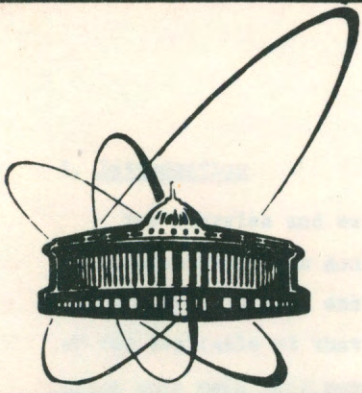

объединенный
институт
ядерных
исследований
дубна

E4-89-211

V.G.Soloviev, N.Yu.Shirikova

DESCRIPTION OF LOW-LYING VIBRATIONAL $K^{\pi} \neq 0^{+}$ STATES OF DEFORMED NUCLEI IN THE QUASIPARTICLE-PHONON NUCLEAR MODEL

Submitted to "Zeitschrift für Physik A"

1989

1. Introduction

The energies and wave functions of two-quasiparticle and one-phonon states in doubly-even deformed nuclei were calculated in 1960-1975. A good enough description was obtained (see [1-4]) of the available at that time experimental data; the predictions were made which were later confirmed experimentally in many cases. It seems to us that new calculations of vibrational states in deformed nuclei are needed. This is necessitated by a large amount of new experimental data in addition to the first quadrupole and octupole states. The experimental data are available on hexadecapole states and on higher-lying collective and weakly collective states. Many experimental data are expected at a new generation of accelerators and the results of calculations may turn out to be useful. Vibrational states are to be calculated on a new basis. As is known, there are particle-hole (p-h) and particle-particle (p-p) effective interactions between quasiparticles. Particle-hole interactions are responsible for the formation of vibrational low-lying states and giant resonances. Therefore, only p-h interactions are usually taken into account. It is necessary to take p-p interactions as well. It is to be mentioned that p-p interactions greatly influence the double β decay [5], probabilities of β^+ decays in spherical [6-8] and deformed [9] nuclei and the strength functions of (n,p) transitions [7]. A new series of calculations is performed within the quasiparticle-phonon nuclear model (QPNM) [10-13] with the wave functions containing one- and two-phonon components and taking account of the Pauli principle in two-phonon components. The role of two-phonon com-

ponents of the wave functions and the influence of the Pauli principle have been studied in [14-16], hexadecapole vibrational states in [17], influence of the quadrupole pairing on the energies of two-quasiparticle states in [18] and the influence of p-p interactions on the properties of $K^{\pi}=2^{+}$ states in [19]. The importance of the monopole pairing is evident and it is interesting to study the influence of the quadrupole pairing.

In recent years, low-lying states in deformed nuclei have been studied within the interacting boson model (IBM). It is important to compare the description of deformed nuclei in the QPNM and IBM and to reveal shortcomings and advantages of each model. This comparison has been made in [20] on the basis of earlier calculations. It showed the necessity of new calculations within the QPNM and IBM and further experimental studies of deformed nuclei.

It is reasonable to develop a general description of non-rotational states of well deformed nuclei with monopole and quadrupole pairing and isoscalar and isovector multipole p-h and p-p interactions between quasiparticles. Then, spin-multipole interactions and states of the magnetic type are to be studied. Further, detailed calculations should be made with the Coriolis interaction. As a first step, the general QPNM Hamiltonian and equations for 0^{+} states in deformed nuclei have been derived in [21]. In the present paper, as a second step, we shall describe in the QPNM vibrational states of the electric type with $K^{\pi}\neq 0^{+}$ and demonstrate its specific features taking ^{168}Er, ^{172}Yb and ^{178}Hf as an example.

2. Equations for $K^{\pi}\neq 0^{+}$ Vibrational States and the Details of Calculations

The mathematical apparatus of the QPNM for deformed nuclei has first been given in [10,11,14,16] and in more detail in

[12,13]. Formulae allowing for p-h and p-p interactions are given in [19,21]; they will be used in the present paper. Therefore, we shall give only several necessary formulae and also formulae that have not been given in [21]. The QPNM Hamiltonian contains an average field of neutron and proton systems as a deformed axial symmetric Saxon-Woods potential, monopole and quadrupole pairing and the effective p-h and p-p interactions. In this paper, we shall use only the multipole interactions. The Hamiltonian is transformed by using the Bogolubov transformation

$$a_{q\sigma}=u_q\alpha_{q\sigma}+\sigma v_q\alpha^{+}_{q-\sigma} \qquad (1)$$

and introducing the RPA phonons $Q^{+}_{\lambda\mu i\sigma}$ and $Q_{\lambda\mu i\sigma}$ where

