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INFLUENCE OF PARTICLE NUMBER FLUCTUATIONS AND VIBRATIONAL MODES ON THERMODYNAMICS CHARACTERISTICS OF A HOT NUCLEUS

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I. Introduction

In the study of thermal effects in finite hot nuclei the finite temperature Bardeen-Cooper-Schriffer (FT-BCS) and finite temperature random phase approximation (FT-RPA) have been widely used. The most important contribution to the thermal nuclear characteristics such as the entropy S, specific heat C, level density ρ and the level density parameter a is given by the quasiparticle (fermionic) effective degrees of freedom in the FT-BCS approximation. The correlated FT-RPA (bosonic) excitations are included in terms of residual interactions. The structure of these degrees of freedom and their evolution at finite temperature have been discussed in detail in many works [1-5]. However, as is well known, the BCS theory breaks the particle-number symmetry. The use of symmetry-breaking wave functions induces quantum fluctuations (QF) in the particle number operator. At finite temperature, because of the thermal averaging, the statistical fluctuations (SF) associated with any operator appear. Attempts have been made at improving the BCS method at zero temperature. There has been a number of approaches to deal with this problem by using various exact or approximate number projection techniques [6-9]. Amongs them we note the method suggested by Lipkin [7] and developing by Nogami and collaborators [8](the LN method) many years ago, which allows one to have calibration of the role of QF in the particle number operator in cold nuclei in a rather simple way. At finite temperature there has been a few papers which have investigated the temperature dependence of QF and SF for the particle number[10], the influence of SF for the particle number on some thermodynamic nuclear characteristics in a model case [11] and in realistic spherical hot nuclei [12]. However, a complete study of the influence of QF and SF for the particle number on the thermodynamic characteristics in realistic hot nuclei is absent so far. The main purpose of the present effort is therefore to give an answer to this open question by extending the LN method to the nonzero temperature case. By averaging the pairing residual interaction, which is not included into the BCS Hamiltonian, over the statistical ensemble we shall evaluate the effect beyond the mean field. By including the residual dipole and quadrupole separable interactions, we also give an estimation for the contribution of the FT-RPA vibrational modes to the specific heat C and the increase of the level density ρ at finite temperature.

The paper is organized in the following way: in Sec.2 we present the theoretical elements of our description; the results are shown in Sec.3; and finally some conclusions are drawn in Sec.4. For the sake of convenience the details of some expressions in the formalism are collected in the Appendix.

2. Formalism

In this section we briefly describe the main features of the particle number fluctuation formulae which are obtained by extending the LN method at finite temperature. In the subsection 2.1 we will obtain the analytic expressions for QF and SF and their contribution to the energy of the system. The expression for the energy $\mathcal{E}$ and the specific heat C of the system are given in Subsec. 2.2, and for the grand partition function Ω, entropy S, level density ρ and its parameter a in Subsec. 2.3. In these subsections the relevant corrections

to these nuclear thermodynamic characteristics due to the particle number fluctuations as well as to the pairing residual interactions are given analytically. In subsec. 2.4 we discuss separately the contribution of the FT-RPA modes.

2.1. Quantum and statistical particle number fluctuations

Let us consider the well-known monopole pairing Hamiltonian

$$H=\sum_{\substack{jm\\ \gtrless 0}} E_j^{\circ} a^+_{jm} a_{jm} - G\sum_{jj'}\sum_{\substack{mm'\\ >0}} a^+_{jm} a^+_{j\tilde{m}} a_{j'\tilde{m}'} a_{j'm'} \qquad (1)$$

where E_j° are the single-particle energies, a^+_{jm} (a_{jm}) are the creation (annihilation) nuclear operators in the single-particle orbits denoted by the index j; G is the monopole pairing constant; and the states denoted by $j\tilde{m}$ are the time reversed single-particle states. In general, for a two-component proton (Z) and neutron (N) system the sum in (1) is done over $j_\tau m_\tau$ $(j'_\tau m'_\tau)$ and also G_τ where $\tau=(Z,N)$. However, hereafter unless a special emphasis is made we omit the subscript τ for simplicity. Using the Bogolubov transformation the Hamiltonian (1) can be written in the quasiparticle basis in terms of the quasiparticle creation (annihilation) α^+_{jm} (α_{jm}) operators as

$$H = H_{00} + H_{11} + H_{20} + H_{22} + H_{31} + H_{40} + H_{11-11} \qquad (2)$$

where H_{mn} consists of terms $\sim(\alpha^+)^m\alpha^n$ or $(\alpha^+)^n\alpha^m$ and H_{11-11} stands for the term $\sim\alpha^+\alpha\alpha^+\alpha$. Their explicit forms are given in Appendix A.

At zero temperature, by considering the Hamiltonian $H+\lambda_1 N+\lambda_2 N^2$ where N is the nucleon number which is set equal to the expectation value of the particle number operator $\hat{N}$ in the quasiparticle vacuum state $|0\rangle$, Nogami has obtained the ground state energy given by averaging this Hamiltonian over $|0\rangle$ as [8]

$$\mathcal{E}_0 = \langle H_{00}\rangle_0 - \lambda_2 \Delta N^2 \tag{3}$$

where ΔN^2 is the particle number fluctuations. The coefficients λ_1 and λ_2 have been calculated in the successive approximation as derivatives of $\langle H_{00}\rangle$.

At finite temperature a statistical ensemble of quasiparticle excitations takes place. The quasiparticle vacuum can no longer serve as a reference state to define normal products. In this case, the thermal vacuum $|0,\beta\rangle$ with β being the inverse temperature $\beta=T^{-1}$ should be used instead of $|0\rangle$. The explicit form of $|0,\beta\rangle$ in terms of $|0\rangle$ has been given in Refs. [13,14]. The averaging procedure is now performed over this thermal vacuum and is equivalent to the average over the statistical ensemble [14]

$$\langle \mathcal{O}\rangle \equiv \langle 0,\beta|\mathcal{O}|0,\beta\rangle = \mathrm{Tr}[\mathcal{O}\exp(-\beta H^{eff})]/\mathrm{Tr}[\exp(-\beta H^{eff})] \tag{4}$$

where $\mathcal{O}$ is arbitrary and H^{eff} is the effective Hamiltonian of the system. The Wick's theorem remains valid for the ensemble average of operators [15,16] such as

$$\langle a_1^+ a_2^+ a_3 a_4\rangle = \langle a_1^+ a_4\rangle\langle a_2^+ a_3\rangle - \langle a_1^+ a_3\rangle\langle a_2^+ a_4\rangle + \langle a_1^+ a_2^+\rangle\langle a_3 a_4\rangle \tag{5}$$

which is the finite temperature Hartree-Fock-Bogolubov (FT-HFB) approximation. The procedure of minimizing $\langle H_{20}\rangle - \lambda_1 N - \lambda_2 N^2$ goes now in parallel with the usual FT-BCS theory. The results are the well-known FT-BCS equations (see, e.g.[1])

$$N = \sum_j \Omega_j \left[1 - \frac{E_j-\lambda}{\varepsilon_j}\tanh\left(\tfrac{1}{2}\beta\varepsilon_j\right)\right] \tag{6}$$

$$\frac{2}{G} = \sum_j \Omega_j \tanh\left(\tfrac{1}{2}\beta\varepsilon_j\right)/\varepsilon_j \tag{7}$$

$$\varepsilon_j = \sqrt{(E_j-\lambda)^2+\Delta_T^2} \tag{8}$$

where ε_j is the quasiparticle energy; Δ is the superfluid energy gap; λ is the chemical potential. The Bogolubov coefficients u_j and v_j are

$$u_j^2 = \tfrac{1}{2}\left(1+\frac{E_j-\lambda}{\varepsilon_j}\right) \qquad v_j^2 = \tfrac{1}{2}\left(1-\frac{E_j-\lambda}{\varepsilon_j}\right) \tag{9}$$

