wave inelasticity parameter η_p and the π-nucleus phase shift δ_p at E = 25 MeV are

$$\eta_p = 0.943 \quad \text{and} \quad \delta_p \approx 7.4^\circ.$$

For the constants $|m_{1,2}|$ we obtain

$$|m_1| = 1.03\ fm^{1/2} \quad \text{and} \quad |m_2| = 0.25\ fm^{1/2}. \qquad (5.18)$$

The obtained values show that the inelastic channel couples with the absoption channel more stronger than with the elastic one.

Taking into account that the 1/v-law for the total cross section is valid near the threshold in the energy range less than 5 MeV (see, fig.13 in ref.[15]), we calculated (5.16) and (5.17) at the energy E = 30 MeV. The obtained values are

$$\delta\sigma_{el} = 1.3\ mb \quad \text{and} \quad \delta\sigma_{abs} = 3.3\ mb. \qquad (5.19)$$

Comparing these values with those for σ_{el} and σ_{abs} at the energy E = 25 MeV: $\sigma_{el} \approx 70$ mb and $\sigma_{abs} \approx 90$ mb (see, ref.[15]), we see that the cusp effect in the absorption channel is ≈ 4% and in the elastic one ≈ 2%.

6. Conclusion

It has been shown that the cusp effects can be visible in the the energy dependence of the pion induced reactions. The absoption channel has the advantage over the elastic scattering channel for the observation the cusp phenomenon. A strong coupling of the inelastic channel with the pion absorption channel is due to the 1/v - law behaviour of the absorption cross section near the threshold.

We do not present here the same analysis for the differential cross sections which will be the subject of a further extented article. We only should like to mention that the cusp effects in the differential cross sections for the considered reactions depend on the scattering angle. This conclusion comes from the fact that the inelastic channels are coupled with the partial P-wave in the elastic channel.

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