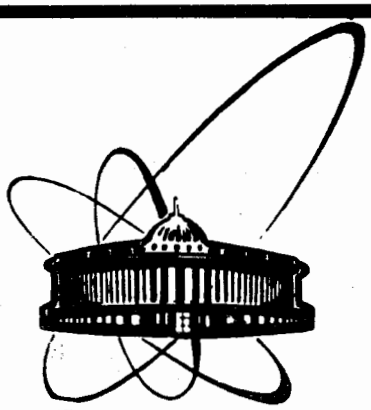

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UNITARY SCATTERING THEORY OF LOW-ENERGY PIONS BY LIGHT NUCLEI: FORMALISM

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Унитарная теория рассеяния пионов
низких энергий на легких ядрах: формализм

Обсуждается унитарный подход к описанию пион-ядерного рассеяния, основанный на эволюционном по константе связи методе. Основные уравнения подхода формулируются непосредственно для расчета пион-ядерных фаз рассеяния. Дается новый вывод унитарного ряда многократного рассеяния, позволяющий представить формализм унитарной теории в виде, аналогичном стандартной теории Ватсона. Рассмотрена итерационная процедура решения основных уравнений, быстросходящегося при низких энергиях /ниже 70 МэВ/. Обсуждается роль эффектов второго порядка в рассеянии низкоэнергетических пионов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Khankhasayev M.Kh. E4-89-443

Unitary Scattering Theory of Low-Energy
Pions by Light Nuclei:Formalism

The unitary approach to the decription of pion-nucleus scattering (UST) based on the method of evolution of the system with respect to the coupling constant is presented. The basic equations are formulated for the direct calculation of the pion-nucleus phase shifts. A new derivation of the unitary multiple scattering series which makes the UST formalism similar to the standard Watson theory is given. An iterative solution of these equations rapidly convergent at low energies (below 70 MeV) is considered. The role of the second-order effects in the pion-nucleus scattering at low energies are discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. Introduction

The unitary scattering theory (UST) of a distinguishable projectile (pion) on a nucleus is based on the Kirznits method of evolution system not with the time as is customary, but with the coupling constant [1]. The UST approach for the description of the pion-nucleus interaction has been developed in several earlier papers [2-5]. We rest heavily on the results reported there, and present here briefly the general formalism. We shall discuss in detail only those aspects of the theory which are treated in a new way.

The evolution method being applied to the problem of scattering of a pion by a nucleus allows one to construct a multiple scattering series conserving the unitarity of the scattering matrix at each step of successive approximations. In this paper we present a new derivation of the unitary multiple scattering series which makes the UST formalism similar to the standard Watson theory[6]. A thorough treatment of the pion absorption effect has been performed in refs.[4,5]. In practical calculations we approximate the absorption correction to the pion-nucleus phase shifts by the ρ^2-term supposing the dominance of two-nucleon mechanism for the pion absorption.

At low energies the iterative series is rapidly converging, and only the lowest iterations need to be taken into account. The range of convergence is established by the microscopical calculations of the second-order corrections. The consistency of the theory with the unitarity provides a correct separation of the potential effects from the non-potential (i.e. true absorption) effects.

Our paper is organized as follows. In sects.2 and 3 we discuss the basic equations of the UST approach. A new derivation of the multiple scattering series is given in sect.4. In sect.5 we present the expressions for the pion-nucleus phase shifts

calculated in the first- and second-order approximations. The convergence of the iterative scheme is investigated. The inclusion of pion absorption is discussed in sect.6. In sect.7 the procedure of taking into account the Coulomb effects is presented. In the concluding section 8 we summarise the main results of our study.

2. Basic equations

We assume for a moment that the π-nucleus interaction has a potential nature*, and write the Hamiltonian in the form

$$H = K_\pi + H_A + \lambda V, \quad V = \sum_{i=1}^{A} v_{\pi N}^i, \qquad (2.1)$$

where H_A is the Hamiltonian of a nucleus, K_π is the kinetic energy operator of the pion, $v_{\pi N}^i$ is the pairwise potential of the πN-interaction, and λ plays the role of the πN-coupling constant. We assume to have solved the solution of the nuclear many-body problem with the channel Hamiltonian

$$h = K_\pi + H_A . \qquad (2.2)$$

Let us consider the evolution of the system as λ varies from 0 to the real value $\lambda = 1$. The parameter λ is here introduced formally and is allowed to rise to unity at the end of the calculations.

A central part in the present approach is played by the matrix elements $V_{\mu\nu} = \langle\mu| V |\nu\rangle$ of the interaction potential taken between the eigenfunctions $|\mu\rangle$, $|\nu\rangle$, etc., of the Hamiltonian (2.1). In terms of these matrix elements the observables such as phase shifts are expressed.