The difference between these equations and those of the conventional FT-BCS theory is the appearance of the terms $\sim 4\lambda_2$ in E_j as in the zero temperature case [8]

$$E_j = E_j^0 + (4\lambda_2 - G)v_j^2 \tag{10}$$

$$\lambda = \lambda_1 + 2\lambda_2(N+1) \tag{11}$$

The presence of the term $\sim 4\lambda_2 v_j^2$ nearly suppresses the self-energy correction $\sim G v_j^2$. This merit of the LN method allows one to put in fact $E_j \approx E_j^0$ [8]. Following the LN method we can calculate the coefficient λ_2 at finite temperature T as $\lambda_2=(n!)^{-1} d^n\langle H_0\rangle/dN^n$ having in mind the average (4). The evaluations at T=0 have shown that the second derivative (n=2) turns out to be sufficient for a good approximate value of λ_2 . At $T \neq 0$ we therefore also restrict ourselves to the second derivative n=2. Differentiating both the sides of Eq. (6) we find

$$\lambda_2^{(2)} = \frac{1}{2}\frac{d\lambda}{dN} = \left[\sum_j \Omega_j \varepsilon_j^{-3} f(\beta,\varepsilon_j)\right]\Big\{\Delta_T^2\left[\sum_j \Omega_j \varepsilon_j^{-3} f(\beta,\varepsilon_j)\right]^2 + \tag{12}$$

$$+\left[\sum_j \Omega_j (E_j-\lambda)\varepsilon_j^{-3} f(\beta,\varepsilon_j)\right]^2 + \frac{1}{2}\beta\left[\sum_j \Omega_j \operatorname{sech}^2(\tfrac{1}{2}\beta\varepsilon_j)\right]\left[\sum_j \Omega_j \varepsilon_j^{-3} f(\beta,\varepsilon_j)\right]\Big\}^{-1}$$

where

$$f(\beta,\varepsilon_j) \equiv \tanh(\tfrac{1}{2}\beta\varepsilon_j) - \tfrac{1}{2}\beta\varepsilon_j \operatorname{sech}^2(\tfrac{1}{2}\beta\varepsilon_j) \tag{13}$$

$$\Omega_j \equiv j + \tfrac{1}{2}$$

Eq. (12) is the formula for the coefficient $\lambda_2^{(2)}$ of the extended LN method at finite temperature. In the following we shall omit the superscript (2) in $\lambda_2^{(2)}$ without confusion. In the limit $T \to 0$ we have $\lim_{T\to 0} f(\beta,\varepsilon_j) = 1$ and $\lim_{T\to 0} \frac{1}{2}\beta\varepsilon_j \operatorname{sech}^2(\frac{1}{2}\beta\varepsilon_j) = 0$; so Eq. (12) transforms into the formula for $\lambda_2^{(2)}$ obtained by Nogami in Ref.[8].

The particle number fluctuations ΔN^2 are defined as

$$\Delta N^2 = \langle \hat{N}^2 \rangle - \langle \hat{N} \rangle^2 \tag{14}$$

with $\hat{N} = \sum_{jm} a_{jm}^+ a_{jm}$. In the quasiparticle basis we find

$$\Delta N^2 = \Delta N_{QF}^2 + \Delta N_{SF}^2 \tag{15}$$

where ΔN_{QF}^2 are QF of the particle number

$$\Delta N_{QF}^2 = 4\sum_j \Omega_j u_j^2 v_j^2 (1-2n_j) = \Delta_T^2 \sum_j \Omega_j \varepsilon_j^{-2}(1-2n_j) \tag{16}$$

and ΔN_{SF}^2 are SF of $\hat{N}$

$$\Delta N_{SF}^2 = 2\sum_j \Omega_j (u_j^2 - v_j^2)^2 n_j(1-n_j) + 8\sum_j \Omega_j u_j^2 v_j^2 n_j^2 = \tag{17}$$

$$= 2\sum_j \Omega_j \varepsilon_j^{-2} n_j \left[(E_j-\lambda)^2(1-n_j) + n_j \Delta_T^2\right]$$

In Eqs. (16) and (17) n_j is the quasiparticle occupation number

$$n_j = \left[\exp(\beta\varepsilon_j) + 1\right]^{-1} \tag{18}$$

The reasons for the indentification with ΔN_{QF}^2 and ΔN_{SF}^2 are the following [10]: (1) In the high temperature limit, $T \to \infty$ and $n_j \to 1/2$, QF vanish as they are supposed to do, (2) In the zero temperature limit, $n_j \to 0$, SF go to zero and QF go to:

$$\Delta N_{QF}^2 \Big|_{T\to 0} = \Delta_T^2 \sum_j \Omega_j \varepsilon_j^{-2} \tag{19}$$

as one expects (See, Ref.[17]). In the phase transition at $T_{crit} = 0.567\,\Delta_{T=0}$ the FT-BCS energy gap Δ_T from Eqs. (6)-(8) collapses and QF of $\hat{N}$ will vanish after the transition has taken place. In general, as has been pointed out in Ref.[10], QF of the operator associated with the conserved symmetry are always zero.

It is noteworthy that, as has been shown and investigated thoroughly in many papers[18-20] the pairing gap Δ_T in realistic nuclei does not vanish at $T = T_{crit}$ because of the thermal fluctuations due to the finiteness of nuclear systems.

In fact, by considering the average gap

$$\langle\Delta_T\rangle = \int_0^\infty \Delta_T \exp(\Omega)\, d\Delta_T \Big/ \int_0^\infty \exp(\Omega)\, d\Delta_T \qquad (20)$$

where Ω is the grand partition function (see below) no phase transition occurs, and the pairing gap $\langle\Delta_T\rangle$ is not zero even at $T > T_{crit}$ but decreases with increasing T. In this work we shall take into account both the cases: 1) with the FT-BCS gap Δ_T

$$\Delta_T = \sum_j \Omega_j \tanh(\tfrac{1}{2}\beta\varepsilon_j)/\varepsilon_j \qquad (21)$$

obtained from the equations (6)-(8), and 2) with the average gap $\langle\Delta_T\rangle$ given by Eq. (20).

2.2. Energy and specific heat of the superfluid system at finite temperature

By averaging the Hamiltonian (2) over the statistical ensemble (4) and using the FT-HFB approximation (5), we obtain the total energy of the system

$$\mathcal{E} = \langle H_0\rangle + \langle H\rangle_{res} \qquad (22)$$

where $\langle H_0\rangle$ is the FT-BCS energy

$$\langle H_0\rangle = \sum_j \Omega_j E_j \left[1 - \frac{E_j-\lambda}{\varepsilon_j}\tanh(\tfrac{1}{2}\beta\varepsilon_j)\right] - \Delta^2/G \qquad (23)$$

The energy

$$\langle H\rangle_{res} = \frac{1}{2} G \sum_j \Omega_j \frac{E_j-\lambda}{\varepsilon_j} n_j \left[1 - \frac{E_j-\lambda}{\varepsilon_j}\tanh(\tfrac{1}{2}\beta\varepsilon_j)\right] \qquad (24)$$

arisen from the residual interactions in the pairing Hamiltonian (2) cannot be included in the FT-BCS approximation. In the

form (24) it leads to a correction for the FT-BCS energy besides the correction due to the particle number fluctuations $\lambda_2 \Delta N^2$ from Eqs. (3),(14)-(17). The total energy reads

$$\mathcal{E}_{tot} = \mathcal{E} - \lambda_2 \Delta N^2$$

The excitation energy $\mathcal{E}^*$ is defined as

$$\mathcal{E}^* = \mathcal{E}_{(tot)}(T) - \mathcal{E}_{(tot)}(0) \qquad (25)$$

In the thermodynamic limit the specific heat C of the superfluid system at finite temperature can be defined as

$$C_{BCS} = \frac{d\langle H_0\rangle}{dT} = \frac{1}{2}\sum_j \Omega_j \operatorname{sech}^2(\tfrac{1}{2}\beta\varepsilon_j)\left[\beta^2\varepsilon_j^2 - \beta\Delta_T d\Delta_T/dT\right] \qquad (26)$$

The derivative $d\Delta_T/dT$ has been given in Ref. [18] as

$$\frac{d\Delta_T}{dT} = \frac{1}{2}\,\frac{\beta^2 \sum_j \Omega_j \operatorname{sech}^2(\tfrac{1}{2}\beta\varepsilon_j)}{\Delta_T\left[\tfrac{1}{2}\beta\sum_j \Omega_j \varepsilon_j^{-2}\operatorname{sech}^2(\tfrac{1}{2}\beta\varepsilon_j) - \sum_j \Omega_j \varepsilon_j^{-3}\tanh(\tfrac{1}{2}\beta\varepsilon_j)\right]} \qquad (27)$$

By including the particle number fluctuations we obtain the specific heat

$$\tilde{C} = C_{BCS} - \lambda_2 \frac{d\Delta N^2}{dT} - \Delta N^2 \frac{d\lambda_2}{dT} \qquad (28)$$