$$Q^{+}_{\lambda\mu i\sigma}=\frac{1}{2}\sum_{qq'}\left[\psi^{\lambda\mu i}_{qq'}A^{+}(qq';\mu\sigma)-\varphi^{\lambda\mu i}_{qq'}A(qq';\mu-\sigma)\right], \qquad (2)$$

$$A^{+}(qq';\mu\sigma)=\sum_{\sigma'}\delta_{\sigma'\sigma'(K-K'),\mu\sigma}\alpha^{+}_{q\sigma'}\alpha^{+}_{q'-\sigma'} \text{ or } \sum_{\sigma'}\delta_{\sigma'\sigma'(K-K'),\mu\sigma}\alpha^{+}_{q\sigma'}\alpha^{+}_{q'\sigma'}.$$

Here $q\sigma$ are quantum numbers of one-particle states, q equals K^{π} and asymptotic quantum numbers introduced by S.G.Nilsson, $\sigma=\pm 1$; K is the projection of the angular momentum onto the nuclear symmetry axis, $\alpha^{+}_{q\sigma}$ and $\alpha_{q\sigma}$ are the creation and absorption operators of quasiparticles.

The QPNM Hamiltonian is written in the form

$$H_{QPNM}=\sum_{q\sigma}\tilde{\varepsilon}_q\alpha^{+}_{q\sigma}\alpha_{q\sigma}+H_{v}+H_{vq} \qquad (3)$$

where $\tilde{\varepsilon}_q$ is the quasiparticle energy with the monopole and quadrupole pairing [18,21]. The explicit form of the Hamiltonian and notation are given in [21]. To derive the RPA equations the following variational principle is used:

$$\delta\left\{\langle Q_{\lambda\mu i\sigma}\left|\left[\sum_{q\sigma}\tilde{\varepsilon}_q\alpha^{+}_{q\sigma}\alpha_{q\sigma}+H_v\right]\right|Q^{+}_{\lambda\mu i\sigma}\rangle-\frac{\omega_{\lambda\mu i}}{2}\left[\sum_{qq'}g^{\lambda\mu i}_{qq'}w^{\lambda\mu i}_{qq'}-2\right]\right\}=0, \qquad (4)$$

where

$$g^{\lambda\mu i}_{qq'} = \psi^{\lambda\mu i}_{qq'} + \varphi^{\lambda\mu i}_{qq'}\ ,\quad w^{\lambda\mu i}_{qq'} = \psi^{\lambda\mu i}_{qq'} - \varphi^{\lambda\mu i}_{qq'}\ .$$

The RPA equations for $K^\pi = 0^+$ states are given in [21]. The RPA equations for multipole states $\lambda\mu$ with $\lambda\mu \neq 20$, i.e. with $K^\pi \neq 0^+$ states have the following form:

$$\widetilde{\varepsilon}_{qq'} g^{\lambda\mu i}_{qq'} - \omega_{\lambda\mu i} w^{\lambda\mu i}_{qq'} - (\varkappa_0^{\lambda\mu} + \varkappa_1^{\lambda\mu}) f^{\lambda\mu}(qq') u^{(+)}_{qq'} D^{\lambda\mu i}_{\tau} -$$

$$- (\varkappa_0^{\lambda\mu} - \varkappa_1^{\lambda\mu}) f^{\lambda\mu}(qq') u^{(+)}_{qq'} D^{\lambda\mu i}_{-\tau} - G^{\lambda\mu}_{\tau} f^{\lambda\mu}(qq') v^{(-)}_{qq'} D^{\lambda\mu i}_{g\tau} = 0, \qquad (5)$$

$$\widetilde{\varepsilon}_{qq'} w^{\lambda\mu i}_{qq'} - \omega_{\lambda\mu i} g^{\lambda\mu i}_{qq'} - G^{\lambda\mu}_{\tau} f^{\lambda\mu}(qq') v^{(+)}_{qq'} D^{\lambda\mu i}_{w\tau}\ . \qquad (5')$$

Here $\varkappa_0^{\lambda\mu}$ and $\varkappa_1^{\lambda\mu}$ are the isoscalar and isovector constants of the p-h interaction of multipolarity λ with projection μ, $G^{\lambda\mu}_{\tau}$ is the p-p interaction constant; $\widetilde{\varepsilon}_{qq'} = \widetilde{\varepsilon}_q + \widetilde{\varepsilon}_{q'}$; the single-particle matrix elements $f^{\lambda\mu}(qq') = \langle q | R_\lambda(r) Y_{\lambda\mu}(\theta,\varphi) | q' \rangle$ are taken with $R_\lambda(r) = \frac{\partial V(r)}{\partial r}$ where $V(r)$ is the central part of the Saxon-Woods potential.

