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ON THE INFLUENCE OF THE QUADRUPOLE PAIRING ON THE ENERGIES OF THE TWO-QUASIPARTICLE STATES IN DEFORMED NUCLEI

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The effect of the quadrupole pairing on the energies and other characteristics of 0^+ – states in deformed nuclei was studied in a number of papers, e.g. /1,2/. The monopole and quadrupole pairing affects also the two-quasiparticle energies of the states with $K^{\pi} \neq 0^+$. The energies of the two-quasiparticle states in even-even deformed nuclei were calculated in many works by taking into account the monopole pairing. The two-quasiparticle energies are strongly affected by the blocking effect /3/. Due to the latter the energies of a number of two-quasiparticle states go bellow the gap, which has been demonstrated for the first time in ref. /4/. It is of interest to study the effect of the quadrupole pairing on the two-quasiparticle states in deformed nuclei and to find the upper limit for the quadrupole pairing constant G_{τ}^{20} from the experimental energies of the two-quasiparticle states with $K \geq 3$. The present paper is devoted to the solution of this problem.

Let us take the Hamiltonian of the proton (or neutron) system in the form:

$$H_0 = \sum_{q\sigma} \{E'(q) - \lambda_{\tau}\} a^{+}_{q\sigma} a_{q\sigma} - \sum_{qq'} \{G_{\tau} + G_{\tau}^{20} f(q) f(q')\} a^{+}_{q+} a^{+}_{q-} a_{q'-} a_{q'+}, \quad (1)$$

where $E(q) = E'(q) - \frac{1}{2}[G_{\tau} + G_{\tau}^{20} f^2(q)] v_q^2$ are the single-particle energies and $q\sigma$, $\sigma = \pm 1$ are the corresponding quantum numbers; $a^{+}_{q\sigma}$ and $a_{q\sigma}$ are the nucleon creation and annihilation operators; $f(q) = \langle q \| R_2(r) Y_{20}(\theta,\varphi) \| q \rangle$.

First, we make the Bogolubov canonical transformation

$$a_{q\sigma} = u_q \alpha_{q\sigma} + \sigma v_q \alpha^{+}_{q-\sigma}$$

and then find the mean value of H_0 with respect to the quasiparticle vacuum. By using the variational principle we derive the equations

$$\left. \begin{aligned} & 2[E(q) - \lambda] u_q v_q - (u_q^2 - v_q^2) \sum_{q'} \{G_{\tau} + G_{\tau}^{20} f(q) f(q')\} u_{q'} v_{q'} = 0, \\ & u_q^2 + v_q^2 = 1 . \end{aligned} \right\} \quad (2)$$

The monopole C_{τ} and quadrupole $C_{2\tau}$ pairing correlation functions are defined in the following way:

$$C_{\tau} = G_{\tau} \sum_{q} u_q v_q, \quad C_{2\tau} = G_{\tau}^{20} \sum_{q} f(q) u_q v_q . \quad (3)$$

Further, we introduce the state-depending functions

$$\Delta_q = C_\tau + f(q)\, C_{2\tau} \,,$$

$$u_q^2 = \frac{1}{2}\left\{1 + \frac{E(q)-\lambda}{\tilde{\mathcal{E}}_q}\right\}, \quad v_q^2 = \frac{1}{2}\left\{1 - \frac{E(q)-\lambda}{\tilde{\mathcal{E}}_q}\right\} \tag{4}$$

and using eq. (2) we find the value for the quasiparticle energy

$$\tilde{\mathcal{E}}_q = \left[\, \Delta_q^2 + \left(E(q)-\lambda\right)^2 \,\right]^{1/2}. \tag{5}$$

