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WHERE DO WE STAND WITH CALCULATIONS OF THE ($2\nu2\beta$)-DECAY RATES?

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The recent direct experimental observation of the two-neutrino mode of the double beta decay of ^{82}Se [1] has increased interest in this process. The two neutrino emitting mode of the double beta decay ($2\nu 2\beta$):

$$(A,Z) \rightarrow (A,Z+2) + 2e^- + 2\tilde{\nu}_e \quad , \qquad (1)$$

occurs as the second-order weak interaction process within the standard model of electroweak interactions.

At present, two possible mechanisms of the (2ν 2β) decay are usually considered. The first is the two nucleon mechanism (2n-mechanism) in which two neutrons in a nucleus undergo the beta decay successively. The second is the Δ - isobar mechanism [2]. This, however, is forbidden [3,4] for the energetically most favoured nuclear transition $0^+ \rightarrow 0^+$. It is then taken for granted that for such nuclear transitions the 2n-mechanism plays the dominant role.

The early calculations [3,4] have systematically overestimated the ($2\nu 2\beta$) amplitudes. In order to come to an agreement with the data, theorists are searching for a suppression of the ($2\nu 2\beta$) amplitude. P.Vogel and M.R.Zirnbauer [5] in their quasiparticle RPA calculations of the ($2\nu 2\beta$) nuclear matrix elements have found out that the value of the Gamow - Teller matrix element is strongly sensitive to particle-particle interaction in the spin-isospin polarization force. For the relevant coupling constant consistent with the experimental β^+ ft values P.Vogel and M.R.Zirnbauer have observed a strong suppression of the nuclear matrix elements for a number of ($2\nu 2\beta$) active nuclei. Similar calculations have been performed by O.Civitarese, A.Faessler and T.Tomoda [6] using a more realistic nuclear interaction. As a result, the same suppression effect has been demonstrated. C.R.Ching and T.H.Ho [7] have proposed a new method for calculations of the $(2\nu 2\beta)$ amplitude in which the closure approximation is not used explicitly. They expanded the $(2\nu 2\beta)$ effective interaction in a series of commutators of two axial vector currents and the nuclear Hamiltonian. The leading term in the expansion vanishes, which indeed leads to a suppression of the nuclear matrix elements. In particular, if the strengths of the spin and isospin nuclear forces are chosen to be equal, the $(2\nu 2\beta)$ decay is forbidden in their calculations.

In view of these results it is tempting to speculate that possibly an underlying deeper explanation should exist for all of them. The aim of the present Letter is to show, that the $(2\nu 2\beta)$ amplitude is actually subjected to a rather general, nuclear--model-independent suppression.

In the standard analysis of the $(2\nu 2\beta)$ amplitude we assume that the beta decay Hamiltonian has the form,

$$\mathcal{H}^{\beta} = \frac{G_F}{\sqrt{2}} 2 \left(\bar{e}_L \gamma_\alpha \nu_{eL}\right) j_\alpha + h.c. \quad , \tag{2}$$

where j_α is the strangeness conserving charged hadron current and e_L and ν_{eL} are operators of the left components of fields of the electron and neutrino, respectively. For the amplitude of the $(2\nu 2\beta)$ process we have

$$\begin{aligned} \langle f | S^{(2)} | i \rangle = & \frac{(-i)^2}{2} \left(\frac{G_F}{\sqrt{2}}\right)^2 \frac{1}{(2\pi)^6} \frac{1}{\sqrt{16\, p_{10}\, p_{20}\, k_{10}\, k_{20}}} \times \\ & \times \bar{u}(p_1) \gamma_\alpha (1+\gamma_5) u^c(k_1) \cdot \bar{u}(p_2) \gamma_\beta (1+\gamma_5) u^c(k_2)\, J_{\alpha\beta} - \\ & - (p_1 \rightleftarrows p_2) - (k_1 \rightleftarrows k_2) \quad + (p_1 \rightleftarrows p_2)(k_1 \rightleftarrows k_2) \quad , \end{aligned} \tag{3}$$

where,

$$J_{\alpha\beta}=\int e^{-i(p_1+k_1)x_1} e^{-i(p_2+k_2)x_2} \langle p_f|T(J_\alpha(x_1)J_\beta(x_2)|p_i\rangle\, dx_1\, dx_2 \quad , \tag{4}$$

Here, p_1 and p_2 (k_1 and k_2) are the four-momenta of the electrons (antineutrinos), p_i and p_f are the four-momenta of the initial and final nucleus and the nuclear matrix element is

$$\langle p_f|T(J_\alpha(x_1)J_\beta(x_2)|p_i\rangle=\langle p_f|T(j_\alpha(x_1)\, j_\beta(x_2)e^{-i\int \mathcal{H}(x')dx'})|p_i\rangle \quad , \tag{5}$$

where $J_\alpha(x)$ is the weak charged nuclear hadron current in the Heisenberg representation and $\mathcal{H}(x)$ is the strong interaction Hamiltonian. In this way in (5) the strong interaction is taken into account exactly.