It is convenient to introduce the eigenfunctions of the channel Hamiltonian (2.2): $|\psi_\mu\rangle$, $|\psi_\nu\rangle$, etc. In this space of the asymptotic, λ-independent states, the equation for the π-nucleus scattering T-matrix takes the form[3,4]

$$\frac{d}{d\lambda} T(E,\lambda) = V(\lambda) - 2\pi i\, T(E,\lambda)\, \delta(E - h)\, V(\lambda) \qquad (2.3)$$

*The problem of taking into account the pion absorption channel is discussed in sect. 6.

with the boundary condition $T(E,\lambda=0)=0$. The Hermitian operator $V(\lambda)$ is defined so that

$$V_{\mu\nu} = \langle\mu| V|\nu\rangle \equiv \langle\psi_\mu| V(\lambda) |\psi_\nu\rangle, \qquad (2.4)$$

or equivalently

$$V(\lambda) = \Omega^{(+)^+}(\lambda)\, V\, \Omega^{(+)}(\lambda),$$

where $\Omega^{(+)}$ is the Möller operator for the given Hamiltonian H: $|\mu\rangle = \Omega^{(+)}|\psi_\mu\rangle$. The evolution equations for $\Omega^{(+)}(\lambda)$ and $V(\lambda)$ have the form

$$\frac{d}{d\lambda} \Omega^{(+)}(\lambda) = \sum_\sigma \Omega^{(+)}(\lambda)\, G^{(+)}(E_\sigma)\, V(\lambda)\, \hat{P}_\sigma \qquad (2.5)$$

and

$$\frac{d}{d\lambda} V(\lambda) = \sum_\sigma [\, \hat{P}_\sigma V(\lambda)\, G^{(-)}(E_\sigma)\, V(\lambda) + \text{h.c.}\,] \qquad (2.6)$$

with the obvious boundary conditions $\Omega^{(+)}(\lambda=0)=1$ and $V(\lambda=0)=V$. Here $G^{(\pm)}$ is the Green function of the channel Hamiltonian h

$$G^{(\pm)}(E) = (E - h \pm i\delta)^{-1}, \qquad (2.7)$$

and $\hat{P}_\sigma \equiv |\psi_\sigma\rangle\langle\psi_\sigma|$ is the operator of projection onto the given state of h. Thus, $\hat{P}_\sigma$ in (2.6) fixes the space of the state vectors from which the evolution of the system starts.

Eqs. (2.3), (2.5) and (2.6) are the basic equations of the approach providing a complete description of the pion-nucleus scattering (both the elastic and inelastic).

3. Elastic scattering: optical model

The standard procedure to treat the elastic scattering is to introduce nuclear ground state and excited state projection operators, $\hat{P}_0 = |0\rangle\langle 0|$ and $\hat{Q} = 1 - \hat{P}_0$, into the basic equations. It can be shown[3,4] that the submatrix $T_0 \equiv \hat{P}_0 T \hat{P}_0$ describing the elastic scattering satisfies the equation

$$\frac{d}{d\lambda} T_0(E,\lambda) = U_0(E,\lambda) - 2\pi i\, T_0(E,\lambda)\, \hat{P}_0 \delta(E - h)\, U_0(E,\lambda). \qquad (3.1)$$

The effective energy-dependent operator $U_0(E,\lambda)$ is determined by the system of equations

$$U_0(E,\lambda) = V(\lambda) - 2\pi i\, K(E,\lambda)\, \hat{Q}\, \delta(E - h)\, V(\lambda) \qquad (3.2)$$

$$\frac{d}{d\lambda} K(E,\lambda) = U_0(E,\lambda) + 2\pi i\, \hat{P}_0\, \delta(E - h)\, K(E,\lambda) \qquad (3.3)$$

with the boundary condition for the auxiliary operator $K(E,\lambda)$: $K(E,\lambda=0)=0$.

In the matrix representation eq. (3.1) is written in the form

$$\frac{d}{d\lambda}\langle k',0|\, T_0(E,\lambda)\,|k,0\rangle = \langle k',0|\, U\,(E_0,\lambda)\,|k,0\rangle - 2\pi i \int \frac{dk''}{(2\pi)^3}\langle k',0|\, T_0(E,\lambda)\,|k'',0\rangle\,\delta(E - E_0(k''))\,\langle k'',0|\, U_0(E,\lambda)\,|k,0\rangle . \tag{3.4}$$

Here k and k′ are the pion momenta before and after the collision, $E = E_0(k) = \omega_\pi(k) + \omega_A(k)$ is the collision energy in the pion-nucleus c.m. , measured from the ground state of the nucleus, $\omega_\pi(k) = (k^2 + m_\pi^2)^{1/2}$, $\omega_A(k) = (k^2 + M_A^2)^{1/2}$, and m_π and M_A are the masses of the pion and the nucleus, respectively. We have introduced here a more detailed notation for the asymptotic pion-nucleus states: $|\psi\rangle = |n,k\rangle$, where n denotes the quantum numbers of the states of the nucleus (n=0 corresponds to the g.s.) We take into account relativistic effects at the kinematical level.

For the partial-wave phase shifts of the pion-nucleus scattering eq.(3.4) reads*

$$\frac{d}{d\lambda}\,\delta(k,\lambda) = -\pi\,\varepsilon_{\pi A}(k)\,\langle k',0|\, U_0(E,\lambda)\,|k,0\rangle, \tag{3.5}$$

where $\varepsilon_{\pi A}(k) = k^2/[2\pi^2\, dE_0(k)/dk\,]$ is the level density of the scattering states. In the non-relativistic case $\varepsilon_{\pi A}(k)=M_{\pi A}k/2\pi^2$, where $M_{\pi A}$ is the reduced mass of the π-nucleus system.

