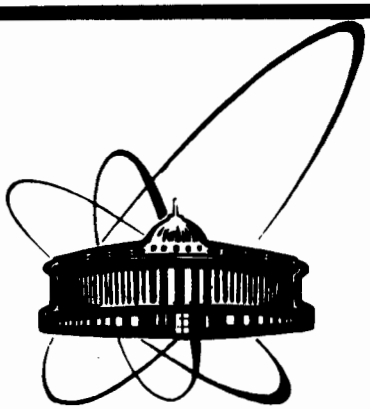

объединенный
институт
ядерных
исследований
дубна

E4-89-312

I.V.Amirkhanov, I.V.Puzynin, T.P.Puzynina,
B.N.Zakhariev

POTENTIAL RECONSTRUCTION FROM R-MATRIX RESONANCE POSITIONS AND REDUCED WIDTHS

Submitted to "Theory of Quantum Systems
with Strong Interactions"

1989

1. INTRODUCTION

The formalism of the one-dimensional inverse Sturm-Liouville problem in the finite interval was originally proposed by Gelfand and Levitan [1]. It was applied to scattering problems with the finite-range potentials (R-matrix scattering theory) by Zhigunov et al[2]. See also the paper [3] concerning the R-matrix as the input information in Marchenko's inverse problem approach. The multi-channel generalization of the R-matrix inverse problem and different exactly solvable models for finite range interactions were considered in [4,5].

Up to now, there were no numerical calculations in the R-matrix inverse problem approach. In this paper we give an example of this type potential reconstruction. This is our first step to the numerical solution of the three-body inverse problem[6].

We also propose the idea of a simple model to test the solution of the multi-channel inverse problem in the R-matrix formalism (using two kinds of Bargmann potentials and wave functions: on the half-axis - for direct problem solutions, and in the finite interval - for approximate inverse problem solutions).

2. ANALYTICAL SOLUTIONS IN THE FINITE INTERVAL $0 \le r \le a$

The formalism, to be used here is a particular one-channel case of the general multi- channel R-matrix inverse problem, given in [5].

The R-matrix Bargmann wave functions built of M pairs (E_ν, γ_ν) of the spectral parameters of the lowest M levels of an unknown potential are the elements of the left half of the row-vector:

$$\Psi^T(x) = \Psi^T_{\cdot}(r)\ \hat{P}^{-1}(r), \qquad (1)$$

with the elements $(\Psi(E_\nu, r), \ldots; \Psi(E_\mu, r) \ldots)$ where $\nu, \mu = 1, .., M$; E_ν and E_μ - are the levels of the unknown and unperturbed potential wells; $\Psi^T_{\cdot}(r) = (\Psi_{\cdot}(E_\nu, , r), \ldots; \Psi_{\cdot}(E_\mu, , r), \ldots)$ is the row-vector of the solutions for the initial i finite rectangular potential well; γ_ν and $\gamma_{\cdot\mu}$ are the reduced widths of the bound states in the unknown and initial wells, respectively (they are equal to the derivatives of the normalized bound states at the right infinite wall); the superscript T means transposition; $\hat{P}(r)$ is the 2×2-matrix with the block-elements[5]:

$$
\begin{aligned}
P_{\nu\cdot\nu}(r) &= \delta_{\nu\cdot\nu} + \int_r^a \Psi(E_{\nu\cdot},r\cdot)\,\Psi(E_{\nu},r\cdot)\,d\,r\cdot \\
P_{\nu\cdot\mu}(r) &= \int_r^a \Psi(E_{\nu\cdot},r\cdot)\,\Psi(E_{\mu},r\cdot)\,d\,r\cdot \\
P_{\mu\cdot\nu}(r) &= -\int_r^a \Psi(E_{\mu\cdot},r\cdot)\,\Psi(E_{\nu},r\cdot)\,d\,r\cdot \\
P_{\mu\cdot\mu}(r) &= \delta_{\mu\cdot\mu} - \int_r^a \Psi(E_{\mu\cdot},r\cdot)\,\Psi(E_{\mu},r\cdot)\,d\,r\cdot
\end{aligned}
\qquad (2)
$$