Traditionally, for integrating over the time variables in eq. (4) one uses the definition of the time-ordered product of two operators in the form

$$T(J_\alpha(x_1)J_\beta(x_2))=\theta(x_{10}-x_{20})\, J_\alpha(x_1)\, J_\beta(x_2)+\theta(x_{20}-x_{10})\, J_\beta(x_2)\, J_\alpha(x_1) \quad . \tag{6}$$

After integration one obtains

$$\begin{aligned} J_{\alpha\beta} = {} & -i2\pi\delta(E_f-E_i+p_{10}+k_{10}+p_{20}+k_{20})\times \\ & \times\int e^{-i(\vec{p}_1+\vec{k}_1)\vec{x}_1} e^{-i(\vec{p}_2+\vec{k}_2)\vec{x}_2}\sum_n\Big[\frac{\langle p_f|J_\alpha(0,\vec{x}_1)|p_n\rangle\langle p_n|J_\beta(0,\vec{x}_2)|p_i\rangle}{E_n-E_i+p_{20}+k_{20}}+ \\ & +\frac{\langle p_f|J_\beta(0,\vec{x}_2)|p_n\rangle\langle p_n|J_\alpha(0,\vec{x}_1)|p_i\rangle}{E_n-E_i+p_{10}+k_{10}}\Big]d\vec{x}_1\, d\vec{x}_2 \; , \end{aligned} \tag{7}$$

Here, $|p_n\rangle$ is an eigenvector of the intermediate nucleus with energy E_n and E_f and E_i are energies of the initial and final nucleus. Using the symbol $\sum_n$ we mean summation over the discrete states and integration over the continuum states of the intermediate nucleus; $\sum_n$ includes the complete set of these states.

The numerical evaluation of the sum over the states $|p_n\rangle$ represents, however, a difficult practical problem. As we have mentioned above, a substantial sensitivity to the details of the nuclear models has been observed in refs. [5, 6] and [7]. The procedure is definitely not well controlled.

We have observed that by using a different though fully equivalent formula for the T product of the hadron currents

$$T(J_\alpha(x_1)J_\beta(x_2)) = J_\alpha(x_1)J_\beta(x_2) + \theta(x_{20}-x_{10})[J_\beta(x_2), J_\alpha(x_1)] \quad , \tag{8}$$

we can obtain more information about the $(2\nu2\beta)$ amplitude. As a matter of fact, by inserting eq. (8) into eq. (4) we obtain

$$J_{\alpha\beta} = \sum_n 2\pi\delta(E_f - E_n + p_{10} + k_{10})\, 2\pi\delta(E_n - E_i + p_{20} + k_{20}) \times$$
$$\times \int e^{-i(\vec{p}_1+\vec{k}_1)\vec{x}_1} e^{-i(\vec{p}_2+\vec{k}_2)\vec{x}_2} \langle p_f|J_\alpha(0,\vec{x}_1)|p_n\rangle\langle p_n|J_\beta(0,\vec{x}_2)|p_i\rangle\, d\vec{x}_1 d\vec{x}_2 +$$
$$+ \int e^{-i(p_1+k_1)x_1} e^{-i(p_2+k_2)x_2} \theta(x_{20}-x_{10})\langle p_f|[J_\beta(x_2), J_\alpha(x_1)]|p_i\rangle dx_1 dx_2 \,. \tag{9}$$

We can see that the first term in the r.h.s. in eq. (9) corresponds to two subsequent nuclear beta decay processes provided the beta transition from the parent nucleus (A, Z) to the intermediate nucleus (A,Z+1) is energetically allowed. We know, however, that for the nuclei in which the double beta decay is experimentally studied such transitions are forbidden. In that case, the argument of the second delta function in eq. (9) is always positive and the first term in eq. (9) is equal to zero. The second term in the r.h.s. of eq. (9) corresponds to the $(2\nu2\beta)$ process and the amplitude of the $(2\nu2\beta)$ decay, is proportional to the nuclear matrix element of the non-equal-time commutator of the nuclear hadron currents,

$$\theta(x_{20}-x_{10}) \langle p_f|\,[J_\beta(x_2), J_\alpha(x_1)]\,|p_i\rangle \quad . \tag{10}$$

Analysing this nuclear matrix element, we distinguish two cases.

In the first case, the two nucleon beta decays in the nucleus are connected by a space-like interval, $(x_1 - x_2)^2 < 0$, i.e. only uncorrelated nucleon beta decays in the nucleus are considered. In that case the commutator of the nuclear hadron currents in (10) is equal to zero and the ($2\nu\ 2\beta$) decay is forbidden.

In the second case the two beta decays in the nucleus are connected by a time-like interval, $(x_1-x_2)^2 > 0$, which means that they are correlated. We note that both the nuclear hadron currents in (10) are of the same charge. This implies that the commutator in (10) is indeed equal to zero if the nuclear currents $J_\alpha(x_1)$ and $J_\beta(x_2)$ are approximated as one-body operators and do not contain the exchange currents. Further, the ($2\nu\ 2\beta$) amplitude of the two-nucleon mechanism is strongly suppressed if the contribution of the exchange currents is small as usually expected. We note that a rough estimate based on the calculations by M.Ericson and J.Vergados [8] of a class of the exchange-current diagrams leads to a suppression by a factor of up to one million as compared with the earlier calculations [3,4]. Recently we have also seen an estimate of another exchange-current mechanism [9], there the suppression is only by a factor of about one hundred.

In the summary, we face now the following situation. Standard calculations as in eq. (7) are based on the <u>one-body</u> Hamiltonian (see eq. (2)). The meson exchanges are only contained via the nuclear strong Hamiltonian used in the construction of the eigenstates $| p_n \rangle$ of the intermediate nucleus. The works performed until now along this line necessarily contain several approximations which are difficult to control. The procedure can hardly be considered as a consistent development of the matrix element of the genuine two-body currents $J_\alpha(x_1)$ and $J_\beta(x_2)$ needed for the non-zero result in eq. (9). This provides an additional insight and completes the discussion started in refs. [5 , 6], and [7] of the difficulties and instability met in the traditional ($2\nu 2\beta$) calculations. We suggest that an alternative approach could be based on eq. (9). This would include an explicit construction of the effective Hamiltonian corresponding to the exchange currents. Then, however, the alternatives to the two-nucleon mechanism, like the $\Delta - \Delta$ mechanism of the ($2\nu 2\beta$) decay [4], should clearly be considered first.

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