As we can see from Eq. (12) the coefficient λ_2, in general, depends upon temperature T. Therefore, there is a derivative of λ_2 over T in the last term of Eq. (27). We will see later in Sec.3 that this temperature dependence of λ_2 is however very weak. The derivative $d\Delta N^2/dT$ is obtained here in the form

$$\frac{d\Delta N^2}{dT} = \Delta_T \frac{d\Delta_T}{dT} \sum_j \Omega_j \tanh(\tfrac{1}{2}\beta\varepsilon_j)\left[\beta\varepsilon_j^{-1}\operatorname{sech}^2(\tfrac{1}{2}\beta\varepsilon_j)(\varepsilon_j^{-2}\Delta_T^2 - 1/2) + \right.$$
$$\left. + 2\varepsilon_j^{-4}(E_j-\lambda)^2\tanh(\tfrac{1}{2}\beta\varepsilon_j)\right] -$$
$$- \beta^2\sum_j \Omega_j\, \varepsilon_j(\varepsilon_j^{-2}\Delta_T^2 - \tfrac{1}{2})\tanh(\tfrac{1}{2}\beta\varepsilon_j)\operatorname{sech}^2(\tfrac{1}{2}\beta\varepsilon_j) \qquad (29)$$

By including also the term $\langle H\rangle_{res}$ we obtain the total specific heat C_{tot}

$$C_{tot} = \tilde{C} + d\langle H\rangle_{res}/dT \qquad (30)$$

with

$$\frac{d\langle H\rangle_{res}}{dT} = \tfrac{1}{4}G\beta^2\sum_j \Omega_j(E_j-\lambda)\operatorname{sech}^2(\tfrac{1}{2}\beta\varepsilon_j)\left\{\varepsilon_j^{-1}(E_j-\lambda)\left[\tanh(\tfrac{1}{2}\beta\varepsilon_j) - \right.\right. \qquad (31)$$
$$\left.\left. -\tfrac{1}{2}\right] - \tfrac{1}{2}\right\} + G\Delta_T\frac{d\Delta_T}{dT}\sum_j \Omega_j\varepsilon_j^{-2}(E_j-\lambda)\left\{\tfrac{1}{2}\varepsilon_j^{-1}\left[1-\tanh(\tfrac{1}{2}\beta\varepsilon_j)\right]\times\right.$$
$$\left.\times\left[\tfrac{1}{2} - \varepsilon_j^{-1}(E_j-\lambda)\tanh(\tfrac{1}{2}\beta\varepsilon_j)\right] - \tfrac{1}{4}\beta\operatorname{sech}^2(\tfrac{1}{2}\beta\varepsilon_j)\left[\varepsilon_j^{-1}(E_j-\lambda)\left(\tanh(\tfrac{1}{2}\beta\varepsilon_j) - \tfrac{1}{2}\right)\right]\right\}$$

2.3. Grand partition function, entropy, level density and level density parameter

The grand partition function Ω^{BCS}, entropy S_{BCS}, level density ρ_{BCS} and its parameter a_{BCS} in the FT-BCS approximation are calculated in the standard way [18,21]. Namely, for the grand partition function Ω^{BCS} we have

$$\Omega^{BCS} \equiv -\beta(\langle H_0\rangle - \lambda N - TS) =$$
$$= -\beta\sum_j \Omega_j(E_j-\lambda-\varepsilon_j) + 2\sum_j \Omega_j \ln[1+e^{-\beta\varepsilon_j}] - \beta\Delta_T^2/G \qquad (32)$$

The level density ρ_{BCS} which is the inverse Laplace transform of the grand partition function, can be written as a thermal projection on the system with fixed N (neutron) and Z (photon) numbers [21]

$$\rho_{BCS}(E,N,Z) = (2\pi i)^{-3}\oint d\beta\oint d\alpha_N\oint d\alpha_Z \exp(S_{BCS}) \qquad (33)$$

where the entropy S_{BCS} is

$$S_{BCS} = \beta\langle H_0\rangle + \Omega_N^{BCS} + \Omega_Z^{BCS} - \alpha_N N - \alpha_Z Z \qquad (34)$$

with $\alpha=\beta\lambda$. The quantity S_{BCS} presents a saddle point at

$$N = \partial\Omega_N^{BCS}/\partial\alpha_N \;;\quad Z = \partial\Omega_Z^{BCS}/\partial\alpha_Z \;;\quad E = -\partial\Omega^{BCS}/\partial\beta \qquad (35)$$

The level density is approximately

$$\rho_{BCS} = (2\pi)^{-3/2} D^{-1/2}\exp(S_{BCS}) \qquad (36)$$

for the two- component system with protons and neutrons. The determinant D has the form

$$D = D_Z\,\partial^2\Omega_N^{BCS}/\partial\alpha_N^2 + D_N\,\partial^2\Omega_Z^{BCS}/\partial\alpha_Z^2 \qquad (37)$$

with

$$D_\tau = \begin{vmatrix} \partial^2\Omega_\tau^{BCS}/\partial\alpha_\tau^2 & \partial^2\Omega_\tau/\partial\alpha_\tau\partial\beta \\ \partial^2\Omega_\tau^{BCS}/\partial\alpha_\tau\partial\beta & \partial^2\Omega_\tau/\partial\beta^2 \end{vmatrix} \qquad (38)$$

The explicit expressions for the derivatives in Eq. (38) are obtained in Ref. [21] and we do not repeat them here.

Taking the particle number fluctuations and the residual part $\langle H\rangle_{res}$ in the pairing Hamiltonian (2) into account we have respectively

$$\tilde{\Omega} = \Omega^{BCS} + \beta\lambda_2 \Delta N^2 \tag{39}$$

$$\Omega_{tot} = \tilde{\Omega} - \beta \langle H\rangle_{res} \tag{40}$$

Inserting Eqs. (39) and (40) into Eqs. (18) we obtain the corresponding level densities $\tilde{\rho}$ and ρ_{tot} . The relevant expressions for the derivatives $\partial^2(\Delta N^2)/\partial\alpha^2$, $\partial^2(\Delta N^2)/\partial\beta\partial\alpha$, $\partial^2(\Delta N^2)/\partial\beta^2$ arisen by deriving $\tilde{\Omega}$ in Eq. (38) are given in the Appendix B. By the reason discussed later in Sec.3, we represent only the formulae for $\tilde{\rho}$.

The level density parameter a is defined as usual

$$a = \frac{1}{2}\frac{dS}{dT} = -\frac{\beta^2}{2}\frac{dS}{d\beta} \tag{41}$$

According to the above-mentioned notation, we obtain respectively

$$a^{BCS} = \tfrac{1}{2}\beta C_{BCS} \tag{42}$$

$$\tilde{a} = a^{BCS} - \tfrac{1}{2}\beta^2\lambda_2\Delta N^2 + \tfrac{1}{2}\beta\lambda_2 d\Delta N^2/dT \tag{43}$$

$$a_{tot} = \tilde{a} - \frac{\beta}{2} d\langle H\rangle_{res}/dT + \frac{\beta^2}{2}\langle H\rangle_{res} \tag{44}$$

2.4. FT–RPA contribution

To study the contribution of the vibrational modes to the specific heat and the level density, we adopt the FT–BCS plus FT–RPA treatment.

The Hamiltonian in this case takes the form

$$H^{FT\text{-}RPA} = H + H_{ph} \tag{45}$$

with the multipole interactions

$$H_{ph} = \frac{1}{2}\sum_{\lambda\mu}\sum_{\tau\rho=\pm1}(\varkappa_0^{(\lambda)} + \rho\varkappa_1^{(\lambda)})M^+_{\lambda\mu}(\tau)M_{\lambda\mu}(\rho\tau) \tag{46}$$

$$\{\tau \leftrightarrow -\tau\} \equiv \{N \leftrightarrow Z\}$$

In Eq. (46) $\varkappa_0^{(\lambda)}$ and $\varkappa_1^{(\lambda)}$ stand for the isoscalar and isovector constants of multipolarity $\lambda \geqslant 1$, respectively. The multipole moment operators $M^+_{\lambda\mu}(\tau)$ and $M_{\lambda\mu}(\tau)$ are expressed in terms of the quasiparticle operators. Their explicit formula can be found in [22]. By introducing thermal phonon operators $Q^+_{\lambda\mu i}(T)$ and $Q_{\lambda\mu i}(T)$ (see the Appendix C) and employing the method of linearising the equations of motion, we have obtained in Ref. [5] the FT–RPA equations for the energies $\omega_{\lambda i}(T)$ of one-phonon excitations at finite temperature. A system like that has been obtained also by other authors using different methods [3,14] . Using these thermal one-phonon energies $\omega_{\lambda i}(T)$ we can evaluate the contribution δC of the FT–RPA modes to the specific heat as

$$\delta C^{FT\text{-}RPA} = C^{FT\text{-}RPA}\Big|_{\omega_{\lambda i}(T)} - C^{FT\text{-}RPA}\Big|_{\omega^0_{\lambda i}} \tag{47}$$

where
$$C^{\text{FT-RPA}} = \frac{d\langle H^{\text{FT-RPA}}\rangle}{dT} =$$
$$= \sum_{\lambda i} \Omega_{\lambda} \left[1 - \operatorname{cth}\left(\tfrac{1}{2}\beta\omega_{\lambda i}\right)\right] \left\{ \frac{d\omega_{\lambda}}{dT}\Big|_{\omega_{\lambda i}} \left[\tfrac{1}{2}\beta\omega_{\lambda i}\left(1 + \operatorname{cth}\left(\tfrac{1}{2}\beta\omega_{\lambda i}\right)\right) - 1\right] - \right. \tag{48}$$
$$\left. - \tfrac{1}{2}\beta^{2}\omega_{\lambda i}^{2}\left[1 + \operatorname{cth}\left(\tfrac{1}{2}\beta\omega_{\lambda i}\right)\right] \right\}$$