The potential, which is reconstructed in the M-level approximation has the form[5]:

$$
V(r) = 2\{\sum_{\nu\nu\cdot}\Psi(E_{\nu},r)\,P^{-1}_{\nu\nu\cdot}(r)\,\Psi(E_{\nu\cdot},r) - \sum_{\nu\mu\cdot}\Psi(E_{\nu},r)\,P^{-1}_{\nu\mu\cdot}(r)\Psi_{\circ}(E_{\mu'},r)+
$$

$$
+\sum_{\mu\nu\cdot}\Psi_{\circ}(E_{\mu},r)\,P^{-1}_{\mu\nu\cdot}(r)\,\Psi(E_{\nu\cdot},r) - \sum_{\mu\mu\cdot}\Psi_{\circ}(E_{\mu},r)\,P^{-1}_{\mu\mu\cdot}(r)\,\Psi_{\circ}(E_{\mu'},r)\}\cdot \qquad (3)
$$

3. NUMERICAL RESULTS

We have chosen the infinite potential well with the inclined linear bottom as a potential to be reconstructed from its spectral R-matrix parameters $\{E_\nu,\gamma_\nu\}$. The last were calculated numerically by using the finite-difference approximation and the Newton iteration scheme[7,8].

Figs. 1-5 demonstrate the shapes of the Bargmann-type potentials built of the 2; 6; 10; 12; 18 pairs of the spectral parameters $\{E_\nu,\gamma_\nu\}$ for the lowest R-matrix states.

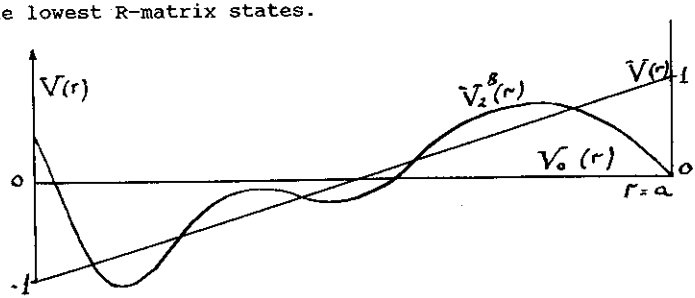

Fig.1 The potential V(r) to be approximately reconstructed is the infinite well with the inclined bottom. The reference potential $V_\circ$ is the infinite rectangular well. The oscillating line represents the Bargmann-type potential $V_2^B(r)$ with two lower levels having the same $\{E_\nu,\gamma_\nu\}$ as V(r) and all others as $V_\circ(r)$. This $V_2^B(r)$ is the two-level approximation of the inverse problem solution.

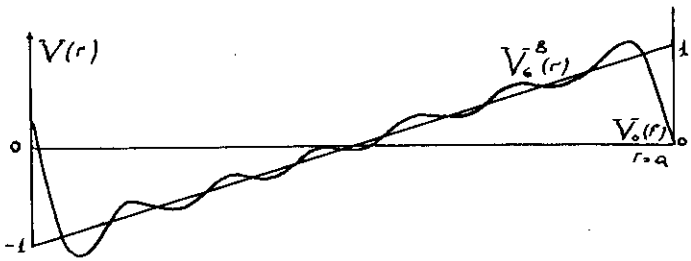

Fig.2. The six-level approximation of the inverse problem solution.

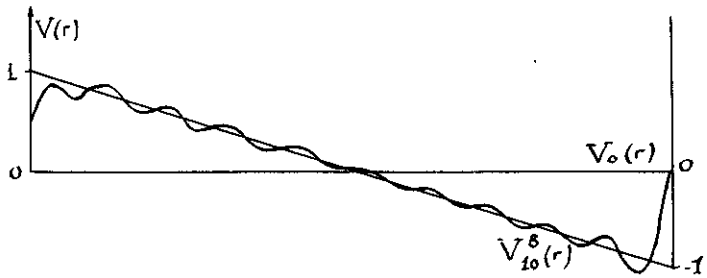

Fig.3. The ten-level approximation obtained with the spectral parameters calculated with the h/2 finite-difference step.

