89-56

объединенный
институт
ядерных
исследований
дубна

E4-89-56

V.A.Nikolaev, O.G.Tkachev

NEW SOLITONS IN THE SKYRME MODEL

Submitted to "Physics Letters B"

1989

It is well known that the stationary solution in the unit topological charge sector of the Skyrme model is the hedgehog configuration or the so-called Skyrme - Witten solution

$$U(\vec{r})_{SW} = \cos(F(r)) + i(\vec{\tau}\cdot\vec{N})\,\sin(F(r)), \tag{1}$$

where $\vec{N}$ determines a direction in the isotopic space and for hedgehog configuration is specified by the vector $\vec{N}=\vec{r}/r$. In Eq.(1) $F(r)$ is the chiral angle, describing the absolute value of the pion field. Function $F(r)$ fulfills the following boundary conditions $F(0) = N\pi$, $F(\infty) = 0$. These conditions insure finiteness of the energy for a soliton with the topological number N, which is equal to baryon number B. In /1/ it was shown that the only configuration that provides minimum the energy of the soliton with N=1 is that given by Eq.(1). However, for other sectors such a form is not obligatory. For example, in /2,3/ the solutions, defined by the "kϕ" configuration

$$\vec{N} = (\cos(k\phi)\cdot\sin(\vartheta),\ \sin(k\phi)\cdot\sin(\vartheta),\ \cos(\vartheta)), \tag{2}$$

where (ϑ,ϕ) are the angles of vector r in the spherical coordinate system,have been considered. In Eq.(2) k is an integer determining also the topological charge. Some interesting properties of states generated by these solutions were described in /2,3/. In the sector with baryon charge B=2 this form of the solution gives us low mass states in the range of around two nucleon masses. Quantization procedure generates rich spectra of rotational bands.

In the present paper we propose a new form of the solution given by the next vector

$$\vec{N} = (\cos(\Phi(\phi))\cdot\sin(T(\vartheta)),\ \sin(\Phi(\phi))\cdot\sin(T(\vartheta)),\ \cos(T(\vartheta))), \tag{3}$$

where $\Phi(\phi), T(\vartheta)$ are some arbitrary functions.

It will be shown that this ansatz is the generalization of the hedgehog and "kϕ" -configurations. In some sense our ansatz gives an explanation of the origin and approximate character of the last. As it will be seen, Eq.(3) leads to a series of new solutions in baryon and topologically trivial sectors. Some of these new states are classically stable.

Let us consider the Lagrangian density $\mathcal{L}$ for the stationary solution

$$\mathcal{L} = \frac{F_\pi^2}{16} \, \mathrm{Tr}(L_k L_k) + \frac{1}{32e^2} \, \mathrm{Tr}\, [L_k, L_i]^2 . \qquad (4)$$

Here $L_\mu = U^+ \partial_\mu U$ are the left currents. After some tedious algebra Eqs. (1),(3) and (4) lead to the expression

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 . \qquad (5)$$

where

$$\mathcal{L}_2 = -\frac{F_\pi^2}{8} \cdot \left\{ (F')^2 + \left[\frac{\sin^2 T}{\sin^2 \theta} \cdot (\Phi')^2 + (T')^2 \right] \cdot \frac{\sin^2 F}{r^2} \right\} \qquad (6)$$

and

$$\mathcal{L}_4 = -\frac{1}{2e^2} \cdot \frac{\sin^2 F}{r^2} \cdot \left\{ \frac{\sin^2 T}{\sin^2 \theta} \cdot (T')^2 \cdot (\Phi')^2 \cdot \frac{\sin^2 F}{r^2} + \right.$$

$$\left. + \left[\frac{\sin^2 T}{\sin^2 \theta} \cdot (\Phi')^2 + (T')^2 \right] \cdot (F')^2 \right\} . \qquad (7)$$

In Eqs.(6-7) we use the symbol prime to denote the following derivatives

$$\Phi' = \frac{d\Phi}{d\phi} \; ; \; T' = \frac{dT}{d\theta} \; ; \quad F' = \frac{dF}{dr} .$$

Variation of Eq.(5) with respect to $\Phi(\phi)$ gives us

$$\Phi'' = 0, \qquad (8)$$

that is $\Phi(\phi) = k\phi + \mathrm{Const}$.

We consider only solutions with a vanishing value of this constant. The number k must be an integer in order to obtain a single-value solution $U(\mathbf{r})$ in the whole $\mathbf{r}$-space.

