89 - 630

объединенный
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исследований
дубна

E4-89-630

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SIMPLE MODEL OF A RAPIDLY ROTATING HOT NUCLEUS

Submitted to "Physics Letters B"

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1989

1. Introduction

The study of thermal effects in finite nuclear systems has been the subject of many publications in the last 15 years. Among them the various thermal mean field approximations have been done [1], temperature-dependent collapse of the pairing correlations has been investigated [2], the RPA solutions at $T \neq 0$ have been obtained [3],[2b] and finite-temperature HFB cranking equations have been derived and applied [4]. The model of the cranked harmonic oscillator has been solved [5] in the semiclassical limit, which imposed ,however, some restrtiction on the value of angular velocity. Further the same problem has been analyzed in [6] at the arbitrary value of the cranking angular velocity ω. Temperature dependent many fermions problem in the spherically symmetric harmonic oscillator well has been studied in [7].

In the present work we examine the system of A nucleons in the cranking deformed harmonic oscillator potential as the grand canonical ensemble which is described by the temperature T and the chemical potential μ .We treat the nucleus as a system with the heat source which one can identify with the experimental instrument , say a particle beam used to excite the targed plus other particles - - products of the reaction (photons, nucleons, etc.). Our task is to investigate the properties of nuclei such as deformation parameters , angular momentum, moment of inertia , level-densities, etc. as the functions of temperature and cranking velocity ω .

In this approach we have to remember that for the finite nuclei the statistical fluctuations of the number of particles (and other quantities also) are important even when the system is not near to the critical point which is the consequence of application of grand canonical ensemble without the thermodynamic limit. This fluctuations have been investigated in [1], where it has been shown that in the mass region A>100 the relative particle number fluctuation $\Delta A/A$ is of order of 5% (the energy fuctuation is of order of 3%). This problem has been discussed also more recently with the similar results.

2. Formulation of the problem

Let A independent nucleons move in an assymetric harmonic oscillator potential well which rotate with frequency ω around the x-axis in the body - system. The hamiltonian has the form:

$$H_0^\omega = \sum_{i=1}^{A} (h_0^\omega)_i \quad , \tag{1}$$

where h_0^ω is the single particle hamiltonian

$$h_0^\omega = h_0 - \hbar\omega l_x =$$
$$= \frac{p^2}{2m} + \frac{m}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) - \hbar\omega l_x \quad , \tag{2}$$

is the angular momentum along the axis of rotation.

Diagonalization (cf.[8]) of hamiltonian (2) gives the one - body wave functions $|n_x n_+ n_-\rangle$ where n_x, n_+, n_- are the quantum numbers of the new normal modes , and the one - particle energies of h_0^ω are:

$$\varepsilon_\alpha^\omega = \varepsilon(n_x, n_+, n_-) =$$
$$= \hbar\omega_x(n_x + \tfrac{1}{2}) + \hbar\omega_+(n_+ + \tfrac{1}{2}) + \hbar\omega_-(n_- + \tfrac{1}{2}) \, , \tag{3}$$

where

$$\omega_\pm^2 = \frac{\omega_y^2 + \omega_z^2}{2} + \omega^2 \pm \frac{1}{2}((\omega_y^2 - \omega_z^2)^2 + 8\omega^2(\omega_y^2 + \omega_z^2))^{1/2} . \tag{4}$$

The total energy of the system depends on the given configuration :

$$E^\omega = \sum_\alpha \varepsilon_\alpha^\omega \rho_\alpha = \hbar\omega_x \Sigma_x + \hbar\omega_+ \Sigma_+ + \hbar\omega_- \Sigma_- \quad , \tag{5}$$

where

$$\Sigma_\chi = \sum_\alpha (n_\chi + \tfrac{1}{2})\rho_\alpha \, , \qquad \chi = +, -, x, \tag{6}$$

and ρ_α represents the occupation numbers of the one - particle states (ρ_α = 0 or 1). In the framework of the thermal approach the statistical averaging over the grand canonical ensemble has to be performed. In this way the dependence on the fixed quantum configuration is removed. Instead of that the average value of particle number agrees with the number of nucleons A. The grand canonical potential Ω is expressed by the partition function Z:

$$\Omega = -\frac{1}{\beta}\ln Z \, , \tag{7}$$

where $\beta = \frac{1}{kT}$ and

$$Z = \mathrm{Tr}\,(\exp(-\beta(\hat{H}_0^\omega - \mu\hat{N}))) . \tag{8}$$

The occupation number of the individual fermion level

$$f_\alpha = -\frac{1}{\beta}\frac{\partial \ln Z}{\partial \varepsilon_\alpha} = \frac{1}{e^{\beta(\varepsilon_\alpha - \mu)} + 1} \quad . \tag{9}$$