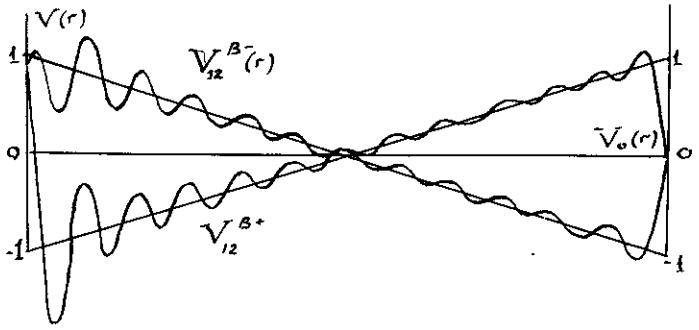

Fig.4. The twelve-level approximations $V_{12}^{B\pm}(r)$ for two potentials with slopes in different directions. The errors in $\{E_\nu, \gamma_\nu\}$ calculated numerically and the ill-posedness of the inverse problem led to greater deviations of $V_{12}^{B\pm}(r)$ from V(r) than in the ten-level approximation. Pay attention to the better approximation in the $V_{12}^{B-}(r)$ case.

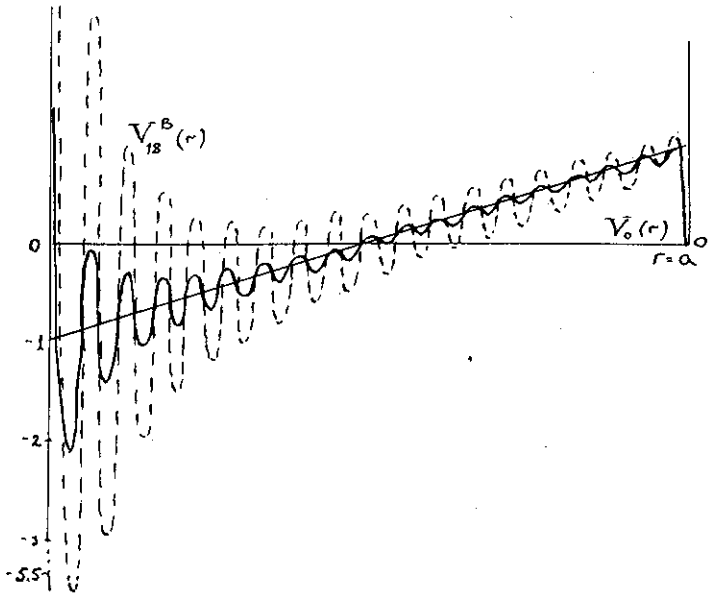

Fig.5.The 18-level approximations. Dashed line - for input spectral parameters calculated with the finite-difference step h. Solid oscillating line - for step h/2. It is remarkable that there is a memory about the original potential in this oscillations.

4. A MODEL CYCLE OF DIRECT-INVERSE SOLUTION

Bargmann-type solutions on the half axis (the Gelfand-Levitan and Marchenko approach) can be used for determination of direct eigenvalue R-matrix solutions on a finite interval. So for solutions regular at the origin we need only to find the energy values at which they are zero on the other end of the interaction region r = a. This is easy to do with the functions in the analytical form. Numerical integration is however required for calculation of normalising constants. The inverse problem can be solved using another type of Bargmann solutions (in the R-matrix approach). Due to minimum errors it is convenient to use this model to investigate quantitatively the measure of stability of the inverse problem solutions.

REFERENCES

1. Gelfand I.M., Levitan B.M.//Inv.Acad.Nauk.,ser.phys.-1951- 15, p.309
2. Zhigunov V.P., Zakhariev B.N., Nijazgulov S.A., Suzko A.A. Preprint JINR P4-7815, Dubna 1974.
3. Chadan K., Montes A.// J.Math.Phys.-1968-v.9, p.1898.
4. Zakhariev B.N., Suzko A.A. Potentials and Quantum Scattering. Direct and Inverse Problems.Energoatomizdat,1985 (to be translated in English by Springer Verlag).
5. Amirkhanov I.V. et al in Proc.Conf."Schroedinger Operator. Standart and Nonstandart" Dubna,1988 World Scientific, 1989
6. Zakhariev B.N.// Few-Body Systems.-1988-v.4,p.25 .
7. Puzynin I.V., Puzynina T.P. // In.: Algorithms and programms for solution of some problems in physics.- 1974 - KFKI-74-34, Budapest, p.93
8. Puzynin I.V., Puzynina T.P., Strizh T.A.// Communic. JINR - 1978 P11-87-332, Dubna

Received by Publishing Department
on May 4, 1989.