Now we have the following expression for the mass of a soliton

$$M = M_2 + M_4 , \qquad (9)$$

$$M_2 = \frac{\gamma}{4} \int_0^\infty dx \cdot x^2 \int_0^\pi d\theta \, \sin\theta \left\{ (F')^2 + \left[\frac{\sin^2 T}{\sin^2 \theta} \, k^2 + (T')^2 \right] \frac{\sin^2 F}{x^2} \right\}, \qquad (10)$$

$$M_4=\gamma\cdot\int_0^\infty dx x^2 \int_0^\pi d\theta\ \sin\theta \left\{ \left[\frac{\sin^2 T}{\sin^2\theta} k^2 + (T')^2 \right]\cdot(F')^2 + \right.$$

$$\left. + \frac{\sin^2 F}{x^2}\ \frac{\sin^2 T}{\sin^2\theta} k^2\cdot(T')^2 \right\} \frac{\sin^2 F}{x^2}\ , \qquad (11)$$

where $\gamma = \pi\cdot F_\pi/e$ and $x = F_\pi\cdot e\cdot r$.

In order to minimize the functional M the functions $T(\theta)$ and $F(x)$ have to obey the following equation

$$\frac{\delta M}{\delta T} = 0\ ;\ \frac{\delta M}{\delta F} = 0, \qquad (12)$$

or more strictly

$$[\ x^2 + 2a\cdot\sin^2 F\ ]\cdot F'' + 2\cdot x\cdot F' + a\cdot(F')^2 \sin(2F) - \frac{a}{4}\sin(2F)\ -$$

$$-2\cdot b\cdot \frac{\sin^2 F}{x^2}\sin(2F) = 0, \qquad (13)$$

$$2\cdot\left[A + k^2 B \frac{\sin T}{\sin^2\theta} \right]\cdot T'' + k^2 B \frac{\sin(2T)}{\sin^2\theta}\cdot(T')^2 +$$

$$+2\cdot T'\cdot \mathrm{ctg}\theta\cdot\left[A - k^2 B \frac{\sin^2 T}{\sin^2\theta} \right] - k^2 A \frac{\sin(2T)}{\sin^2\theta} = 0. \qquad (14)$$

The coefficients a,b, and A,B in Eq.(13-14) are the following integrals:

$$a = \int_0^\pi \left\{ k^2 \frac{\sin^2 T}{\sin^2\theta} + (T')^2 \right\}\cdot\sin\theta\ d\theta, \qquad (15)$$

$$b = k^2 \int_0^\pi \frac{\sin^2 T}{\sin^2\theta}(T')^2 \sin\theta\ d\theta, \qquad (16)$$

$$A = \int_0^\infty \sin^2 F \left[\frac{1}{4} + (F')^2\right] dx, \tag{17}$$

$$B = \int_0^\infty \frac{\sin^4 F}{x^2}\, dx. \tag{18}$$

From Eqs.(9-11) and (15-18) we conclude that the function $T(\vartheta)$ has to be some integer factor of π for $\vartheta = 0$ and $\vartheta = \pi$. We consider only the configurations with finite masses, that is why we have $F(0) = N\pi$ with integer N. Without loss of generality we take $F(\infty) = 0$. It is not difficult to prove that the asymptotic behavior of F is presented by

$$F(x) \underset{x \to \infty}{\to} \frac{1}{x^{p+1}}, \quad \text{with} \quad p = \frac{\sqrt{1+2a} - 1}{2}. \tag{19}$$

In the vicinity of the coordinate system origin

$$F(x) \underset{x \to 0}{\to} \pi \cdot N - \alpha \cdot x^p, \tag{20}$$

where

$$p = \frac{\sqrt{1+2a} - 1}{2}, \quad \text{if} \quad a \geqslant 4$$

and

$$p = \frac{1 + \sqrt{1 + 16b/a}}{4}, \quad \text{for} \quad a \geqslant 2b,$$

α is some numerical factor.

It is clear that $T(\vartheta)$ has the next behavior near the boundary of its definition domain

$$T(\vartheta) \underset{\vartheta \to 0}{\to} \vartheta^k \tag{21}$$

and

$$T(\vartheta) \underset{\vartheta \to \pi}{\to} \pi \cdot l - (\pi - \vartheta)^k. \tag{22}$$

Here l is an integer number. The solutions of the Eqs.(13-14) are graphically represented in Fig.1, 2. for some values of k and l.

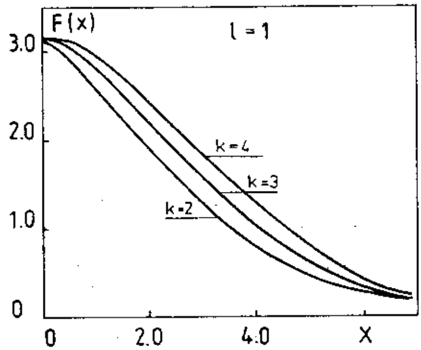
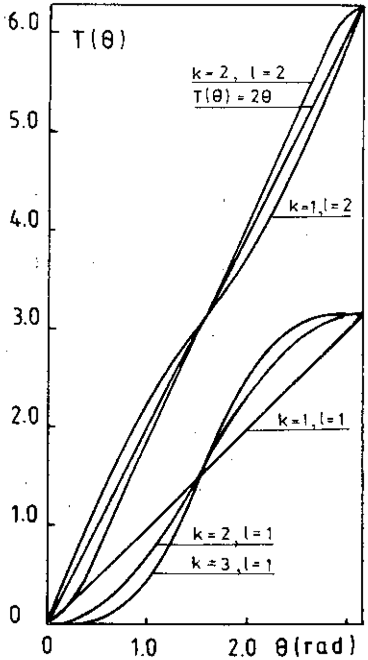


Fig.1. Solution F(x) of Eq.(13) for $l=1$ and k=2,3,4.