The problem of finding the π-nucleus phase shifts has been reduced to the calculation of the matrix element $\langle k',0|U_0(E,\lambda)|k,0\rangle$. In the present approach the non-Hermitian operator $U_0(E,\lambda)$ plays the role of an optical potential. In the low-energy limit, when the pion energy is less than the energy of the first excited state of the nucleus, the operator $U_0(E,\lambda)=V(\lambda)$

*In relations, like (3.5), which have a general nature, we do not indicate explicitly the angular momentum, spin, etc., indices.

is Hermitian, and the scattering T - matrix satisfies the two - body unitarity condition.

3.1. The iterative solution

The simplest method of solving the system of eqs. (3.2) and (3.3) is the expansion of $U_0(E,\lambda)$ in powers of the Hermitian operator $V(\lambda)$. For the phase shifts (3.5) we obtain

$$\delta(k) = -\pi\,\varepsilon_{\pi A}(k)\int_0^1 d\lambda\,\Big\{\langle k',0|V(\lambda)|k,0\rangle - 2\pi i \sum_{n>0}\int \frac{dk''}{(2\pi)^3}\int_0^\lambda d\lambda_1\langle k',0|V(\lambda_1)|k'',n\rangle\,\delta(E_0(k) - E_n(k''))\,\langle k'',n|V(\lambda)|k,0\rangle + \ldots\Big\}, \tag{3.6}$$

where $E_n(k) = E_0(k) + \varepsilon_n$, ε_n is the energy of the first excited state of the nucleus,measured from the ground state. At low energies, as we shall show below, it is sufficient to consider only the terms linear and quadratic in $V(\lambda)$.

4. Multiple scattering series

In the standard Watson theory[6], the multiple scattering series is constructed on the scattering T - matrix. In the present approach an analogous expansion is derived for the many-body Hermitian operator $V(\lambda)$ (2.4). In this section we present a new derivation (cf. ref.[2]) of the series for $V(\lambda)$.

4.1. Pion-bound nucleon v-matrix

Let us introduce an auxiliary operator $v^i(\lambda)$ which is determined by the equation

$$\frac{d}{d\lambda}\,v^i(\lambda) = \sum_\sigma \Big[\hat P_\sigma\, v^i(\lambda)\,\hat A\, G^{(-)}(E_\sigma)\, v^i(\lambda) + \text{h.c.}\Big]. \tag{4.1}$$

Here, $\hat A$ is the operator of antisymmetrization of nuclear wave functions,and $\hat P_\sigma = |\psi_\sigma\rangle\langle\psi_\sigma|$ is the operator of projection onto the given state of the channel Hamiltonian h (2.2). Formally, eq.(4.1) coincides with (2.6), but the boundary condition for $v^i(\lambda)$ is $v^i(\lambda=0)=v^i_{\pi N}$. The operator $v^i(\lambda)$ is analogous to the Watson operator τ_i that describes the interaction of a pion with a bound nucleon.

Representing $V(\lambda)$ in the form

$$V(\lambda) = \sum_{i=1}^{A} v^i(\lambda) + V_c(\lambda), \qquad (4.2)$$

where $V_c(\lambda)$ is some connected part, and substituting it into eq.(2.6) one can obtain an inhomogeneous equation for V_c:

$$\frac{d}{d\lambda} V_c(\lambda) = \sum_\sigma \Big\{ \hat{P}_\sigma \Big[\sum_{i\neq j} v^i(\lambda)\, \hat{A}\, G^{(-)}(E_\sigma)\, v^j(\lambda) + \qquad (4.3)$$
$$+ \sum_i \Big(v^i(\lambda)\, G^{(-)}(E_\sigma)\, \hat{A}\, V_c(\lambda) + V_c(\lambda)\, G^{(-)}(E_\sigma)\, \hat{A}\, v^i(\lambda) \Big) +$$
$$+ V_c(\lambda)\, G^{(-)}(E_\sigma)\, V_c(\lambda) \Big] + \text{h.c.} \Big\}$$

with the boundary condition $V_c(\lambda=0)=0$.

The very form of this equation suggests the iterative procedure for its solution. Thus, we obtain

$$V(\lambda) = \sum_i v^i(\lambda) + \sum_{i\neq j} \Big(\sum_\sigma \hat{P}_\sigma \int_0^\lambda d\lambda_1\, v^i(\lambda_1)\, \hat{A}\, G^{(-)}(E_\sigma)\, v^j(\lambda_1) + \text{h.c.} \Big) + \ldots . \qquad (4.4)$$

It is important that at each stage of this iteration procedure $V(\lambda)$ remains to be Hermitian. Therefore, the T-matrix is unitary at each step of the successive approximations.

4.2. Free two-body u-matrix

The second step of the derivation of the multiple-scattering series for $V(\lambda)$ consists in its expression in terms of the matrix $u^i(\lambda)$ describing the pion interaction with a free nucleon. This matrix is determined by the equation /3,4/

$$\frac{d}{d\lambda} u^i(\lambda) = \sum_s \Big[\hat{P}_s\, u^i(\lambda)\, g^{(-)}(E_s)\, u^i(\lambda) + \text{h.c.} \Big], \qquad (4.5)$$

where $\hat{P}_s = |\chi_s\rangle\langle\chi_s|$ is the operator of projection onto the eigenstates of the free Hamiltonian $H_0 = K_\pi + K_A$, K_A is the kinetic-energy operator of the nuclear nucleons, $g^{(\pm)}(E)$ is the free Green function

$$g^{(\pm)}(E) = (E - H_0 \pm i\delta)^{-1}. \qquad (4.6)$$

The boundary condition for $u^i(\lambda)$: $u^i(\lambda=0) = v^i_{\pi N}$ is the same as for $v^i(\lambda)$ in eq. (4.1). In eq.(4.5) the matrix u^i acts in the $(A+1)$-particle space.

