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EFFECT OF PARTICLE-PARTICLE INTERACTIONS ON EXCITED STATES OF SPHERICAL AND DEFORMED NUCLEI

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I. Introduction

Within the quasiparticle-phonon nuclear model (QPNM)[1-4] one can calculate excited states of spherical and deformed nuclei. The QPNM Hamiltonian includes an average field of neutron and proton systems as the Saxon-Woods potential, the monopole and quadrupole pairing and the effective isoscalar and isovector multipole, spin-multipole and tensor interactions between quasiparticles. They include charge-exchange interactions. Only particle-hole (p-h) interactions are usually taken into account. In this report I give a review of the papers devoted to the study of the influence of particle-particle (p-p) interactions on nuclear characteristics and to the QPNM generalisation to effective separable interactions of a finite rank.

2. The QPNM Equations with Separable Interactions of a Finite Rank

Expansion of central interactions over multipole λ and spin-multipole $L\lambda$ include the radial functions $R^{\lambda}(r_1 r_2)$ and $R^{\lambda L}(r_1 r_2)$ that are usually written in a simple separable form. Nucleon-nucleon potentials are sometimes represented in a separable form. Thus, separable representations of rank $n_{max} \leq 5$ for the Paris and Bonn potentials provide a satisfactory approximation for these potentials. Calculations of nuclear characteristics depending on the matrix elements of effective interactions over single-particle states are less sensitive to the radial dependence of forces in comparison with the calculations of few-nucleon systems where the use is made of separable representations of nucleon-nucleon potentials. Therefore, the use of separable interactions of a finite rank in calculating characteristics of complex nuclei is justified.

The QPNM equations with effective separable interactions of a finite rank have been derived in ref.[5]. The radial functions for p-h and p-p separable interactions of a rank n_{max} are

$$R^{\lambda}(r_1 r_2) = \sum_{n=1}^{n_{max}} R_n^{\lambda}(r_1) R_n^{\lambda}(r_2) , \quad R^{\lambda L}(r_1 r_2) \approx \sum_{n=1}^{n_{max}} R_n^{\lambda L}(r_1) R_n^{\lambda L}(r_2). \tag{I}$$

We perform the Bogolubov transformation and introduce the operators of quasiparticles α_{jm}^{+}, α_{jm} and phonons $Q_{\lambda\mu i}^{+}$, $Q_{\lambda\mu i}$. As a result of transformation the QPNM Hamiltonian is

$$H_{QPNM} = \sum_{jm} \varepsilon_j \alpha^+_{jm} \alpha_{jm} + H_v + H_{vq}, \qquad (2)$$

where ε_j is the quasiparticle energy on the subshell j. The first two terms describe quasiparticles and phonons in the RPA and H_{vq} describes the quasiparticle-phonon interaction.

For the energies $\omega_{\lambda i}$ and wave functions $Q^+_{\lambda\mu i}\Psi_0$ of one-phonon states of the electric and magnetic type we get the RPA secular equations as a determinant equal to zero. The rank of the determinant is $12 \cdot n_{max}$, i.e. n_{max} times larger than simple separable interactions with $n_{max} = 1$. If p-p interactions are neglected, the rank of the determinant is $4 \cdot n_{max}$.

For the solutions of the RPA secular equations when the energies $\omega_{\lambda i}$ and phonon amplitudes $\psi^{\lambda i}_{jj'}$ and $\varphi^{\lambda i}_{jj'}$ are found, Hamiltonian (2) appears to be determined. It contains no any free parameters and no unfixed constants.

In the QPNM the wave functions of excited states are represented as a series in the number of operators; in odd nuclei each term is multiplied by the operator of quasiparticles. The approximation implies break off this series. In most of the calculations performed earlier, the wave function consisted of one- and two-phonon terms.