In $C^{\text{FT-RPA}}\big|_{\omega^{o}_{\lambda i}}$ the two-quasiparticle poles $\omega^{o}_{\lambda i}$ (see Appendix) at finite temperature must be used instead of $\omega_{\lambda i}$. The formula for $d\omega_{\lambda}(T)/dT$ is given in Appendix C. The coefficient K_{vib} for the increase of the level density due to the vibrational modes is given by [1]

$$K_{vib} \approx \prod_{\lambda i} \left\{ \left[1 - \exp(-\beta\omega^{o}_{\lambda i})\right] / \left[1 - \exp(-\beta\omega_{\lambda i})\right] \right\}^{2\lambda+1} \tag{49}$$

In this work we shall calculate K_{vib} for the dipole modes in comparison with the quadrupole ones. The former has not been evaluated so far and has usually been neglected in calculations.

3. Numerical results

The calculations have been made for the hot $^{58}Ni^{*}$ nucleus. The single-particle energies E_j have been calculated in the Woods-Saxon potential describing the mean field at zero temperature. The parameters of this potential are taken from Ref.[23]. In general, at finite temperature the single-particle spectrum does depend upon temperature. However, the calculations performed in Refs. [24] have shown that this temperature dependence turns out

to be rather smooth and very weak up to $T \sim 6$ MeV. Therefore, in the temperature region of interest we neglect this dependence by putting $E_j(T) = E_j(0)$. The pairing constants G are chosen to be $G_N = 0.28$ for neutrons and $G_Z = 0.302$ for protons. The pairing gap at zero temperature is therefore found to be $\Delta_N(T{=}0) \approx 1.4$ MeV and $\Delta_Z(T{=}0) = 0$. The critical temperature T_{crit} where the collapse of the FT-BCS pairing gap takes place is $T_{crit} \approx 0.79$ MeV. In the FT-RPA calculations we use the temperature independent dipole and quadrupole isoscalar and isovector $\varkappa^{(1,2)}_{0,1}$ constants. They have been defined following the procedure described in detail in Refs. [22,25].

The particle number fluctuations ΔN^2 are depicted in Fig.1 as functions of the temperature. At $T < T_{crit}$ QF dominate. They vanish at the critical point T_{crit} in the case of phase transition with the FT-BCS gap Δ_T(21) for the neutron component (Fig.1a). At temperature T larger than T_{crit} only SF remain. For protons, as $\Delta_Z(T)$ is always zero, there are only SF for the particle number operator as shown by the dotted-dashed curve in Fig.1a. By using the averaging gap $\langle\Delta_N\rangle$ (20), which is not zero at $T = T_{crit}$, QF does not vanish but decreases fastly at $T > T_{crit}$ and becomes negligibly small at T about ~ 3 MeV (Fig.1b). The phase transition is smeared out and the discontinuous point in $(\Delta N^2)_N$ at $T = T_{crit}$ disappears.

The coefficients λ_2 calculated from Eq. (12) for N and Z are shown in Fig.2 against the temperature. The temperature dependences for both $\lambda_2^{(N)}$ and $\lambda_2^{(Z)}$ are rather weak except the region $T < 2$ MeV for the proton component. In general, one therefore can put roughly $\lambda_2^{(N)} \approx 0.1$ MeV, $\lambda_2^{(Z)} \approx 0.15$ MeV for all temperature in hot ^{58}Ni.

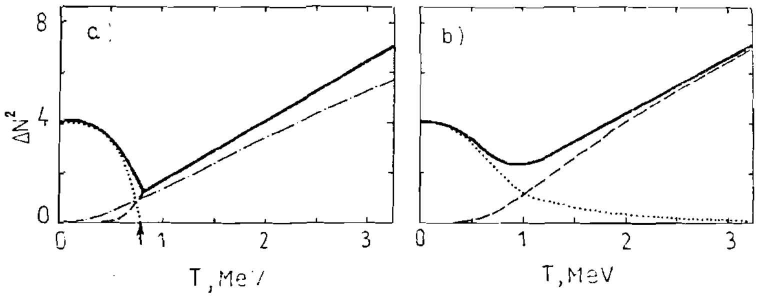

Fig.1. Particle number fluctuations ΔN^2 versus temperature.
a) Results obtained by using the FT-BCS pairing gap Δ_T (21)
b) Results obtained by using the average pairing $\langle\Delta_T\rangle$(20)
The dotted curves denote QF. The dashed curves describe the SF. The dotted-dashed curve stands for $(\Delta N^2)_Z$ of protons. The arrow points out the critical temperature.

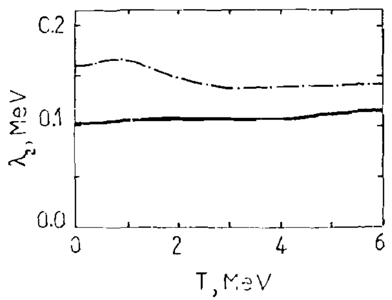

Fig.2. Coefficients $\lambda_2^{(2)}$ plotted against the temperature. The full curve is λ_2 for the neutron component while the dotted-dashed curve stands for the proton one.

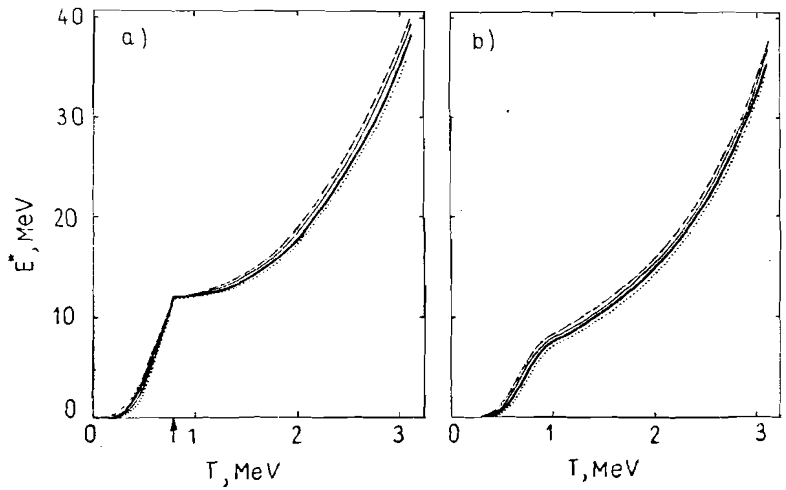

Fig.3. Excitation energy versus temperature. The thin curve corresponds to the value obtained in the FT-BCS approximation. The dotted curve is obtained by including the particle number fluctuations in the FT-BCS Hamiltonian. The dashed curve is calculated under the influence of the residual part $\langle H\rangle_{res}$ in the monopole pairing Hamiltonian. The thick curve is the total one.
a) Results with using the FT-BCS gap Δ_T
b) Results with using the average gap $\langle\Delta_T\rangle$

We now consider the influence of the particle number fluctuations and the residual part $\langle H\rangle_{res}$ on the thermodynamic characteristics of $^{58}Ni^*$.