$$D^{\lambda\mu i}_{\tau} = \sum^{\tau}_{qq'} f^{\lambda\mu}(qq') u^{(+)}_{qq'} g^{\lambda\mu i}_{qq'}\ ,$$

$$D^{\lambda\mu i}_{g\tau} = \sum^{\tau}_{qq'} f^{\lambda\mu}(qq') v^{(-)}_{qq'} g^{\lambda\mu i}_{qq'}\ ,$$

$$D^{\lambda\mu i}_{w\tau} = \sum_{qq'} f^{\lambda\mu}(qq') v^{(+)}_{qq'} w^{\lambda\mu i}_{qq'}\ ,$$

$$u^{(\pm)}_{qq'} = u_q v_{q'} \pm u_{q'} v_q\ ,$$

$$v^{(\pm)}_{qq'} = u_q u_{q'} \pm v_q v_{q'}\ .$$

Summation over single-particle states of the neutron and proton systems is denoted by $\sum_{qq'}$; $\sum^{\tau}_{qq'}$ implies summation

over the levels of the neutron at $\tau = n$ and proton at $\tau = p$ systems. Eqs. (5) and (5') are used to derive the secular equation for the energies $\omega_{\lambda\mu i}$ of one-phonon states as an equality to zero of the determinant of rank 6.

To describe deformed nuclei in the QPNM we can use more complex interactions. Thus, in [22] the QPNM equations with effective separable interactions of a finite rank are derived for spherical nuclei. For separable interactions of the rank n_0 the RPA secular equation is given as an equality to zero of a determinant of the rank $6n_0$.

Let us give formulae for nonrotational states with $K^\pi \neq 0^+$ of doubly even deformed nuclei within the QPNM taking account of p-h and p-p interactions whose wave functions consist of one and two-phonon terms, namely

$$\Psi_\nu(K_0^{\pi_0} \sigma_0) = \Big\{ \sum_{i_0} R^\nu_{i_0} Q^+_{\lambda_0\mu_0 i_0 \sigma_0} + \sum_{\substack{\sigma_1\sigma_2 \\ \lambda_1\mu_1 i_1 \\ \lambda_2\mu_2 i_2}} \frac{(1+\delta^{\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2})^{\frac{1}{2}} \delta_{\sigma_1\mu_1 + \sigma_2\mu_2, \sigma_0 K_0}}{2[1+\delta_{K_0,0}(1-\delta_{\mu_1,0})]^{\frac{1}{2}}} \cdot \qquad (6)$$

$$\cdot P^\nu_{\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2} Q^+_{\lambda_1\mu_1 i_1 \sigma_1} \cdot Q^+_{\lambda_2\mu_2 i_2 \sigma_2} \Big\} \Psi_0\ ,$$

where $\mu_0 = K_0$. Its normalisation condition is

$$\Big[\sum_{i_0} (R^\nu_{i_0})^2 + \sum_{(\lambda_1\mu_1 i_1) \geq (\lambda_2\mu_2 i_2)} (P^\nu_{\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2})^2 (1 + \mathcal{H}^{K_0}(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)) \Big] = 1\ . \qquad (7)$$

The function $\mathcal{H}^{K_0}(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)$ is responsible for the effect of the Pauli principle in two-phonon terms of the wave function (6); its form is presented in [13,14,16].

Using the variational principle we get the following equations

$$(\omega_{\lambda_0\mu_0 i_0} - \eta_\nu) R^\nu_{i_0} - \sum_{(\lambda_1\mu_1 i_1) \geq (\lambda_2\mu_2 i_2)} (1+\delta^{\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2})^{-\frac{1}{2}} (1+\delta_{K_0,0}(1-\delta_{\mu_1,0}))^{-\frac{1}{2}}.$$

$$\cdot P^{\nu}_{\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2} U^{\lambda_0\mu_0 i_0}_{\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2}\left(1+\mathcal{K}^{K_0}(\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2)\right)=0,$$

$$\left(\omega_{\lambda_1\mu_1 i_1}+\omega_{\lambda_2\mu_2 i_2}+\Delta\omega(\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2)-\eta_\nu\right)P^{\nu}_{\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2}- \tag{8}$$

$$-\sum_{i_0}\left(1+\delta_{\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2}\right)^{\frac{1}{2}}\left(1+\delta_{K_0,0}\left(1-\delta_{\mu_1,0}\right)\right)^{-\frac{1}{2}}R^{\nu}_{i_0}U^{\lambda_0\mu_0 i_0}_{\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2}=0,$$