For doubly even nuclei the secular equation is written as an equality to zero of the determinant whose rank equals the number of one-phonon terms in the wave function. Effect of the Pauli principle in two-phonon terms of the wave function leads to the factor $1 + \mathcal{K}^J(\lambda_1 i_1, \lambda_2 i_2)$ and shifts the two-phonon poles $\Delta\omega(\lambda_1 i_1, \lambda_2 i_2)$. With the use of the finite rank $n_{max} > 1$ of separable interactions the rank of the determinant does not increase in comparison with $n_{max} = 1$. The inclusion of separable interactions with $n_{max} > 1$ makes the expressions for $\Delta\omega(\lambda_1 i_1, \lambda_2 i_2)$ and $U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)$, caused by the quasiparticle-phonon interaction, more complicated. This complication of the functions turns out to be unessential in computer calculations.

Thus, the basic QPNM equations have been derived in ref./5/ for p-h and p-p isoscalar and isovector multipole, spin-multipole and tensor separable interactions of a finite rank. The finite rank of separable interactions makes the RPA equations more complicated, which is unimportant in computer calculations. Most important is the fact that allowance for the finite rank separable equations does not result in any essential complication of equations for calculating the fragmentation of quasiparticle and collective motion. This implies that the QPNM may serve as a basis for calculating many properties of atomic nuclei and spectroscopic factors of nuclear reactions.

3. β^+ Decay of Neutron-Deficit Nuclei, Strength Functions of (n,p) Transitions and Renormalisation in Nuclei of the Constant of the Axial-Vector Weak Interaction

Monopole pairing and p-h interactions play the basic role in describing low-lying vibrational states and giant resonances of different types. As concerns p-p interactions, as has been shown in refs./6-10/, their role is important in describing double β decay, Gamow-Teller (GT) β^+ decays of neutron-deficit spherical and deformed nuclei and the strength functions of GT (n,p) transitions.

In the RPA with p-h and p-p interactions the summed ft values determined as $(\widetilde{ft})^{-1} = \sum_i (ft)^{-1}_i$ have been calculated for GT $0^+_{g.s.} \to 1^+$ transitions where the summation was over the states i in the energy interval $\Delta\mathcal{E}$. In this interval $\Delta\mathcal{E}$ there are nuclear levels to which the experimentally observed decays proceed. The single-particle energies and wave functions of the Saxon-Woods potential, monopole pairing, p-h and p-p GT interactions were used in the calculations. The results of calculations /7,8,10/ and experimental data are listed in Table 1. According to the calculations, the low-lying states occupied by intensive β^+ decays are separated by the gap of 3-4 MeV from higher-lying states. Therefore, the results of calculations are almost independent of the position of low-lying states. The p-p interactions decrease the total GT strength and shift a part of the GT strength to the higher-lying part of the spectrum. In the calculations, the main role is played by the $\sigma\tau$ matrix elements, and the use of more complex interactions/9/ with the same number of constants fixed from the experimental data, as in refs./7,8/, has a symbolic meaning as has been demonstrated in ref./11/.

The agreement of the calculated $\log \widetilde{ft}$ values with the experimental ones, demonstrated in Table I, takes place at $|G_A/G_V| = 1.26$ and under renormalisation $|G_A/G_V| = 1$. In ref./8/ the total S_+ strength of the GT(n,p) transition on ^{54}Fe has been calculated. At $G_1^{\sigma\tau} = -\frac{7.5}{A}$ MeV it was obtained that $S_+ = 4.2$ which is in agreement with $S_+ = 3.8$ measured in ref./12/ in the $^{54}Fe(n,p)^{54}Mn$ reaction. With increasing renormalisation of the constant of the axial-vector weak interaction G_A one should decrease $|G_1^{\sigma\tau}|$ for describing $\log \widetilde{ft}$ of the GT β^+ decays, which will make the agreement of the calculated value of S_+ with the experimental one worse.