The excitation energies E^* (25) are presented in Fig.3 versus the temperature. The shifts caused by the particle number fluctuations and $\langle H\rangle_{res}$ are systematic and very weak. The same situation is observed for the entropy S as a function of temperature shown in Fig.4. For the specific heat C the shifts caused by these corrections are not systematic (Fig.5). This behaviour takes place for $T < T_{crit}$ in calculations with the FT-BCS gap Δ_T (Fig.5a) and for $T > T_{crit}$ in those with the average gap $\langle\Delta_T\rangle$ (Fig.5b). The particle number fluctuations influence very weakly the logarithm of the level density(Fig6). The effect of $\langle H\rangle_{res}$ on $\ln\rho$ is so small that we neglect it in this characteristic. This is why we did not represent the formula for the influence of $\langle H\rangle_{res}$ on ρ in Subsec. 2.3. Here the shift caused by ΔN^2 in the $\ln\rho$ is not systematic at $1\ \mathrm{MeV} \leqslant T \leqslant 2\ \mathrm{MeV}$ in calculations with the FT-BCS gap Δ_T (Fig.6a).

The level density parameters a are shown in Fig.7. In the phase transition case (Fig.7a) we see that the parameter a is about of the Fermi gas value only in the region $1\ \mathrm{MeV} \leqslant T \leqslant 3.5\ \mathrm{MeV}$.

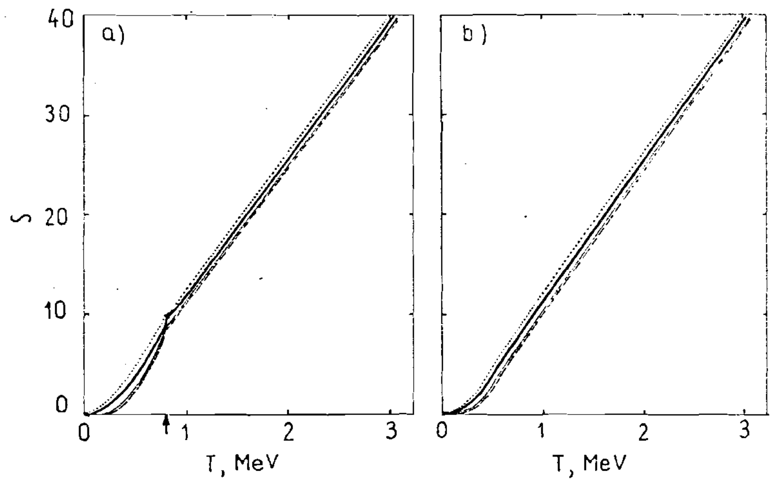

Fig.4. Entropy versus temperature.
The notation as in Fig.3.

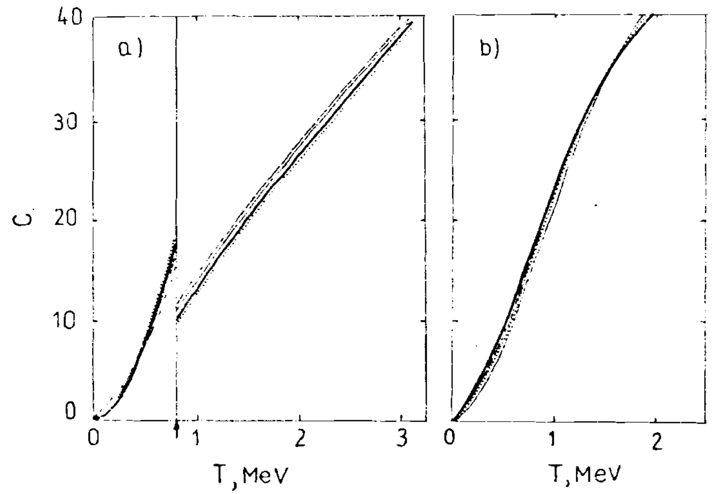

Fig.5. Specific heat versus temperature. The notation as in Fig.3.

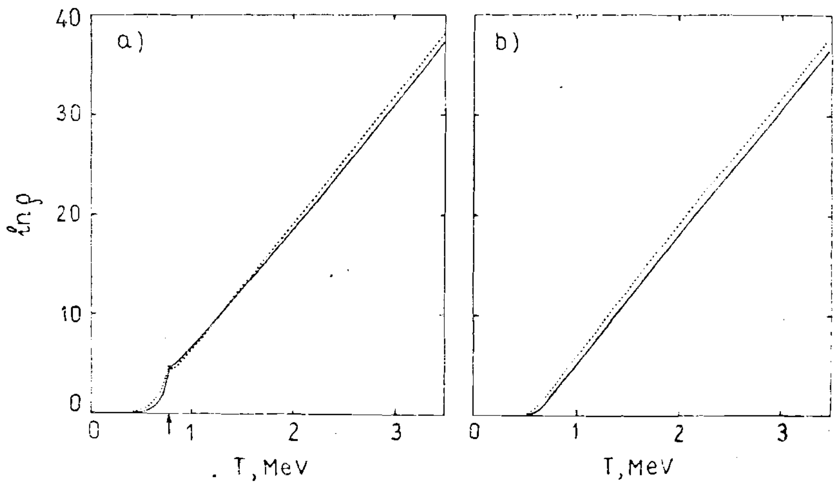

Fig.6. Logarithm of the level density versus temperature. The notation as in Fig.3.

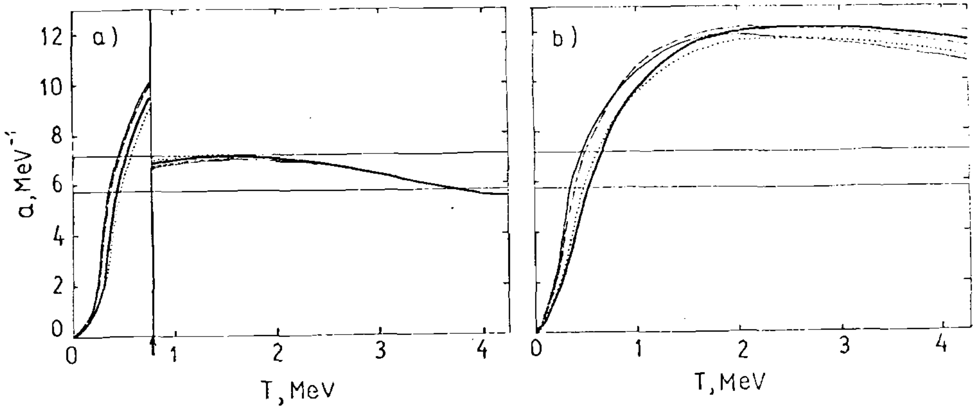

Fig.7. Level density parameter versus temperature. The notations as in Fig.3. The horizontal lines correspond to the Fermi-gas values a = A/8 and A/I0, respectively.

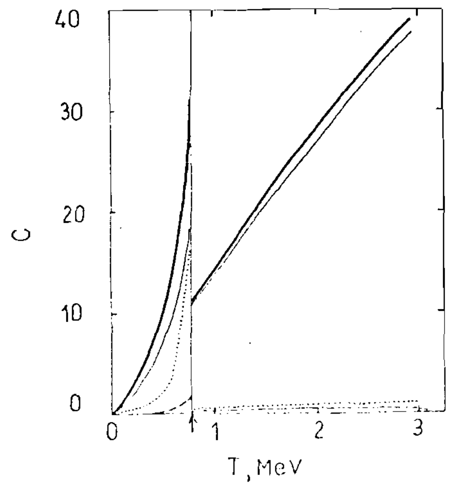

Fig.8. The contribution of FT-RPA modes to the specific heat C.
The thin curve denotes the specific heat in the FT-BCS theory. The dashed curve is $\delta C^{FT\text{-}RPA}$ given by the dipole vibrations. The dotted curve is the contribution of the quadrupole modes. The total specific heat is described by the thick curve.

In the case with the average pairing gap $\langle\Delta_T\rangle$ (Fig.7b) the parameter a is larger than the Fermi gas value and reaches this values only at sufficiently high temperatures. The effect of ΔN^2 and $\langle H\rangle_{res}$ are more noticeable in this case as compared with the case in Fig.7a or with other thermodynamic characteristics. In all the cases performed in Figs. 4-7 the phase transition is smeared out by using $\langle\Delta_T\rangle$ due to the statistical fluctuations arisen as a manifestation of the finiteness of the nucleus.

Table . Contribution δC of dipole and quadrupole FT-RPA modes to the specific heat and the corresponding coefficients K_{vib} for the increase of the level density at several temperatures. The calculations have been made with using the FT-BCS pairing gap Δ_T from Eq. (21).