To derive the expansion of $v^i(\lambda)$ in terms of $u^i(\lambda)$ we write $v^i(\lambda)$ in the form (cf. with (4.2))

$$v^i(\lambda) = u^i(\lambda) + v^i_b(\lambda), \qquad (4.7)$$

where $v^i_b(\lambda)$ is the correction caused by the binding of the nucleon in the nucleus. Substituting (4.7) into (4.1) we obtain the equation for $v^i_b(\lambda)$ in a form similar to eq. (4.3). The iterative solution of this equation in powers of $u^i(\lambda)$ yields the desired expansion :

$$v^i(\lambda) = u^i(\lambda) + \int_0^\lambda d\lambda_1 \Big\{ \Big[\sum_\sigma \hat{P}_\sigma\, u^i(\lambda_1)\, \hat{A}\, G^{(-)}(E_\sigma)\, u^i(\lambda_1) - \sum_s \hat{P}_s\, u^i(\lambda_1)\, g^{(-)}(E_s)\, u^i(\lambda_1) \Big] + \text{h.c.} \Big\}. \qquad (4.8)$$

Finally, one can obtain the multiple scattering series for $V(\lambda)$ in terms of $u^i(\lambda)$ by substituting (4.8) into (4.4). The first two terms of this series can be written in the form

$$V(\lambda) = V^{(1)}(\lambda) + V^{(2)}(\lambda), \quad V^{(2)}(\lambda) = V_1^{(2)}(\lambda) + V_1^{(2)}(\lambda), \qquad (4.9)$$

where

$$V^{(1)}(\lambda) = \sum_{i=1}^{A} u^i(\lambda), \qquad (4.10)$$

$$V_1^{(2)}(\lambda) = \sum_{i\neq j}^{A} \int_0^\lambda d\lambda_1 \Big[\sum_s \hat{P}_s\, u^i(\lambda_1)\, g^{(-)}(E_s)\, u^j(\lambda_1) + \text{h.c.} \Big], \qquad (4.11)$$

$$V_2^{(2)}(\lambda) = \sum_{i,j=1}^{A} \int_0^\lambda d\lambda_1 \Big\{ \Big[\sum_s \hat{P}_s\, u^i(\lambda_1)\, g^{(-)}(E_s)\, u^j(\lambda_1) - \sum_\sigma \hat{P}_\sigma\, u^i(\lambda_1)\, G^{(-)}(E_\sigma)\, \hat{A}\, u^j(\lambda_1) \Big] + \text{h.c.} \Big\}. \qquad (4.12)$$

Here $\hat{P}_\sigma$ and $\hat{P}_s$ are the operators of projection onto the eigenstates of the Hamiltonian H and h, respectively. The terms $V^{(1)}$ and $V_1^{(2)}$ describe the pion scattering by two nucleons of the nucleus in the impulse approximation. The corrections to the impulse approximation caused by the nuclear binding and the Pauli exclusion principle are included in the second-order term $V^{(2)}$.

5. Pion-nucleus phase shifts

Substituting (4.9) into (3.6) we obtain expansion for the π-nucleus phase shifts. For the real parts of the π-nucleus phase shifts the first two iterations are

$$\mathrm{Re}\,\delta(k) = \delta^{(1)}(k) + \delta^{(2)}_1(k) + \delta^{(2)}_2(k), \qquad (5.1)$$

where the first-order approximation $\delta^{(1)}$ reads

$$\delta^{(1)}(k) = -\pi\,\varepsilon_{\pi A}(k) \int_0^1 d\lambda\, \langle k',0|\sum_{i=1}^{A} u^i(\lambda)\,|k,0\rangle, \qquad (5.2)$$

and the second-order corrections $\delta^{(2)}_1$ and $\delta^{(2)}_2$ are

$$\delta^{(2)}_{1,2}(k) = -\pi\,\varepsilon_{\pi A}(k) \int_0^1 d\lambda\, \langle k',0|\, V^{(2)}_{1,2}(\lambda)\,|\,k,0\rangle. \qquad (5.3)$$

The operators $V^{(2)}_{1,2}$ are defined in (4.11) and (4.12).

The imaginary parts of the π-nucleus phase shifts arise in the second-order approximation (see (3.6))

$$\mathrm{Im}\,\delta^{(2)}(k) = 2\pi^2\,\varepsilon_{\pi A}(k) \int_0^1 d\lambda \int_0^\lambda d\lambda_1 \langle k',0|\, V^{(1)}(\lambda_1)\,\hat{Q}\,\delta(E-h)\times V^{(1)}(\lambda)\,|\,k,0\rangle, \qquad (5.4)$$

where $V^{(1)}$ is defined in (4.10), and $\hat{Q}$ is the operator of projection onto the excited states of the nucleus which provides a correct threshold behaviour of the inelasticity parameters.

The structure of the matrix elements in (5.2) and (5.3) is analogous to that of the expressions for the first- and second-order optical potentials (see,e.g. /7,8/).