After some simple transformations we obtain the following system of equations

$$1 = \frac{G_\tau}{2C_\tau} \sum_q \frac{C_\tau + f(q)\, C_{2\tau}}{\tilde{\mathcal{E}}_q}\,, \tag{6}$$

$$1 = \frac{G^{20}_\tau}{2C_{2\tau}} \sum_q f(q) \frac{C_\tau + f(q)\, C_{2\tau}}{\tilde{\mathcal{E}}_q}\,, \tag{6^I}$$

$$N = \sum_q \left\{1 - \frac{E(q)-\lambda}{\tilde{\mathcal{E}}_q}\right\}, \tag{6^{II}}$$

where N is the number of nucleons. Equations (6)-(6^{II}) are identical to the equations derived in ref./1/ . The ground-state energy of the system consisting of an even number of nucleons and its wave-function has the form:

$$\mathcal{E}_0 = \sum_q 2E(q)\, v_q^2 - \frac{C_\tau^2}{G_\tau} - \frac{C_{2\tau}^2}{G^{20}_\tau} = \tag{7}$$

$$= \sum_q 2E(q)\, v_q^2 - \frac{1}{4}\sum_{qq'} \left[\, G_\tau + G^{20}_\tau f(q) f(q')\right] \frac{\Delta_q\, \Delta_{q'}}{\tilde{\mathcal{E}}_q\, \tilde{\mathcal{E}}_{q'}}\,,$$

$$\Psi_{00} = \prod_q \left(u_q + v_q\, a^+_{q-}\, a^+_{q+}\right) \Psi_0^0\,, \tag{7^I}$$

where $a_{q\sigma}\,\Psi_0^0 = 0$.

The values of C_n and C_{2n} (for the neutron system of ^{168}Er) given in table 1 are calculated by the values of the constants G_n and G_n^{20} obtained from the experimental pairing energies.

Table I

The influence of the quadrupole pairing on the energies of the two-quasiparticle states of ^{168}Er nucleus

K^π	Configuration	Energy, MeV						
		Experiment	without blocking			with blocking		
			$G_\tau^{20}=0$	$G_\tau^{20}=0.5\,\varkappa_o^{20}$ without nondiag. terms	$G_\tau^{20}=0.5\,\varkappa_o^{20}$ with non-diag. terms	$G_\tau^{20}=0$	$G_\tau^{20}=0.5\,\varkappa_o^{20}$	$G_\tau^{20}=0.75\,\varkappa_o^{20}$
4^-	nn 633↑ + 521↓	1.09	1.71	1.72	1.79	I.10	I.06	I.04
6^-	nn 633↑ + 512↑	1.77	1.90	1.90	1.72	1.63	1.53	1.40
5^+	nn 513↓ + 512↑	2.27	2.36	2.16	2.20	2.14	2.02	1.89
4^-	pp 523↑ + 411↓	1.91	2.23	2.18	2.22	1.51	1.47	1.46
	G_n , MeV		0.122	0.113	0.113	0.122	0.113	0.102
	C_n , MeV		0.85	0.71	0.71	0.85	0.71	0.53
	$_n$, MeV		-5.21	-5.14	-5.12	-5.21	-5.14	-5.05
	C_{2n}, MeV/barn		0	3.2	3.7	0	3.2	6.7

It is seen from table 1 that when the constant G_n^{20} increases, parameter C_{2n} increases too, C_n decreases and the chemical potential λ_n remains almost unchanged. A similar tendency is observed in other nuclei under investigation.

The description of the 0^+ - states in deformed nuclei is given in ref. /5/, where the quadrupole particle-particle interaction with both the diagonal $f(q)$ and nondiagonal $f(qq')$ matrix elements is taken into account. The condition for elimination of the spurious solutions of the RPA-equations leads to two equations determining the functions C_τ and $C_{2\tau}$. The first one coincides with eq. (6) and the second one is:

$$1 = G_\tau^{20}\left\{\sum_q \frac{f(q)\,C_\tau}{2C_{2\tau}\tilde{\mathcal{E}}_q} + \sum_{qq'} \frac{\left[f(qq')(u_q u_{q'} + v_q v_{q'})\right]^2}{\tilde{\mathcal{E}}_q + \tilde{\mathcal{E}}_{q'}}\right\}. \qquad (8)$$

If one neglects the nondiagonal matrix elements $f(qq')$, eq. (8) transforms into eq. (6'). Our calculations of the two-quasiparticle energies $\tilde{\mathcal{E}}_q + \tilde{\mathcal{E}}_{q'}$ are given in table 1. As can be seen from the table, the presence of the nondiagonal matrix elements leads to unsignificant changes in the quantities $\tilde{\mathcal{E}}_q + \tilde{\mathcal{E}}_{q'}$: even for the greatest values of G_τ^{20} the change is of an order of

100 keV. Thus, the nondiagonal matrix elements should be neglected and one should solve the system (6), (6^{I}) and (6^{II}).