Parameter μ is determined by the condition

$$A = \sum_\alpha \frac{1}{e^{\beta(\varepsilon_\alpha - \mu)} + 1} \quad . \tag{10}$$

Assumption that the system is in the thermal equilibrium leads to the following system of equations which are equivalent with the condition for the extremum of potential Ω :

$$\frac{\partial\Omega}{\partial\omega_x} = 0 \, , \quad \frac{\partial\Omega}{\partial\omega_y} = 0 \, , \quad \frac{\partial\Omega}{\partial\omega_z} = 0 \, . \tag{11}$$

These three conditions are not independent. They are connected via the potential volume conservation condition:

$$\omega_x \cdot \omega_y \cdot \omega_z = \omega_0^3 . \tag{12}$$

This last condition displays , in the simplest manner , the consequences of nuclear forces. As it has been shown in [7] ω_0 could also depend on temperature. For the sake of simplicity we assume ω_0 to be constant. After elementary calculations we find that equations (11) together with the condition (12) are equivalent to the following equations:

$$\omega_x^2 \langle x^2\rangle = \omega_y^2 \langle y^2\rangle = \omega_z^2 \langle z^2\rangle \, , \tag{13}$$

where $\langle\ldots\rangle$ means the statistical averaging.

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$$\omega_x^2 \langle x^2 \rangle = \omega_y^2 \langle y^2 \rangle = \omega_z^2 \langle z^2 \rangle \, , \tag{13}$$

where $\langle \ldots \rangle$ means the statistical averaging.

We have:

$$\langle x^2\rangle = \frac{\langle n_x+\frac{1}{2}\rangle}{m\,\omega_x}\,, \tag{14a}$$

$$\langle y^2\rangle = \frac{1}{2m}\left\{\frac{\langle n_+ +\frac{1}{2}\rangle}{\omega_+} + \frac{\langle n_- +\frac{1}{2}\rangle}{\omega_-} + \right.$$
$$\left. + \frac{\omega_y^2-\omega_z^2+4\omega^2}{\omega_+^2-\omega_-^2}\left(\frac{\langle n_+ +\frac{1}{2}\rangle}{\omega_+} - \frac{\langle n_- +\frac{1}{2}\rangle}{\omega_-}\right)\right\}, \tag{14b}$$

$$\langle z^2\rangle = \frac{1}{2m}\left\{\frac{\langle n_+ +\frac{1}{2}\rangle}{\omega_+} + \frac{\langle n_- +\frac{1}{2}\rangle}{\omega_-} + \right.$$
$$\left. + \frac{\omega_z^2-\omega_y^2+4\omega^2}{\omega_+^2-\omega_-^2}\left(\frac{\langle n_+ +\frac{1}{2}\rangle}{\omega_+} - \frac{\langle n_- +\frac{1}{2}\rangle}{\omega_-}\right)\right\}. \tag{14c}$$

To calculate $\langle x^2\rangle, \langle y^2\rangle, \langle z^2\rangle$ as the functions of T and μ we have to perform the following summations:

$$\langle n_x+\tfrac{1}{2}\rangle = \sum_{n_x,n_+,n_-}(n_x+\tfrac{1}{2})\,\frac{1}{e^{\beta(\varepsilon_\alpha-\mu)}+1}\,, \tag{15a}$$

$$\langle n_+ +\tfrac{1}{2}\rangle = \sum_{n_x,n_+,n_-}(n_+ +\tfrac{1}{2})\,\frac{1}{e^{\beta(\varepsilon_\alpha-\mu)}+1}\,, \tag{15b}$$

$$\langle n_- +\tfrac{1}{2}\rangle = \sum_{n_x,n_+,n_-}(n_- +\tfrac{1}{2})\,\frac{1}{e^{\beta(\varepsilon_\alpha-\mu)}+1}\,. \tag{15c}$$

Unfortunately, these summations cannot be calculated analytically. Note ,however, that it is the similar summation problem as in the quantum Hall effect calculations or in the de'Haas van Alphen effect theory [9]. In spite of these two cases in our present problem we have not small parameter (since in our case $T/\mu \sim 1$).The explicit summation in Eqs(15)is possible in the high temperature limit (when we deal with the Boltzmann distribution). This limit is ,however, noninteresting since with growing temperature the nuclear shells and other quantum effects disappear.