5.1 First-order approximation

To evaluate the ground state expectation value in (5.2) we use the so-called "semi-factored" approximation following ref./7/. In this approximation the expression for $\delta^{(1)}$ reads/3/

$$\delta^{(1)}(k) = -\pi\,\varepsilon_{\pi A}(k)\,\rho_{00}(q) \int_0^1 d\lambda\,\bar{u}(k',k;\lambda), \qquad (5.5)$$

where $\rho_{00}(q)$ is the nuclear (matter) form factor, and $\bar{u}$ is the free pion-nucleon u-matrix averaged with respect to the single-particle density $F^{(1)}_{00}(p,p)$

$$\bar{u}(k',k;\lambda) = \int \frac{dp}{(2\pi)^3}\, F^{(1)}_{00}(p,p)\ \langle k,p+p_0|u(\lambda)|k',p+p_0-q\rangle. \qquad (5.6)$$

Here, $p_0 = -k/A + [(A-1)/2A]\cdot q$, $q = k'-k$ is the transfer momentum.

In the first-order approximation the π-nucleon phase shifts are purely real. Hence,we must consider the second-order terms in the series.

5.2. Second-order approximation: real parts of the phase shifts

The real parts of the second-order term contain two terms (5.3). The first term $\delta^{(2)}_1$ describes the pion rescattering by two nuclear nucleons in the impulse approximation, and the second one $\delta^{(2)}_2$ contains the nuclear binding effects.

The correction $\delta^{(2)}_1$ can be calculated using the approximation of factorization/3/

$$\delta^{(2)}_1(k) = -2\pi A(A-1)\varepsilon_{\pi A}(k) \int \frac{dk''}{(2\pi)^3}\ \hat{P}\left(\frac{1}{E_0(k) - E_0(k'')}\right) C_{00}(q',q'')\times \int_0^1 d\lambda \int_0^\lambda d\lambda_1\ \bar{u}(k,k'';\lambda_1)\ \bar{u}(k'',k';\lambda_1), \qquad (5.7)$$

where $q' = k'' - k$, $q = k'' - k'$, the operator $\hat{P}$ denotes the principle value integration, $\bar{u}$ is defined in (5.6),and

$$C_{00}(q_1,q_2) = \langle 0|\exp(iq_1r_1 + iq_2r_2)|0\rangle \qquad (5.8)$$

is the two-body correlation function. An accuracy of the approximation (3.7) can be estimated as to be of about 20%/3/.

The expression for $\delta^{(2)}_2$ like (5.7) is obtained if both the approximations of factorization and completeness are used

$$\delta^{(2)}_2(k) = -2\pi\ A(A-1)\varepsilon_{\pi A}(k) \int \frac{dk''}{(2\pi)^3}\ \hat{P}\left(\frac{1}{E_0(k) - E_0(k'') - \Delta} - \frac{1}{E_0(k) - E_0(k'')}\right)\cdot\left[\frac{1}{A-1}\,\rho_{00}(q) + C_{00}(q',q'') - \frac{A}{A-1}\,\rho_{00}(q')\rho_{00}(q'')\right]\times \int_0^1 d\lambda \int_0^\lambda d\lambda_1 u(k,k'';\lambda_1)u(k'',k';\lambda_1). \qquad (5.9)$$

Here, the parameter Δ is a certain mean excitation energy of a nucleus measured from the ground state. In the derivation of (5.9)

we have assumed that all excited states are degenerate, i.e. $E_n=E_o+\Delta$ for $n > 0$. The parameter Δ is a free parameter of the theory. In general it depends on the nucleus and the energy of the projectile.

5.2.1. On the binding effects in $Re\delta_{\pi A}$. We shall show here that the correction $\delta_2^{(2)}$ is small at least at low energies, and a good approximation for $Re\ \delta_{\pi A}$ is given by the sum

$$Re\ \delta_{\pi A}(k) = \delta^{(1)}(k) + \delta_1^{(2)}(k). \qquad (5.10)$$

From (5.9) it follows that $\delta_2^{(2)}$ vanishes as $\Delta \to 0$. In other words, one can say that $\delta_2^{(2)}$ disappears in the static limit of the theory, i.e. as $m_\pi/M \to 0$ (see, ref/3/). There is also an additional reason which provides the smallness of this correction. Let us suppose that the functions ρ_{oo}, C_{oo} and $\bar{u}$ in the integrand of (5.9) are independent on the variable k''. For this case we obtain

$$\delta_2^{(2)} \sim \int_0^\infty dk''^2\, k'' \left(\frac{1}{k^2 - k''^2 - k_0^2} - \frac{1}{k^2 - k''^2} \right) = 0$$

for an arbitrary values of $k_0^2 \equiv 2 M_{\pi A} \Delta$ ($M_{\pi A}$ is the reduced mass of the π-nucleus system). Hence, the main contribution in the integral (5.9) stems from the domains where integrand functions vary substantially. For ρ_{oo} and C_{oo} these domains are determined approximately as

$$k'' \geq 1/(\lambda_1)^{1/2},\ \lambda_1 \equiv a^2/2,$$

where a is the nuclear size parameter, and for $\bar{u}$

$$k'' \geq 1/(\lambda_2)^{1/2},\ \lambda_2 \equiv 2\alpha,$$

where α is the parameter determining the range of the πN-interaction $\alpha \approx 0.2$ fm^2. To estimate the off-shell behaviour of the u-matrix in (5.9) analytically we use a rank-one separable potential with the Gaussian form factor /9/

$$g_\nu(k) = k^\ell \exp(-\alpha_\nu k^2).$$

To estimate the contribution of $\delta_2^{(2)}$ to the sum (5.1), it is convenient to compare the term proportional to $(C_{oo} - \rho_{oo}\,\rho_{oo})$ in (5.9) with $\delta_1^{(2)}$, and the term linear in ρ_{oo} in (3.9) with $\delta^{(1)}$.