DIPOLE $(\lambda^{\pi}=1^-)$			QUADRUPOLE $(\lambda^{\pi}=2^+)$		
T MeV	$\delta C^{FT\text{-}RPA}$	K_{vib}	T MeV	$\delta C^{FT\text{-}RPA}$	K_{vib}
0.20	0.01	1.00	0.24	0.01	1.00
0.40	0.50	1.26	0.48	1.88	1.11
0.60	0.27	1.20	0.55	2.50	1.21
0.70	0.23	1.15	0.63	4.58	1.50
0.79	1.04	1.24	0.71	15.30	1.33
0.90	0.01	1.11	0.79	11.00	1.49
1.00	0.01	1.01	1.00	0.15	1.56
2.00	0.03	1.00	2.40	0.55	1.60

The contribution of the thermal dipole and quadrupole one-phonon vibrations to the specific heat is shown in Fig.8, where the calculations have been made in the phase transition case with the FT-BCS gap Δ_T (21). It is also given in the Table together with the coefficients K_{vib} . It is evident from Fig.8 that the influence of the dipole modes on the specific heat is very weak. The total specific heat obtained a sum of C^{BCS}, $\delta C^{FT-RPA}_{\lambda=1}$ and $\delta C^{FT-RPA}_{\lambda=2}$ has a slight increase as compared to C^{BCS} due to more important contribution of the quadrupole modes. The coefficient K_{vib} for the increase of the level density is nearly ~ 1 for the dipole modes, but increases remarkably with the temperature for the quadrupole vibrations, as shown in the Table (Cf. Refs.[1,26]) Of course, if one uses the average gap $\langle\Delta_T\rangle$ in calculations, the discontinuity at $T = T_{crit}$ in Fig.8 should be smoothed out as in the preceding figures 3-7 (b). Therefore, we do not consider this case here.

4. Conclusions

In the present work we have extended to the finite temperature case the LN method that is a rather simple and good approximation for improving the BCS theory at zero temperature taking into account the particle number fluctuations ΔN^2 . This extension has been applied here to study the influence of QF and SF for the particle number operator $\hat{N}$ on some thermodynamic characteristics in the hot $^{58}Ni^*$ nucleus up to $T \sim 5$ MeV. We have also evaluated the contributions of the residual part usually

neglected in the monopole pairing Hamiltonian and of the FT-RPA modes to the specific heat and the coefficients for the increase of the level density.

Summarizing the numerical results, we find:

1) If QF for the particle number appear due to the symmetry breaking of the FT-BCS method, SF for $\hat{N}$ is a physical quantity characterizing all thermodynamic finite nuclear systems. Below the critical temperature T_{crit} QF for the particle number dominate. Above T_{crit} the most important contribution to ΔN^2 is given by SF which increase with T. By using the FT-BCS pairing gap Δ_T in calculations QF for the particle number vanish at $T = T_{crit}$ where the gap Δ_T collapses. With taking into account the average gap $\langle\Delta_T\rangle$ due to the finiteness of the nucleus, QF remain nonzero for $T > T_{crit}$ but decreases fastly to be negligibly small at $T \gtrsim 2$ MeV.

2) The effects of the particle number fluctuations and of the residual part in the monopole pairing Hamiltonian on the thermodynamic characteristics of $^{58}Ni^*$ exist although they are small. In some characteristics like the specific heat C, the level density ρ and its parameter a , the shift caused by particle number fluctuations is not systematic. The influence of ΔN^2 on the level density parameter a is more noticeable.

3) While the quadrupole vibrations give a remarkable contribution to the specific heat C and to the coefficient for the increase of the level density, the influence of dipole modes is small.

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Appendix

A) The monopole pairing Hamiltonian

The explicit form of the monopole pairing Hamiltonian (2) has been given in many works [18,27]. Here we represent it in the form convenient to the context of the present work

$$H_{00} = 2\sum_j \Omega_j E_j^0 v_j^2 - G\Big(\sum_j \Omega_j u_j v_j\Big)^2 - G\sum_j \Omega_j v_j^4 \qquad \text{(A1)}$$

$$H_{11} = \sum_j \Big[E_j^0(u_j^2 - v_j^2) + 2G\sum_{j'} \Omega_{j'} u_j v_j u_{j'} v_{j'} + G v_j^4\Big] \mathcal{N}_j \qquad \text{(A2)}$$

$$H_{20} = \sum_j \sqrt{\Omega_j}\Big[2E_j^0 u_j v_j - G\sum_{j'} \Omega_{j'}(u_j^2 - v_j^2) u_{j'} v_{j'} - 2G u_j v_j^3\Big](\mathcal{A}_j^+ + \mathcal{A}_j) \qquad \text{(A3)}$$

$$H_{22} = -G\sum_{jj'} \sqrt{\Omega_j \Omega_{j'}}(u_j^2 u_{j'}^2 + v_j^2 v_{j'}^2)\mathcal{A}_j^+ \mathcal{A}_{j'} \qquad \text{(A4)}$$

$$H_{31} = G\sum_{jj'} \sqrt{\Omega_{j'}}\, u_j v_j (u_{j'}^2 - v_{j'}^2)(\mathcal{N}_j \mathcal{A}_{j'} + \mathcal{A}_{j'}^+ \mathcal{N}_j) \qquad \text{(A5)}$$

$$H_{40} = \frac{1}{2} G\sum_{jj'} \sqrt{\Omega_j \Omega_{j'}}(u_j^2 v_{j'}^2 + v_j^2 u_{j'}^2)(\mathcal{A}_j^+ \mathcal{A}_{j'}^+ + \mathcal{A}_j \mathcal{A}_{j'}) \qquad \text{(A6)}$$

$$H_{11-11} = -G\Big(\sum_j u_j v_j \mathcal{N}_j\Big)^2 \qquad \text{(A7)}$$

The operators $\mathcal{N}_j$, $\mathcal{A}_j^+$, $\mathcal{A}_j$ in these equations are

$$\mathcal{N}_j = \sum_{m \gtrless 0} \alpha_{jm}^+ \alpha_{jm} \qquad \text{(A 8)}$$

$$\mathcal{A}_j^+ = \Omega_j^{-1/2} \sum_{m>0} \alpha_{jm}^+ \alpha_{j\tilde{m}}^+ \qquad \text{(A 9)}$$

$$\mathcal{A}_j = \Omega_j^{-1/2} \sum_{m>0} \alpha_{j\tilde{m}} \alpha_{jm} \qquad \text{(A10)}$$

Their commutation relations are

$$[\mathcal{N}_j, \mathcal{A}_{j'}] = -2\delta_{jj'} \mathcal{A}_j \qquad \text{(A11)}$$

$$[\mathcal{A}_j, \mathcal{A}_{j'}^+] = \delta_{jj'}(1 - \Omega_j^{-1} \mathcal{N}_j) \qquad \text{(A12)}$$

In the averaging procedure the following relations take place in the FT-HFB approximation

$$\langle \mathcal{A}_j^+ \mathcal{A}_{j'} \rangle = n_j^2 \delta_{jj'} \qquad \text{(A13)}$$

$$\langle \mathcal{N}_j \mathcal{N}_{j'} \rangle = 2\delta_{jj'} \Omega_j n_j + (2\Omega_j n_j)^2 - 2\delta_{jj'} \Omega_j n_j^2 \qquad \text{(A14)}$$

B) The derivatives of ΔN^2 over α and β

In calculations of the determinant D_τ (38) with the particle number fluctuations ΔN^2 taken into account, we must calculate the derivatives $\partial^2 \Delta N^2/\partial\alpha^2$, $\partial^2 \Delta N^2/\partial\alpha\partial\beta$ and $\partial^2 \Delta N^2/\partial\beta^2$. They are

$$\partial^2(\Delta N^2)/\partial\alpha^2 = A_1 + A_2 + A_3 + A_4 \qquad \text{(B1)}$$

$$A_1 = \beta \frac{\partial M}{\partial\alpha}\Big\{2\sum_j \Omega_j \varepsilon_j^2 (E_j - \lambda)^2 b_j^2 + \sum_j \Omega_j [\Delta^4 - (E_j - \lambda)^4] a_j b_j\Big\} \qquad \text{(B2)}$$

$$A_2 = M^2 \sum_j \Omega_j \Big\{4b_j(a_j - 2b_j)(E_j - \lambda)^2 + 4\Delta^2 a_j b_j + [\Delta^2 - (E_j - \lambda)^2] \times$$
$$\times a_j(a_j - 5b_j - \beta^2 \varepsilon_j^4 b_j^2)\Big\} \qquad \text{(B3)}$$

$$A_3 = 2M\sum_j \Omega_j (E_j - \lambda)[\Delta^2 - (E_j - \lambda)^2][9a_j b_j - 4b_j^2 - a_j^2 + a_j b_j^2 \beta^2 \varepsilon_j^4] \qquad \text{(B4)}$$