The selfconsistent system of Eqs(13) together with the condition (12) and (10) resolve themselves into the following equation system :

$$f_1(\omega_y,\omega_z,\mu) = \omega_x\langle x^2\rangle - \omega_y^2\langle y^2\rangle = 0\,, \tag{16a}$$
$$f_2(\omega_y,\omega_z,\mu) = \omega_y\langle y^2\rangle - \omega_z^2\langle z^2\rangle = 0\,, \tag{16b}$$
$$f_3(\omega_y,\omega_z,\mu) = A - \langle 1\rangle = 0\,. \tag{16c}$$

As the independent variables we choose ω_y, ω_z and μ ($\omega_x = \omega_0^3/\omega_y\omega_z$ owing to condition (12)).

The explicit form of the system (16) is as follows:

$$\omega_x\langle n_x+\tfrac{1}{2}\rangle = \omega_y^2\,\frac{1}{2}\left\{\frac{\langle n_+ +\frac{1}{2}\rangle}{\omega_+} + \frac{\langle n_- +\frac{1}{2}\rangle}{\omega_-} + \right.$$
$$\left. + \frac{\omega_y^2+\omega_z^2+4\omega^2}{\omega_+^2-\omega_-^2}\left(\frac{\langle n_+ +\frac{1}{2}\rangle}{\omega_+} - \frac{\langle n_- +\frac{1}{2}\rangle}{\omega_-}\right)\right\}, \tag{17a}$$

$$(\omega_y^2-\omega_z^2)\left\{\left(1+\frac{4\omega^2+\omega_y^2+\omega_z^2}{\omega_+^2-\omega_-^2}\right)\frac{\langle n_+ +\frac{1}{2}\rangle}{\omega_+} + \left(1-\frac{4\omega^2+\omega_y^2+\omega_z^2}{\omega_+^2-\omega_-^2}\right)\frac{\langle n_- +\frac{1}{2}\rangle}{\omega_-}\right\} = 0\,, \tag{17b}$$

$$A = \langle 1\rangle\,; \qquad \omega_x = \omega_0^3/\omega_y\omega_z\,. \tag{17c}$$

From the above equations it follows that there exist two types of solutions: 1) $\omega_y = \omega_z$ – axial rotational case; 2) $\omega_y \neq \omega_z$ – nonaxial case. For the first case the system (17) attains the form of only two equations:

$$\omega_x\langle n_x+\tfrac{1}{2}\rangle = \frac{\omega_y}{2}\left\{\langle n_+ +\tfrac{1}{2}\rangle + \mathrm{sgn}(\omega_y-\omega)\,\langle n_- +\tfrac{1}{2}\rangle\right\}, \tag{18a}$$
$$A = \langle 1\rangle\,. \tag{18b}$$

One can consider also the limit $\omega_y \to \omega_z$ for the equation (17a), which leads to the following condition:

$$\frac{\langle n_+ +\frac{1}{2}\rangle}{\omega_y+\omega}\;\frac{\omega_y^2+2\omega_y\omega+2\omega^2}{2\omega_y\omega} - \frac{\langle n_- +\frac{1}{2}\rangle}{|\omega_y-\omega|}\;\frac{\omega_y^2-2\omega_y\omega+2\omega^2}{2\omega_y\omega} = 0\,. \tag{19}$$

This condition , together with Eqs(18a,b), determine the position of the bifurcation point on the curve $\omega_y = \omega_z$ versus ω at each value of T, i.e., the position of the point where the axial solution splits into axial and nonaxial solutions. From the other side, these bifurcation points can be found as the solution of the sufficiency condition for bifurcation [10], applied to the system (16), i.e.,

$$\det\left(1 - \{f_{ik}\}\right) = 0, \tag{20}$$

where

$$\{f_{ik}\} = \begin{pmatrix} \frac{\partial f_1}{\partial \omega_y}, & \frac{\partial f_2}{\partial \omega_y}, & \frac{\partial f_3}{\partial \omega_y} \\ \frac{\partial f_1}{\partial \omega_z}, & \frac{\partial f_2}{\partial \omega_z}, & \frac{\partial f_3}{\partial \omega_z} \\ \frac{\partial f_1}{\partial \mu}, & \frac{\partial f_2}{\partial \mu}, & \frac{\partial f_3}{\partial \mu} \end{pmatrix}, \tag{21}$$

3. Numerical results

The model used for the computer calculations consists from A=100 and $\omega_0=41A^{-1/3}$MeV. We put $\hbar=k=1$, thus all variables are taken in the energetical units ω_0.