Estimating these ratios we obtain the following parameters

$$\varepsilon_1 = \lambda_1 k_0^2 \quad \text{and} \quad \varepsilon_2 = 2\,(\delta_{\pi N} / k)\cdot k_0^2 (\lambda_2)^{1/2},$$

respectively. Here, $\delta_{\pi N}$ is a characteristic value for the πN phase shifts at a given energy of the pion. The numerical values for ε_1 and ε_2 can be obtained by setting Δ=20 MeV (see ref./10/), $\delta_{\pi N}/k \leq 0.3$ fm (for $T_\pi \leq 100$ MeV) and a=1.5 fm: $\varepsilon_1 \leq 0.15$ and $\varepsilon_2 \leq 0.05$.

The above analysis shows that in $Re\ \delta_{\pi A}$ the effects caused by the nuclear binding and the Pauli exclusion principle, which are included in $\delta_2^{(2)}$, play a minor role in $Re\delta_{\pi A}$ at low energies. However, these effects are very important for the inelasticity parameters.

5.3. Inelasticity parameters

Using the approximations of factorization and completeness (as for (5.9)), we obtain the following expression for $Im\delta^{(2)}$ /3/:

$$Im\,\delta_{\pi A}^{(2)}(k) = 2\pi^2 A(A-1)\varepsilon_{\pi A}(k)\varepsilon_{\pi A}(k_\Delta) \int \frac{dn''}{4\pi} \Big\{ \frac{1}{A-1}\,\rho_{oo}(q) + \qquad (5.11)$$

$$C_{oo}(q',q'') - \frac{A}{A-1}\,\rho_{oo}(q')\rho_{oo}(q'') \Big\} \times \int_0^1 d\lambda \int_0^\lambda d\lambda_1\ \bar{u}(k,k'';\lambda_1)\ \bar{u}(k'',k';\lambda_1)$$

where $q = k' - k$, $q' = k' - k''$, $q'' = k'' - k$, $k'' = k_\Delta n''$; n'' is the unit vector, and k_Δ is determined by the equation

$$E_o(k) - E_o(k_\Delta) - \Delta = 0. \qquad (5.12)$$

It is worthwhile to stress that using the approximation of completeness we suppose only excited states of the nucleus (but the g.s.) to be degenerate. Hence, Δ is a certain mean excitation energy of the nucleus.

Unlike $Re\delta_{\pi A}$, the imaginary parts of the π-nucleus phase shifts depend rather strongly on Δ at low energies

$$Im\delta_{\pi A} \sim \varepsilon_{\pi A}(k_\Delta) \sim k_\Delta. \qquad (5.13)$$

An additional dependence on Δ stems from the functions entering into the integral (5.11).

Due to the Pauli principle, there is a certain cancellation between different terms in (5.11). If we neglect the correlations caused by the centre-of-mass motion, than

$$\mathrm{Im}\ \delta_{\pi A} \sim k_\Delta\ [\ \rho_{00}(q) - S_{00}(q',q'')],$$

where S_{00} is the the exchange part of the correlation function.

From this expression it follows that the effects of the nuclear binding and the Pauli principle manifest themselves nontrivially in the inelasticity parameters. The effect of the Pauli principle tends to cancel $\mathrm{Im}\delta_{\pi A}$. On the other hand the nuclear binding effect (the dependence of S on k_Δ) tends to destroy this cancellation. An appropriate value for Δ can be determined from the analysis of the scattering data /10/.

5.4. Two-body matrix elements

The ultimate goal of the calculations is to relate the π-nucleus phase shifts to the πN phase shifts. This problem has been discussed in detail in ref./3/.

Relations (5.1)-(5.9) in ref./3/ complete the determination of the considered lowest iterations for the pion-nucleus phase shifts in terms of the πN phase shifts, the two-particle correlation function and nuclear form factor.

5.5.Convergence of the iterative series

Calculation of the second-order corrections makes it possible to determine the range of convergence of the considered multiple scattering series. For the case of π-^{4}He this has been investigated in/3/. It was shown that it is sufficient to take into account only two terms in the series at energies below 70 - 80 MeV. For more heavier nuclei such as ^{12}C and ^{16}O the problem of convergence become more actual.

In our calculations we use for ^{4}He, ^{12}C and ^{16}O the form factors, correlation functions and single-particle density calculated in the harmonic oscillator model . The nuclear size parameters are determined from the scattering data/11/.