$$A_4 = -2\Delta^2 \sum_j \Omega_j b_j \Big\{b_j[2(E_j - \lambda)^2 + \varepsilon_j^2] + 2(a_j - 3b_j)(E_j - \lambda)^2\Big\} +$$
$$+ \sum_j \Omega_j a_j b_j [\Delta^4 - 5(E_j - \lambda)^4] + \sum_j \Omega_j [\Delta^2 - (E_j - \lambda)^2](E_j - \lambda)^2 \times$$
$$\times a_j [a_j - 5b_j - \beta^2 \varepsilon_j^4 b_j^2] \qquad \text{(B5)}$$

$$\partial^2(\Delta N^2)/\partial\beta\partial\alpha = B_1 + B_2 + B_3 + B_4 \tag{B6}$$

$$B_1 = [M + \beta\partial M/\partial\beta][2\sum_j \Omega_j \varepsilon_j^2 (E_j-\lambda)^2 b_j^2 + \sum_j \Omega_j (\Delta^4 - (E_j-\lambda)^4) a_j b_j] \tag{B7}$$

$$B_2 = KM[\sum_j \Omega_j (E_j-\lambda)^2 (9a_j b_j - 8b_j^2 - a_j^2 + \beta^2 \varepsilon_j^4 a_j b_j^2) + + \Delta^2 \sum_j \Omega_j (a_j^2 - a_j b_j - \beta^2 \varepsilon_j^4 a_j b_j^2) \tag{B8}$$

$$B_3 = 2\Delta^2 \sum_j \Omega_j \varepsilon_j^2 (E_j-\lambda) b_j^2 - \sum_j \Omega_j (E_j-\lambda)[\Delta^4 - (E_j-\lambda)^4] a_j b_j \tag{B9}$$

$$B_4 = K\sum_j \Omega_j (E_j-\lambda)[(E_j-\lambda)^2 - \Delta^2](4b_j^2 + a_j^2 - 5a_j b_j - \beta^2 \varepsilon_j^4 a_j b_j^2) \tag{B10}$$

$$\partial^2(\Delta N^2)/\partial\beta^2 = C_1 + C_2 + C_3 + C_4 \tag{B11}$$

$$C_1 = (K + \beta\partial K/\partial\beta)[\sum_j \Omega_j \varepsilon_j^2 b_j (a_j \Delta^2 + (E_j-\lambda)^2 (2b_j - a_j))] \tag{B12}$$

$$C_2 = K^2 \sum_j \Omega_j \{(a_j - b_j)[a_j \Delta^2 + (E_j-\lambda)^2 (2b_j - a_j)] - a_j b_j [\Delta^2 - (E_j-\lambda)^2][2 + \beta^2 \varepsilon_j^4 b_j] + + 2\varepsilon_j^2 a_j b_j + 2(E_j-\lambda)^2 b_j (a_j - 3b_j)\} \tag{B13}$$

$$C_3 = \sum_j \Omega_j \varepsilon_j^4 a_j b_j [\Delta^2 - (E_j-\lambda)^2] \tag{B14}$$

$$C_4 = K\sum_j \Omega_j \varepsilon_j^2 a_j \{[\Delta^2 - (E_j-\lambda)^2](a_j - b_j - 2\beta^2 \varepsilon_j^4 b_j^2) + 2\varepsilon_j^2 b_j\} \tag{B15}$$

In Eqs. (B.1) – (B15) we use the notation of Ref.[21]

$$a_j = \frac{1}{2}\varepsilon_j^{-2} \operatorname{sech}^2(\tfrac{1}{2}\beta\varepsilon_j) \quad ; \quad b_j = \beta^{-1}\varepsilon_j^{-3} \tanh(\tfrac{1}{2}\beta\varepsilon_j)$$
$$M = \beta\Delta\frac{\partial\Delta}{\partial\alpha} \quad ; \quad K = \beta\Delta\frac{\partial\Delta}{\partial\beta} \tag{B16}$$

and $M = 0$, $K = 0$ if $\Delta = 0$

$$\partial\Delta/\partial\alpha = [\sum_j \Omega_j (E_j-\lambda)(a_j - b_j)][\beta\Delta\sum_j \Omega_j (a_j - b_j)]^{-1} \tag{B17}$$

$$\partial\Delta/\partial\beta = -[\sum_j \Omega_j \varepsilon_j^2 a_j][\beta\Delta\sum_j \Omega_j (a_j - b_j)]^{-1} \tag{B18}$$

Eqs. (B17) and (B18) have been obtained earlier in Refs. [18, 21]

$$\frac{\partial M}{\partial\alpha} = \beta^{-1} F_1^{-1} \{[\sum_j \Omega_j E_j (E_j-\lambda) P_j - M\sum_j \Omega_j E_j P_j] - - F_1^{-1}[\sum_{j'} \Omega_{j'} E_{j'} (a_{j'} - b_{j'})][\sum_j \Omega_j (E_j-\lambda) P_j - M\sum_j \Omega_j P_j]\} \tag{B19}$$

with

$$F_1 = \sum_j \Omega_j (a_j - b_j)$$
$$P_j = [3(a_j - b_j) + \beta^2 \varepsilon_j^4 a_j b_j]\varepsilon_j^{-2} \quad ; \tag{B20}$$

$$\frac{\partial M}{\partial\beta} = \beta^{-1} K F_1^{-2} (F_2 - F_3) \tag{B21}$$

with

$$F_2 = \{\sum_j \Omega_j \varepsilon_j^{-2} [3(a_j - b_j) + \beta^2 \varepsilon_j^4 a_j b_j]\}\{\sum_j \Omega_j (E_j-\lambda)(a_j - b_j)\}$$
$$F_3 = \{\sum_j \Omega_j \varepsilon_j^{-2} (E_j-\lambda)[3(a_j - b_j) + \beta^2 \varepsilon_j^4 a_j b_j]\}\{\sum_j \Omega_j (a_j - b_j)\} \; ; \tag{B22}$$

$$\frac{\partial K}{\partial\beta} = \beta^{-1} K F_1^{-2} (I_1 - I_2) \tag{B23}$$

with

$$I_1 = \beta^2 F_1 [\sum_j \Omega_j \varepsilon_j^4 a_j b_j]$$
$$I_2 = [\sum_j \Omega_j \varepsilon_j^2 a_j][\sum_j \Omega_j \varepsilon_j^{-2} (3(a_j - b_j) + \beta^2 \varepsilon_j^4 a_j b_j)] \tag{B24}$$

C) The FT–RPA equation

Introducing the thermal one-phonon operators [5,28,29]

$$Q^+_{\lambda\mu i} = \frac{1}{2}\sum_{jj'}\left[\psi^{\lambda i}_{jj'}A^+_{\lambda\mu}(jj') - \varphi^{\lambda i}_{jj'}A_{\lambda\tilde{\mu}}(jj') + \xi^{\lambda i}_{jj'}B_{\lambda\mu}(jj') - \zeta^{\lambda i}_{jj'}B^+_{\lambda\tilde{\mu}}(jj')\right]$$

$$Q_{\lambda\mu i} = \left[Q^+_{\lambda\mu i}\right]^+ \qquad \text{(C1)}$$

and linearizing the equations of motion for the Hamiltonian (45) and operators (C1)

$$\langle [H^{FT\text{-}RPA}, Q^+_{\lambda\mu i}]\rangle = \omega_{\lambda i}(T)\,Q^+_{\lambda\mu i}$$
$$\langle [H^{FT\text{-}RPA}, Q_{\lambda\mu i}]\rangle = -\omega_{\lambda i}(T)\,Q_{\lambda\mu i} \qquad \text{(C2)}$$

we have obtained in Ref.[5] the FT–RPA equation in the form:

$$(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})\left[X_N^{(\lambda i)}+X_Z^{(\lambda i)}\right] - 4\varkappa_0^{(\lambda)}\varkappa_1^{(\lambda)}X_N^{(\lambda i)}X_Z^{(\lambda i)} = 1 \qquad \text{(C3)}$$

where

$$X_\tau^{(\lambda i)} = (2\lambda+1)^{-1}\sum_{jj'}^{\tau}\left[f_{jj'}^{(\lambda)}\right]^2\left[\frac{(u^{(+)}_{jj'})^2\varepsilon^{(+)}_{jj'}n^{(+)}_{jj'}}{\varepsilon^{(+)2}_{jj'}-\omega^2_{\lambda i}} - \frac{(v^{(-)}_{jj'})^2\varepsilon^{(-)}_{jj'}n^{(-)}_{jj'}}{\varepsilon^{(-)2}_{jj'}-\omega^2_{\lambda i}}\right] \qquad \text{(C4)}$$

with

$u^{(+)}_{jj'} = u_j v_{j'} + u_{j'}v_j$; $v^{(-)}_{jj'} = u_j u_{j'} - v_j v_{j'}$ being the combinations of the Bogolubov coefficients