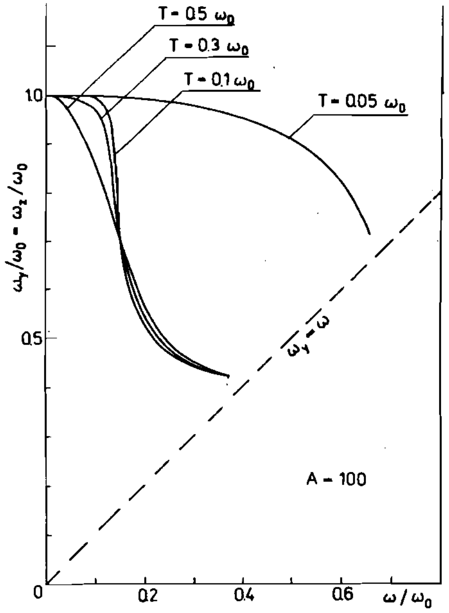
Now we present the preliminary numerical results. More detailed and comprehensive analysis will be published elsewhere in near future.

In Fig. 1 the solutions $\omega_y=\omega_z$ of the system (18) are presented as the functions of ω. One can observe the sharp temperature dependence (esspecially, in the small temperature region) – which displays the fact that near T=0 the nucleus conserves almost sphericál shape even at nonzero angular velocities ω. This is no case in higher temperature. This behaviour is connected with the sharp Pauli principle in the low temperature limit. In Fig. 2 the corresponding solutions for μ as the functions of ω are depicted. It is characteristic that the chemical potential weakly depends on temperature.

In both Figs 1 and 2 the line $\omega_y=\omega$ is also plotted. Along this curve the frequency ω_- in the axial case: $\omega_-=|\omega_y-\omega|$ changes its character , which imposes constraints onto solutions..

In order to find the nonaxial solutions the more thorought analysis is necessary . We confine here to employing of the condition (19) only .The bifurcation points on the curves $\omega_y=\omega_z$ can be found as the solution of Eq.(19) together with Eqs(18a,b). In Fig.3 the interpolated curve of bifurcation points versus temperature is plotted.

Fig.1. The axial solutions for frequencies $\omega_y=\omega_z$ as functions of ω at several temperatures.



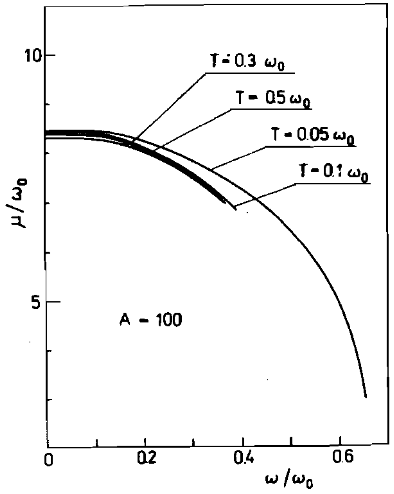


Fig.2. The chemical potential μ versus ω at various temperatures.

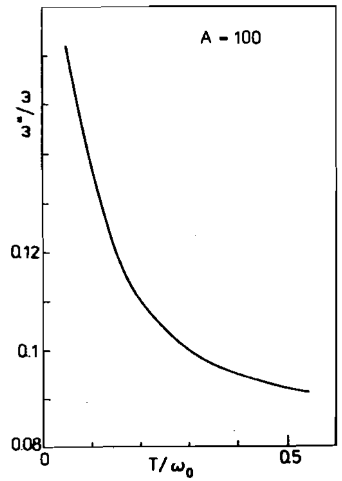


Fig.3. The line of bifurcation points at which the axial solution splits into axial and nonaxial solutions.

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Received by Publishing Department
on August 31, 1989.

Яцак Л., Навроцка В., Назмитдинов Р.Г. E4-89-630

Простая модель быстровращающегося нагретого ядра

Рассмотрены нуклоны, движущиеся в потенциале анизотропного гармонического осциллятора, вращающегося с большой угловой скоростью ω. Самосогласованное решение задачи позволяет исследовать изменение ядерной формы и других характеристик как функции температуры T и ω.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1989

Jacak L., Nawrocka W., Nazmitdinov R.G. E4-89-630

Simple Model of a Rapidly Rotating Hot Nucleus

Nucleons moving in an anisotropic harmonic oscillator potential are considered. The whole system rotates with a large angular velocity ω. A self-consistent solution of the problem makes it possible to study the variation of nuclear shape and other nuclear properties as functions of ω and T.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1989