5.5.1.Convergence for $\mathrm{Re}\delta_{\pi A}$. Table 1 lists the results of calculations of two lowest iterations for $\mathrm{Re}\delta_{\pi A}$ for the π-^{12}C

Table 1. Real parts of the s- and p-wave phase shifts (in degrees) for π -^{12}C : potential calculations

T_π, MeV	$\delta_s^{(1)}$	$\delta_s^{(2)}$	$\delta_s=\delta_s^{(1)}+\delta_s^{(2)}$	$\delta_p^{(1)}$	$\delta_p^{(2)}$	$\delta_p=\delta_p^{(1)}+\delta_p^{(2)}$
14	-3.06 (-37.55)	-4.20	-7.26	4.57	0.74	5.31
50	-5.22 (-52.26)	-6.89	-12.11	23.04	2.94	25.98
68	3.79 (-62.87)	-5.30	-1.51	31.92	2.87	34.79

scattering. We observe a rapid convergence for the p- and higher partial waves, but not for the s-wave.

Taking into account the particular isotopic structure of the first-order term (5.5) for the s-wave

$$\delta_{\pi A,s}^{(1)} \sim \delta_{11}^0 + 2\,\delta_{31}^0, \qquad (5.18)$$

where $\delta_{2I,2j}^{l}$ is the πN phase shifts, we present in table 1 (in brackets) the results of calculations when the sign in (5.18) has been changed. We observe that in this case convergence in the s-wave is restored. It is seen that the problem of convergence for the s-wave is not of the dynamic origin but reflects the effect of isotopic cancellation in the first-order term. The same result has been obtained also for π-^{16}O.

Earlier/3/, it has been shown that for the π-^{4}He scattering the second-order correction in the s-wave becomes dominant only at energies below 30 Mev. From table 1 it follows that for heavier nuclei the pion rescattering effect in the s-wave must be taken into account in the whole energy range up to 70 MeV.

6. Pion absorption correction

The generalization of any multiple scattering theory to the case in which the number of particles is not conserved brings

about complications in the formalism due to an essential increase in the number of basic equations[12,13]. In the UST-approach the problem of taking into account the pion absorption has been considered in refs.[4,5], where the coupled channel method a'la Koltun and Mizutani[12] has been developed. It has been shown that the problem is effectively reduced to the consideration of pion scattering by two potentials. The resulting expression for the π-nucleus phase shifts reads as

$$\delta_{\pi A}(k) = \delta^{pot}_{\pi A}(k) + \delta^{abs}_{\pi A}(k), \qquad (6.1)$$

Here, $\delta^{pot}_{\pi A}$ is the phase shift caused by the pure potential scattering. The method of its calculation is presented above. The absorption correction has the form

$$\delta^{abs}_{\pi A}(k) = -\pi\, \varepsilon_{\pi A}(k) \int_0^1 d\eta \, \langle \psi^{(+)}_{k,0} |\, R_0(E,h)\, | \psi^{(+)}_{k',0} \rangle, \qquad (6.2)$$

where $\psi^{(+)}_{k,0}$ and $\psi^{(+)}_{k',0}$ describe the distortion of the pion wave by the potential interaction before and after the absorption. The operator $R_0(E,\eta)$ describes all processes involving pure nucleonic intermediate states. The dynamic parameter η, which varies from 0 to 1, plays the role of a coupling constant of the channels. The exact system of equations for $R_0(E,\eta)$ has been derived in ref.[4].

Assuming the two-nucleon mechanism for the pion absorption one can approximate $\delta^{abs}_{\pi A}$ by the term proportional to $\rho^2(r)$ (ρ is the nuclear density)[4,5]

$$\delta^{abs}_{\pi A}(k) = A(A-1)\, k\, \frac{1+\xi}{1+2\xi/A}\, \hat{\rho}^2(q)\, [\, \tilde{B}_0(k) + \tilde{C}_0(k)(\boldsymbol{\kappa}\cdot\boldsymbol{\kappa}')], \qquad (6.3)$$

where $\xi = w_\pi(k)/2M$, w_π is the pion energy, M is the mass of a nucleon, $\hat{\rho}^2(q)$ is the Fourier transform of the square of $\rho(r)$ (normalized to unity), $q = k - k'$ is the momentum transfer, and $\boldsymbol{\kappa}$ and $\boldsymbol{\kappa}'$ are the pion momenta in the $(\pi,2N)$ center-of-mass system

$$\boldsymbol{\kappa} = (k - \xi P_0)/(1+\xi), \quad \boldsymbol{\kappa}' = (k' - \xi P_0')/(1+\xi),$$

where P_0 and P_0' are the total momenta of the pair of nucleons (in the approximation in which they are "frozen": $P_0 = -2k/A$ and

$P_0' = P_0 - q$). Expression (6.3) is obtained in the local density approximation supposing the pion absorption operator is of a short range. The lowest s- and p-waves in the π-2N scattering have been taken into account.

One can expect that the parameters $\tilde{B}_0$ and $\tilde{C}_0$ corresponding to the short range part of the π-nucleus interaction are approximately constant in the low energy domain 0 - 50 MeV (see refs. [5,10]).

At low energies the complex parameters $\tilde{B}_0$ and $\tilde{C}_0$ we determine from the experimental data on the π-nucleus scattering lengths ($a_0 = \lim \delta_0(k)/k, k \to 0$) and volumes ($a_1 = \lim \delta_1(k)/k^3$, $k \to 0$). The experimental values for the π-nucleus lengths and volumes can be determined from the data on the strong interaction shifts and widths in the 1s- and 2p-levels of the pionic atoms. The quantities $a^{pot}_{0,1}$ are calculated in the framework of the potential theory.

The parameters $\tilde{B}_0$ and $\tilde{C}_0$ differ from the corresponding parameters of the optical potential by inclusion of the effects associated with distortion of the pion wave in the elastic channel.