$\varepsilon^{(\pm)}_{jj'} = \varepsilon_j \pm \varepsilon_{j'}$ being the two-quasiparticle poles.

and

$$n^{(+)}_{jj'} = 1 - n_j - n_{j'} \quad ; \quad n^{(-)}_{jj'} = n_j - n_{j'}$$

In Eqs. (C4) and (C5) $f^{(\lambda)}_{jj'}$ are the reduced matrix elements of the single-particle operators generating excitations of multipolarity λ; u_j and v_j are the coefficients of the Bogolubov

transformation. Differentiating both sides of Eq. (3) we find the obvious form

$$\frac{d\omega}{dT} = \Big\{4\varkappa_0^{(\lambda)}\varkappa_1^{(\lambda)}\Big[X_N^{(\lambda)}(M_Z^{(+)} - M_Z^{(-)} - 2R_Z\Delta_Z d\Delta_Z/dT) + X_Z^{(\lambda)}(M_N^{(+)} - M_N^{(-)} - 2R_N\Delta_N d\Delta_N/dT)\Big] - (\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})\Big[(M_N^{(+)}+M_Z^{(+)}) - (M_N^{(-)}+M_Z^{(-)}) - 2(R_N\Delta_N d\Delta_N/dT + R_Z\Delta_Z d\Delta_Z/dT)\Big]\Big\}\Big\{\omega\Big[(\varkappa_0^{(\lambda)}+\varkappa_1^{(\lambda)})(\mathcal{D}_N+\mathcal{D}_Z) - 4\varkappa_0^{(\lambda)}\varkappa_1^{(\lambda)}(X_N^{(\lambda)}\mathcal{D}_Z + X_Z^{(\lambda)}\mathcal{D}_N)\Big]\Big\}^{-1} \qquad \text{(C6)}$$

where

$$M_\tau^{(+)} = (2\lambda+1)^{-1}\sum_{jj'}^{\tau}\left[(\varepsilon^{(+)}_{jj'})^2 - \omega^2\right]^{-1}\Big\{(f^{(\lambda)}_{jj'})^2\varepsilon^{(+)}_{jj'}\Big[2u^{(+)}_{jj'}\frac{du^{(+)}_{jj'}}{dT}n^{(+)}_{jj'} - \frac{1}{4}\beta^2(u^{(+)}_{jj'})^2 S^{(+)}_{jj'} + \Delta_\tau\frac{d\Delta_\tau}{dT}(u^{(+)}_{jj'})^2 F^{(+)}_{jj'}\Big]\Big\} \qquad \text{(C7)}$$

$$M_\tau^{(-)} = (2\lambda+1)^{-1}\sum_{jj'}^{\tau}\left[(\varepsilon^{(-)}_{jj'})^2 - \omega^2\right]^{-1}\Big\{(f^{(\lambda)}_{jj'})^2\varepsilon^{(-)}_{jj'}\Big[2v^{(-)}_{jj'}\frac{dv^{(-)}_{jj'}}{dT}n^{(-)}_{jj'} + \frac{1}{4}\beta^2(v^{(-)}_{jj'})^2 S^{(-)}_{jj'} - \Delta_\tau\frac{d\Delta_\tau}{dT}(v^{(-)}_{jj'})^2 F^{(-)}_{jj'}\Big]\Big\} \qquad \text{(C8)}$$

$$R_\tau = (2\lambda+1)^{-1}\sum_{jj'}^{\tau}\varepsilon_j^{-1}\varepsilon_{j'}^{-1}(f^{(\lambda)}_{jj'})^2\Big\{\left[(\varepsilon^{(+)}_{jj'})^2-\omega^2\right]^{-2}\left[u^{(+)}_{jj'}(\varepsilon^{(+)}_{jj'})^3 n^{(+)}_{jj'}\right] - \left[(\varepsilon^{(-)}_{jj'})^2-\omega^2\right]^{-2}\left[v^{(-)}_{jj'}(\varepsilon^{(-)}_{jj'})^3 n^{(-)}_{jj'}\right]\Big\} \qquad \text{(C9)}$$

$$S^{(\pm)}_{jj'} \equiv \varepsilon_j\,\mathrm{sech}^2\left(\tfrac{1}{2}\beta\varepsilon_j\right) \pm \varepsilon_{j'}\,\mathrm{sech}^2\left(\tfrac{1}{2}\beta\varepsilon_{j'}\right) \qquad \text{(C10)}$$

$$F^{(\pm)}_{jj'} \equiv \varepsilon_j^{-1}\varepsilon_{j'}^{-1}n^{(\pm)}_{jj'} + \tfrac{1}{4}\beta\left[\varepsilon_j^{-1}\mathrm{sech}^2\left(\tfrac{1}{2}\beta\varepsilon_j\right) \pm \varepsilon_{j'}^{-1}\mathrm{sech}^2\left(\tfrac{1}{2}\beta\varepsilon_{j'}\right)\right] \qquad \text{(C11)}$$

$$\mathcal{D}_\tau = (2\lambda+1)^{-1}2\sum_{jj'}^{\tau}(f^{(\lambda)}_{jj'})^2\Big\{\left[(\varepsilon^{(+)}_{jj'})^2-\omega^2\right]^{-2}\left[(u^{(+)}_{jj'})^2\varepsilon^{(+)}_{jj'}n^{(+)}_{jj'}\right] - \left[(\varepsilon^{(-)}_{jj'})^2-\omega^2\right]^{-2}\left[(v^{(-)}_{jj'})^2\varepsilon^{(-)}_{jj'}n^{(-)}_{jj'}\right]\Big\} \qquad \text{(C12)}$$

$$\frac{du^{(+)}_{jj'}}{dT} = \begin{cases} \frac{1}{4}\Delta_\tau d\Delta_\tau/dT\left\{\varepsilon_j^{-3}(E_j-\lambda_\tau)(u_{j'}/v_j - v_{j'}/u_j) + \varepsilon_{j'}^{-3}(E_{j'}-\lambda_\tau)(u_j/v_{j'} - v_j/u_{j'})\right\} & \text{if } \Delta_\tau \neq 0 \\ 0 & \text{if } \Delta_\tau = 0 \end{cases} \qquad \text{(C13)}$$

$$\frac{dV^{(-)}_{jj'}}{dT}=\begin{cases}\frac{1}{4}\Delta_\tau d\Delta_\tau/dT\{\varepsilon_j^{-3}(E_j-\lambda)(u_{j'}/u_j+v_{j'}/v_j)+\varepsilon_{j'}^{-3}(E_{j'}-\lambda)(u_j/u_{j'}+v_j/v_{j'})\} & \text{if } \Delta_\tau\neq 0 \\ 0 & \text{if } \Delta_\tau=0\end{cases} \tag{C14}$$

Therefore, we see that although one cannot obtain an obvious form for ω_λ from the FT-RPA equation (C3), the analytic formula for $d\omega_\lambda/dT$ has however been derived by us in Eq. (C6).

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Влияние флуктуаций числа частиц и вибрационных мод на термодинамические характеристики нагретого ядра

Метод Ногами распространен на конечную температуру T. Рассчитано влияние квантовых и статистических флуктуаций числа частиц в модели температурного BCS на энергию возбуждения, энтропию, теплоемкость, плотность уровней и параметр плотности уровней в нагретом ^{58}Ni ядре. Расчеты выполнены с парной щелью теории температурного BCS и со статистически усредненной щелью обусловленной конечностью ядра. В температурном RPA рассчитан вклад дипольных и квадрупольных вибрационных мод в теплоемкость и плотность уровней.

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Influence of Particle Number Fluctuations and Vibrational Modes on Thermodynamics Characteristics of a Hot Nucleus

The Nogami's method is extended to finite temperature T. The effect of quantum and statistical fluctuations for the particle number in the finite temperature BCS model on the excitation energy, entropy, specific heat, level density and level density parameter is calculated in hot ^{58}Ni nucleus. The calculations have been performed with the finite temperature BCS gap and statistical average gap due to the finiteness of nucleus. In the finite temperature RPA the contribution of dipole and quadrupole vibrational modes to the specific heat and the increase of level density is calculated.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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