Based on the calculations of $\log \widetilde{ft}$ for the GT β^+ decays of a large number of neutron-deficit spherical and deformed nuclei, given in Table I, and total strength functions S_+ of the GT (n,p) transitions we can state that in complex nuclei the following condition should hold:

$$|G_A/G_V| \geq 1. \qquad (3)$$

Table I

Gamow–Teller β^+ transitions $0^+_{g.s.} \to 1^+$

β^+-transition	log $\widetilde{ft}$ exp.	log $\widetilde{ft}$, $\lvert G_A/G_V \rvert = 1$	calc. $\lvert G_A/G_V \rvert = 1.25$
Deformed nuclei		$G_1^{11} \cdot A$ = 8.3 MeV	$G_1^{11} \cdot A$ = 8.5 MeV
$^{166}Hf \to {}^{166}Lu$	4.8	4.8	5.0
$^{166}Yb \to {}^{166}Tm$	4.9	4.9	4.9
$^{164}Yb \to {}^{164}Tm$	4.8	4.7	4.7
$^{162}Yb \to {}^{162}Tm$	4.7	4.7	4.8
Spherical nuclei		$G_1^{01} \cdot A$ = 7.5 MeV	$G_1^{01} \cdot A$ = 7.8 MeV
$^{154}Yb \to {}^{154}Tm$	3.6	3.5	3.8
$^{152}Yb \to {}^{152}Tm$	3.4	3.4	3.5
$^{152}Er \to {}^{152}Ho$	3.9	3.5	3.8
$^{150}Er \to {}^{150}Ho$	3.6	3.5	3.6
$^{148}Dy \to {}^{148}Tb$	3.9	3.8	3.9
$^{146}Dy \to {}^{146}Tb$	3.8	4.0	4.1
$^{108}Sn \to {}^{108}In$	3.4	3.5	3.3
$^{106}Sn \to {}^{106}In$	3.2	3.3	3.1
$^{104}Sn \to {}^{104}In$	3.2	3.3	3.2
$^{104}Cd \to {}^{104}Ag$	4.0	3.8	3.8
$^{102}Cd \to {}^{102}Ag$	3.6	3.6	3.5
$^{100}Cd \to {}^{100}Ag$	3.2	3.2	3.1
$^{98}Pd \to {}^{98}Rh$	3.5	3.5	3.4
$^{96}Pd \to {}^{96}Rh$	3.3	3.4	3.3
$^{94}Ru \to {}^{94}Tc$	3.6	3.6	3.5

A further experimental study of β^+ decays of neutron–deficit spherical and deformed nuclei is necessary. Renormalisation of the G_A in nuclei can be determined more accurately by measuring β^+ decay in ^{100}Sn.

4. Low–Lying Vibrational States in Deformed Nuclei

Within the QPNM with monopole and quadrupole pairing and isoscalar and isovector p–h and p–p multipole interactions one can study low–lying vibrational states in deformed nuclei. The calculations are made with the wave function

$$\Psi_\nu(K_0^{\pi_0}\sigma_0) = \Big\{ \sum_{i_0} R^\nu_{i_0} Q^+_{\lambda_0\mu_0 i_0\sigma_0} + \sum_{\substack{\lambda_1\mu_1 i_1\sigma_1 \\ \lambda_2\mu_2 i_2\sigma_2}} \frac{(1+\delta_{\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2})^{-1}}{2[1+\delta_{K_0 0}(1-\delta_{\mu_1 0})]^{1/2}} \delta_{\sigma_1\mu_1+\sigma_2\mu_2,\sigma_0 K_0} P^\nu_{\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2} Q^+_{\lambda_1\mu_1 i_1\sigma_1} Q^+_{\lambda_2\mu_2 i_2\sigma_2} \Big\} \Psi_0 \qquad (4)$$

The Pauli principle is taken into account in the two–phonon terms (4). The variational principle /13,14/ was used to derive equations for the energies η_ν and coefficients $R^\nu_{i_0}$ and $P^\nu_{\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2}$. Phonons are calculated in the RPA with p–h and p–p interactions. For the $K^\pi = 0^+$ states from the condition of eliminating spurious states there were derived equations for monopole and quadrupole pairing which have been investigated in ref. /15/.

Good enough description of the energies, B(Eλ) values and the structure of quadrupole, octupole and hexadecapole states in ^{168}Er, ^{172}Yb and ^{178}Hf has been obtained in ref. /14/. It is shown that nonrotational states with $K^\pi = 0^-, 1^-, 2^\pm$, $3^\pm$ and 4^+ with energies up to 2.5 MeV have dominating one–phonon components.