In calculating the pairing and two-quasiparticle energies one should take into account the blocking effect. So let us write down equations for the two-quasiparticle state energies with blocking. For this, we first calculate the mean value of H_o with respect to the state

$$\alpha^{+}_{q_1\sigma_1} \alpha^{+}_{q_2\sigma_2} \Psi_{oo} \tag{9}$$

and then use the variational principle. In this way, for the quantities

$$C_\tau(q_1q_2) = G_\tau \sum_{q\neq q_1,q_2} u_q v_q \ , \quad C_{2\tau}(q_1 q_2) = \sum_{q\neq q_1,q_2} f(q) u_q v_q \ , \tag{10}$$

$$\Delta_q(q_1q_2) = C_\tau(q_1q_2) + f(q) C_{2\tau}(q_1q_2), \ \tilde{\mathcal{E}}_q(q_1q_2) = \left[\Delta^2_q(q_1q_2) + (E(q)-\lambda)^2\right]^{1/2}$$

we obtain the following equations:

$$1 = \frac{G_\tau}{2} \sum_{q\neq q_1,q_2} \frac{C_\tau(q_1q_2) + f(q)\, C_{2\tau}(q_1q_2)}{C_\tau(q_1q_2)\, \tilde{\mathcal{E}}_q(q_1q_2)} \ , \tag{11}$$

$$1 = \frac{G^{20}_\tau}{2} \sum_{q\neq q_1,q_2} f(q) \frac{C_\tau(q_1q_2) + f(q)\, C_{2\tau}(q_1q_2)}{C_{2\tau}(q_1q_2)\, \tilde{\mathcal{E}}_q(q_1q_2)} \ , \tag{11^{I}}$$

$$N = 2 + \sum_{q\neq q_1,q_2} \left\{ 1 - \frac{E(q)-\lambda}{\tilde{\mathcal{E}}_q(q_1q_2)} \right\} . \tag{11^{II}}$$

The two-quasiparticle energies take now the form

$$\mathcal{E}(q_1q_2) - \mathcal{E}_o \ , \tag{12}$$

where

$$\mathcal{E}(q_1q_2) = E(q_1) + E(q_2) + 2\sum_{q\neq q_1,q_2} E(q) v^2_q(q_1q_2) - \frac{C^2_\tau(q_1q_2)}{G_\tau} - \frac{C^2_{2\tau}(q_1q_2)}{G^{20}_\tau} . \tag{13}$$

Obviously, if $C_{2\tau}=0$, eqs. (11)-(11^{II}) transform into the monopole pairing equations with blocking, see e.g. /6/.

The energies of some two-quasiparticle states of ^{168}Er, ^{172}Yb and ^{178,180}Hf shown in tables 1 and 2 have been calculated with the monopole and quadrupole pairing taking account of the blocking effect.

Table 2

The influence of the quadrupole pairing on the energies of the two-quasiparticle states

	K^π	Configuration	Energy, MeV			
			Experiment	Calculation with blocking		
				$G_\tau^{20}=0$	$G_\tau^{20}=0.25\,\varkappa_0^{20}$	$G_\tau^{20}=0.5\,\varkappa_0^{20}$
^{172}Yb	3^+	nn 521↓+ 512↑	1.17	1.32	1.27	1.22
	6^-	nn 512↑+ 633↑	1.55	1.36	1.32	1.26
	4^-	nn 633↑+ 521↓	1.64	1.69	1.69	1.68
^{178}Hf	8^-	pp 404↓+ 514↑	1.15	1.19	1.25	1.38
^{180}Hf	8^-	pp 404↓+ 514↑	1.14	1.17	1.22	1.35