6.1. On the double counting problem

To avoid the problem of double counting, the two-channel formalism[12,4] prescribes subtraction of the pole term in the elementary πN amplitude in the p_{11}-channel when we calculate the pure potential part of the π-nucleus phase shifts (6.1). However, if we use a local density approximation for calculating $\delta^{abs}_{\pi A}$ then this subtraction procedure is not needed. This conclusion comes from the diagram analysis of the iterative series for calculating both the terms in the two potential formula (6.1). In the expansion of $\delta^{abs}_{\pi A}$ (in terms of the πNN-vertex functions) there is a class of diagrams which can be considered as iterations of the Born pole term of the πN u-matrix in the p_{11}-wave. Separating this

diagrams, which are indeed of a "long-range" type, from $\delta_{\pi A}^{abs}$ and adding them to $\delta_{\pi A}^{pot}$, we get the iterative series for $\delta_{\pi A}^{pot}$ in terms of the full πN u-matrix in the p_{11} -channel.

7. Coulomb effects

A characteristic feature of the present approach is that the formalism is developed for a direct calculation of the phase shifts caused by the strong interaction. Therefore, the allowance for the Coulomb interaction is here the problem inverse to that of obtaining the hadronic phase shifts from the phase shift analysis of the scattering data.

We parameterize the pion-nucleus scattering amplitude as usual

$$f_{\pi A}(\theta) = f_c(\theta) + f_{sc}(\theta). \qquad (7.1)$$

Here, $f_c(\theta)$ is the Coulomb amplitude, and f_{sc} is the nuclear-Coulomb amplitude

$$f_{sc}(\theta) = \frac{1}{2ik} \sum_{\ell=0}^{\infty} e^{2i\hat{\sigma}_\ell^{\pm}} \left(e^{2i\Delta_\ell^{\pm}} - 1 \right), \qquad (7.2)$$

where the Coulomb phases $\hat{\sigma}_\ell^{\pm}$ are calculated for the nonpoint charge distribution in accordance with /14/. The total nuclear phase shifts $\Delta_\ell^{\pm}$ take into account the effect of the Coulomb distortion of the pion wave. We use the formalism of approximate treating for the Coulomb corrections which has been developed in refs./14,15/

$$\Delta_\ell^{\pm} = \delta_{\pi A,\ell} + \delta_{R,\ell}^{(\pm)}, \qquad (7.3)$$

where $\delta_{\pi A,\ell}^{\pm}$ are the pure hadronic phase shifts and the Coulomb corrections are calculated as

$$Re\delta_{R,\ell}^{\pm} = a_\ell^{\pm} [dA_\ell/dk + \sin(2A_\ell)\cosh(2B_\ell)/2k] \qquad (7.4)$$

$$Im\delta_{R,\ell}^{\pm} = a_\ell^{\pm} [dB_\ell/dk + \cos(2A_\ell)\sinh(2B_\ell)/2k].$$

Here, $A_\ell = Re\delta_{\pi A,\ell}$, $B_\ell = Im\delta_{\pi A,\ell}$, and

$$a_\ell^{\pm}(k) = \frac{2}{\pi} \eta^{\pm} k^2 \int_0^{\infty} dk' \frac{k'^2}{k^2 - k'^2} \int_{-1}^{1} dx\, P_\ell(x)\, F_A^{ch}(q^2)\, F_\pi^{ch}(q^2) \qquad (7.5)$$

is the Coulomb factor, $q^2 = k^2 + k'^2 - 2kk'x$, F_A^{ch} and F_π^{ch} are the charge form factors of the nucleus and of the pion, respectively, and $\eta^{\pm}$ is the Sommerfeld parameter : $\eta^{\pm} = \pm Z\alpha/\beta$, $\alpha = 1/137$, β is the pion velocity in the lab. c.m., Z is the charge of the nucleus. The approximation (7.4) is valid if the Coulomb interaction can be considered as a small correction.

8. Conclusion

In this paper we discussed the UST-approach and its applications to the description of pion-nucleus interaction. In view of the formal development of the approach, we presented a new derivation of the unitary multiple scattering series which makes the UST formalism very similar to the standard optical model (in the momentum space) based on the Watson multiple scattering theory.

The calculational scheme of the UST-approach is much simpler in comparison with that of the momentum space optical model /7-9,16/ because the UST equations are formulated on the pion-nucleus phase shifts. This makes it possible to search for the dynamics of their formation in a straightforward way by calculating microscopically second-order corrections.

In the present paper we developed an iterative method for solving the basic equations appropriate for the study of interaction of low-energy pions with light nuclei. Analysing the second -order corrections we determined the range of convergence of the iterative series (below ~ 70 MeV). At these energies it is sufficient to take into account only two lowest iterations.

Calculating the second-order term in $Re\delta_{\pi A}^{pot}$, we demonstrated the importance of the pion rescattering effect in the s-wave in the whole energy range up to ~ 70 MeV. It has been shown that the nuclear binding only slightly affects the real parts of the pion-nucleus phase shifts, but plays very important role in the formation of the inelasticity parameters ($Im\delta_{\pi A}^{pot}$). The effect of

the Pauli principle is also very important for $\mathrm{Im}\delta_{\pi A}^{pot}$ and tends to cancel this quantity at low energies.

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