The study of vibrational states with $K^\pi \neq 0^+$ in well deformed doubly even nuclei has shown that the energy and structure of each state are determined mainly by the single–particle energies and wave functions of the Saxon–Woods potential, monopole pairing and p–h isoscalar multipole interaction. The multipole p–h isovector interaction, quadrupole pairing and multipole p–p interaction are of minor importance. Inclusion of the p–p interaction improves the desoription of vibrational states. Moreover, it justifies the applicability of RPA to describe states with an energy less than 1 MeV.

Phenomenological methods of describing low–lying vibrational states are based on that the first quadrupole and octupole states are collective and then there are no collective states up to those forming giant quadrupole and octupole resonances.

A qualitatively new result has been obtained in studying low–lying vibrational states: the Eλ strength distribution differs in some cases from the generally accepted one. In particular, there are cases when collective is not the first but a higher lying state with a given K^π, or the largest part of the Eλ strength is concentrated not on the first states but in the energy interval 2 ÷ 3 MeV.

Consider now the E3 strength distribution in ^{168}Er shown in Table 2. According to /16/ the first $K^\pi_\nu = 0^-_1$, 1^-_1 and 2^-_1 states are collective. Six collective $K^\pi = 3^-$ states have been observed. On the first three 3^-_1, 3^-_2 and 3^-_3 states there are 1.3 s.p.u.; and on the fourth 3^-_4 state, 4.68 s.p.u.. In the interval from 2.25 to 2.50 MeV 7.9 s.p.u. are concentrated. This distribution of the E3 strength sharply differs from the standard one. Table 2 shows the results of calculations /14/ in the QPNM, in sdf IBM in /16/ and IBM-1 + f boson in /17/. In the calculations /16/, the B(E3) values were normalized to the experimental value of the $3^- 1_1$ state. As a result, for the first 3^-3_1 state the calculated B(E3) value turned out to diverge by a factor of 500 from the experimental one. If most of the Eλ strength is concentrated not on the first K^π_1 state, it is practically impossible to describe it within the IBM. This is confirmed by the calculations in /17/ in the IBM-1 + f boson model in which the first three $K^\pi = 3^-$ levels are omitted. The main part of the E3 strength is concentrated on the 3^-_4 state which is considered by them as a first collective $K = 3^-$ state. The first three $K^\pi = 3^-$ states are neglected as two-quasiparticle ones. The states 3^-_1; 3^-_2 and 3^-_3 in ^{168}Er cannot be thought two-quasiparticle states. According to the experimental data /18/ on (dp) and $(t\alpha)$ reactions, their wave functions contain the sum of two-quasiproton and two-quasineutron terms. According to the experimental data /16/ on $(\alpha\alpha')$ reactions, the B(E3) values for the $K^\pi_\nu = 3^-_1, 3^-_2$ and 3^-_3 states are 30 ÷ 60 times larger than the values for the corresponding two-quasiparticle states.

According to the calculations /14/ in the QPNM, the B(E3) values for 0^-_1, 1^-_1, 2^-_1 and 3^-_1 in ^{168}Er agree with the experimental data. Among the $K^\pi = 3^-$ states the fourth 3^-_4 state has the largest B(E3) value, which is in agreement with experiment. The total octupole E3 strength concentrated on the states with an energy up to 2.6 MeV equals 20 s.p.u., according to the experimental data /16/, and 20.3 s.p.u., according to calculations in /14/.

The Eλ strength distribution differing from the standard one is observed in other nuclei. In ^{172}Yb apart from the first 2^+ state the second λ^+_2 state is also collective. By the experimental data /16/, in ^{172}Yb that concentration of the E2 strength is 1.7 times larger in the interval from 2 to 3 MeV than on the first 2^+ state.

It is to be noted that in some cases the predictions made in the IBM and QPNM strongly differ. Thus, according to the calculations /19/ in sdg IBM in ^{168}Er for the $I^\pi K_\nu = 4^+3_1$ state B(E4)=50.8 s.p.u., and according to the calculations /14/ in the QPNM B(E4)=0.4 s.p.u. It is expedient to check this discrepancy experimentally.