For the monopole and quadrupole pairing the blocking effect turned out to be as large for the states having one quasiparticle on the Fermi level and the other on the following level as for the monopole pairing. Since the constants of the quadrupole pairing G_τ^{20} or quadrupole particle-particle interaction and the constants of the quadrupole particle-hole interaction have the same dimension, the quantities G_τ^{20} can be expressed through the isoscalar constant $\varkappa_0^{20}$ of the particle-hole interaction with $\lambda\mu = 20$. It is seen from the tables that with increasing G_τ^{20} and decreasing G_τ (in order to conserve the pairing energy) the discrepancy between the calculated and experimental values of the energies of two-quasiparticle states increases. The same occurs for the energies of two-quasiparticle states in other nuclei.

According to ref. /7/ the states $K_i^\pi = 4_1^-$ and 4_2^- in nucleus ^{168}Er with energies 1.094 MeV and 1.905 MeV, respectively, are not purely two-quasiparticle ones. It is observed that to the leading neutron component nn 633↑+ 521↓ of the state 4_1^- the proton pp 523↑+ 411↓ component is admixed; in the same way, a neutron component is mixed to the leading proton component of the state 4_2^- . The RPA-calculations with multipole-multipole force of the type $\lambda\mu = 54$ give for the state 4_1^- the following results: for $\varkappa_0^{54} = 0.015$ Fm^2/MeV – $E_{4_1^-} = 1.05$ MeV and its structure is nn 633↑+ 521↓ 93%, pp 523↑+ 411↓ 6 %; for $\varkappa_0^{54} = 0.018$ Fm^2/MeV – $E_{4_1^-} = 1.0$ MeV and the structure is nn 633↑+ 521↓ 86%, pp 523↑+ 411↓ 12%; for $\varkappa_0^{54}$ = 0.021 Fm^2/MeV – $E_{4_1^-}$ = 0.9 MeV and the structure is nn 633↑+ 521↓ 75%, pp 523↑+ 411↓ 22%.

In the second state 4_2^- the situation is just opposite: here the leading component is the proton one pp 523↑+ 411↓ ; the neutron component is slightly admixed to it. About 97-98% from the strength of these configurations is concentrated in the $4_1^-, 4_2^-$ levels. The energy of the 4_2^- state is considerably bellow the experimental value because of the rough description of the single-particle states. As follows from the experimental data /7/ and numerical calculations the multipole-multipole interaction with $\lambda = 2$ affects the structure of the $K^\pi = 4^-$ states, if the neutron and the proton poles are near to each other. Such a 4^--state has a dominating two-quasiparticle component and its energy lies bellow the pole not more than 0.1 MeV. As a rule the energies of such states can be calculated within the independent quasiparticle model with monopole pairing force.

The comparison of the energies of two-quasiparticle states with the corresponding experimental data allows one to conclude that the constant of the quadrupole pairing or that of the particle-particle interaction with $\lambda\mu = 20$ should not exceed the following value:

$$G_\tau^{20} < 0.5\, \varkappa_0^{20} \, . \tag{14}$$

Since the quadrupole pairing with $G_\tau^{20} < 0.5\, \varkappa_0^{20}$ does not affect essentially the energies of two-quasiparticle states, the poles of the secular equation in the RPA for $K^\pi \neq 0^+$-states can be calculated disregarding the quadrupole pairing. The values of the constant G_τ^{20} in the interval from zero to 0.5 $\varkappa_0^{20}$ can be established more exactly in the process of description of the 0^+ – states in deformed nuclei within the quasiparticle-phonon nuclear model /8,9/ .

Based on the above investigations we can conclude that in calculations within the quasiparticle-phonon nuclear model with inclusion of particle-hole and particle-particle interactions of the energies and wave functions of nonrotational states with $K^\pi \neq 0^+$ in even-even deformed nuclei one can neglect the quadrupole pairing and take into account only the monopole pairing in the proton and neutron systems.

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