Fig.2. Solution $T(\theta)$ of Eq.(14) for some values of l and k.



Now consider more carefully the structure of the solitons. For that purpose let's calculate the baryon charge density

$$J_0^B(r) = -\frac{1}{24\pi^2}\,\varepsilon_{0\nu\rho\sigma}\,\mathrm{Tr}\,(L_\nu L_\rho L_\sigma)\,. \tag{23}$$

The straightforward calculation gives

$$J_0^B(r) = \frac{1}{2\pi^2 r^2}\,\sin^2 F\,(F')\,\frac{\sin T(\theta)}{\sin\theta}\,\frac{dT}{d\theta}\,\frac{d\Phi}{d\phi}\,. \tag{24}$$

Here we have used Eqs. (1) and (3) . The expression for the topological charge density, Eq.(24), is the generalization of the one for "kϕ" ansatz from /2/ and the Skyrme - Witten ansatz.

Eq.(24) immediately results in the expression for the corresponding topological charge

$$B = -\frac{1}{4\pi^2}\left[F(x) - \frac{\sin(2F)}{2}\right]\Bigg|_{F(0)}^{F(\infty)} \cdot \cos(T)\Bigg|_{T(0)}^{T(\pi)} \Phi \Bigg|_{\Phi(0)}^{\Phi(\pi)} =$$

$$= \frac{k \cdot N}{2}\left[\, 1 - \cos(\,\pi l\,)\,\right] . \tag{25}$$

One can see now that for even l we have meson- like solitons. On Fig.3 the baryon charge distributions are schematically presented in the (X,Y) - plane for solitons characterized by the number k,N ($F(0)= \pi$ N, $F(\infty) = 0$) and boundary conditions $T(0)=0$, $T(\pi)=\pi l$, $l= 2,3,4$.

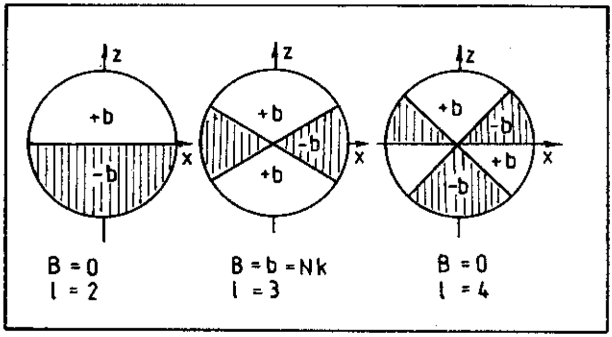


Fig.3. Baryon charge distribution on the (X,Z)-plane for solution characterized by number N,k,l.

Table 1. The classical masses calculated in the present paper with the generalized ansatz. In the last column the results with the "kϕ"-ansatz are presented for comparing with the first column.

k \ l	1	2	3	4	$T(\theta)=\theta$
1	11.605	26.358	46.332	71.169	11.605
2	22.458	45.536	73.533	106.609	24.829
3	34.585	66.701	103.081	144.321	44.369
4	47.675	89.310	134.450		70.176
5	61.569	113.119			102.206

The calculated soliton masses for N=1 and some value of k, l are represented in Table 1 in units ($\pi F_\pi / e$) where F_π=186.4 MeV and $e = 2\pi$.

In conclusion, we should like to emphasize that the soliton spectrum with "$k\Phi$"- multibaryon configurations has been extended up to $N \cdot k$ multi-baryon configurations for odd l. For example, a three-baryon state corresponds to the k=3, N=1, l=1 member of Tabl.1 with the binding energy about 7 MeV per baryon. Moreover, we have got the spectrum meson-like ($N \cdot k/2$-baryon - $N \cdot k/2$-antibaryon) configurations for even l. (See for example the k=2, N=1, l=2 case that corresponds to a two-baryon - two-antibaryon meson-like configuration with mass about 4234 MeV). Some of the obtained configurations are classically stable objects which may be seen from Tabl.1. The mass of this object is less than the sum of the masses of its baryon components. The classical "binding energy" of these states may easily be obtained by using Tabl.1

More complete analysis of the spectra will be published later.

References.

1. Rybakov Y.P. and Sanyuk V.I. Preprint NBI-HE-81-49.
2. Weigel H., Schwesinger B., Holzwart G. Phys.Lett., 1986, p.321.
3. Nikolaev V.A., Tkachev O.G. JINR Communication P4-86-515, 1986, Dubna; Nikolaev V.A. in: Proceedings of the VIII International Seminar on High Energy Physics Problems, Dubna, 1986, DI,2-86-668, Vol.1 p.78.

Received by Publishing Department
on January 30, 1989.