On the basis of calculations in the QPNM it has been concluded /20/ that collective two-phonon states do not exist in deformed

Table 2

E3-Strength Distribution in ^{168}Er

I = 3	Exp. /16/		B(E3)$_{s.p.u.}$, calc.		
K^π_ν	E, MeV	B(E3)$_{s.p.u.}$	QPNM /14/	sdf IBM /16/	IBM+f boson /17/
1^-	1.431	3.92	4.6	3.92	5.5
3^-_1	1.541	0.25	0.14	134.6	-
2^-_1	1.633	4.94	4.6	4.94	8.0
3^-_2	1.828	0.60	0.6	2.30	-
0^-_1	1.913	1.96	3.0	1.53	4.6
3^-_3	1.999	0.42	0.3	0.085	-
1^-_2	2.022	-	0.3	-	1.1
3^-_4	2.269	4.68	2.0	8.5	5.8
2^-_2	2.302	-	0.2	-	0.3
(3^-_5)	2.324	1.53	-	0.034	-
1^-_3	-	-	4.9	-	2.6
(3^-_6)	2.486	1.70	-	0.017	-

nuclei. The two-phonon is the state in which the contribution of the two-phonon term to wave function normalisation exceeds 50 %. The calculations /14/ in the QPNM with p-h and p-p interactions gave the energies and B(Eλ) values for the 2^+_1 and first octupole states which are in reasonable agreement with experimental data. With inclusion of p-p interactions the collectivity of these states decreases thus decreasing the shifts $\Delta\omega$ of two-phonon poles. The shifts turned out to be equal to 0.1-1.5 MeV. The energy centroids of the lowest collective two-phonon states calculated in /14/ are 2.5 - 4.0 MeV. Therefore, the conclusion about the absence of collective two-phonon states is valid.

The QPNM formed the basis for calculating the energies and structure of states of deformed nuclei. It is possible to calculate the Eλ transition probabilities between excited states. Of great interest is the experimental study of excited states of deformed nuclei with an energy 2÷3 MeV. We hope that these experiments will be carried out at a new generation of accelerators with a large energy resolution.

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Влияние частично-частичных взаимодействий на возбужденные состояния сферических и деформированных ядер

Продемонстрировано, что в рамках квазичастично-фононной модели ядра можно проводить расчеты с эффективными конечного ранга n_{max} сепарабельными взаимодействиями. Учет сепарабельных взаимодействий с $n_{max} > 1$ не приводит к существенному усложнению вычислений фрагментации квазичастичных и коллективных состояний. Изучена роль частично-частичных взаимодействий. Показано, что они оказывают большое влияние на гамов-теллеровские β^+ распады и (n,p) переходы. На основе хорошего их описания утверждается, что перенормировка в ядрах константы аксиально-векторного слабого взаимодействия невелика и $|G_A/G_V| \geq 1$. В КФМЯ с p-h и p-p взаимодействиями получено достаточно хорошее описание низколежащих квадрупольных, октупольных и гексадекапольных вибрационных состояний в ряде деформированных ядер. Исследовано распределение Eλ-силы среди низколежащих состояний и показано, что в отдельных случаях оно отлично от стандартного - основная часть Eλ-силы сконцентрирована не на первом, а на более высоких состояниях.

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Effect of Particle-Particle Interactions on Excited States of Spherical and Deformed Nuclei

It is shown that in the quasiparticle-phonon nuclear model one can perform calculations with effective separable interactions of a finite rank n_{max}. Inclusion of separable interactions with $n_{max} > 1$ does not lead to essential complication of the calculations of fragmentation of quasiparticle and collective states. The role of particle-particle interactions is studied. They are shown to affect greatly the Gamow-Teller β^+ decays and (n,p) transitions. Their good description allows us to make a conclusion that renormalization in nuclei of the constant of the axial-vector weak interaction is not large and $|G_A/G_V| \geq 1$. A good enough description of low-lying quadrupole, octupole and hexadecapole states in some deformed nuclei has been obtained in the QPNM with p-h and p-p interactions. The Eλ strength distribution among low-lying states is studied; it is shown that in some cases it differs from the standard one: the main part of the Eλ strength is concentrated not on the first but on higher-lying states.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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