where

$$\Delta\omega(\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2)=\sum_{i'}\left\{\mathcal{K}^{K_0}(\lambda_2\mu_2 i_2,\lambda_1\mu_1 i'|\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2)W^{\lambda_1\mu_1}_{i_1 i'}+\right.$$

$$\left.+\mathcal{K}^{K_0}(\lambda_2\mu_2 i'\lambda_1\mu_1 i_1|\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2)W^{\lambda_2\mu_2}_{i_2 i'}\right\}, \tag{9}$$

$$U^{\lambda_0\mu_0 i_0}_{\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2}\left[1+\mathcal{K}^{K_0}(\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2)\right]=-\frac{1}{2}\sum_{\sigma_1\sigma_2}\delta_{\sigma_1\mu_1+\sigma_2\mu_2,\sigma_0 K_0}\cdot$$

$$\cdot\left\{\langle Q_{\lambda_0\mu_0 i_0\sigma_0}H_{\upsilon q}Q^{+}_{\lambda_1\mu_1 i_1}Q^{+}_{\lambda_2\mu_2 i_2}\rangle+\langle Q_{\lambda_2\mu_2 i_2}Q_{\lambda_1\mu_1 i_1}H_{\upsilon q}Q^{+}_{\lambda_0\mu_0 i_0\sigma_0}\rangle\right\}.$$

From eqs. (8), taking account of condition (7), we calculate the energies η_ν and functions $R^{\nu}_{i_0}$ and $P^{\nu}_{\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2}$. Equations (8) coincide with those of [13,16,23]. These formulae will be used in further calculations.

The calculations are performed with the single-particle energies and wave functions of the Saxon-Woods potential with the parameters for the zones A = 165,173 and 181 fixed in 1968--73 and presented in [4,24]. For all nuclei of the zone the same deformation parameters β_2 and β_4 are used. Thus, for

^{178}Hf β_2=0.24 and β_4= -0.03, the parameter of the hexadecapole deformation β_4 differs from the one proposed in [25] where β_4 =-0.166. This difference of β_4 influences the energies of some two-quasiparticle poles. The single-particle spectrum is taken from the bottom of the well up to +5 MeV. The monopole and quadrupole pairing constants were fixed [18] by the pairing energies and those of two-quasiparticle states with $K>4$. The energies of two-quasiparticle poles were calculated taking account of the blocking effect and the Gallagher-Moszkowski corrections. As has been shown in [18], the inclusion of the quadrupole pairing does not practically improve the description of the energies of two-quasiparticle states with $K>4$. The isovector interaction constants are equal to $\varkappa_1^{\lambda\mu}=-1.5\,\varkappa_0^{\lambda\mu}$; with this relation a good description of the isovector quadrupole and octupole giant resonances in deformed nuclei was obtained. The calculations performed with $\varkappa_1^{\lambda\mu}=-\varkappa_0^{\lambda\mu}$, when the neutron--proton interactions are increased, do not provide considerable effect. The constants $\varkappa_0^{\lambda\mu}$ and $G_\lambda^{\lambda\mu}$ were chosen from the experimental energies of the first $K^{\pi}_{\nu=1}$ nonrotational states described by the wave function (6). The dependence of the characteristics of $K^{\pi}=2^{+}$ states on G_τ^{22} has been studied in [19]; it was shown that for $G_\tau^{22}<0.5\,\varkappa_0^{22}$ the influence of the p-p interaction is negligible and it can be neglected, at $G_\tau^{22}=(0.8\div 1.0)\,\varkappa_0^{22}$ the best description of the energies and B(E2) values is achieved, for $G_\tau^{22}>1.1\,\varkappa_0^{22}$ the discrepancy with the experimental data is observed and for $G_\tau^{22}>1.2\,\varkappa_0^{22}$ RPA becomes inapplicable.

The dependence of p-p interactions on the truncation of the space of single-particle states has been studied in [19]; it was shown that the decrease in the number of single-particle states for the p-p interaction can be compensated by

renormalisation of the constant G_τ^{22}. In our calculations we have used the same single-particle basis for p-h and p-p interactions and assumed $G_p^{\lambda\mu} = G_n^{\lambda\mu} \equiv G^{\lambda\mu}$ and $G^{\lambda\mu} = 0.9\,\varkappa_0^{\lambda\mu}$.

In this paper we study the states with $K^\pi \neq 0^+$. As a basis we take into account phonons with multipolarities $\lambda\mu$ =20,22, 30,31,32,33, 43 and 44, and for each $\lambda\mu$ we use 10 RPA phonons. Phonons with $K^\pi \neq 20$ are calculated with p-h and p-p interactions. Phonons with $\lambda\mu$ =20 are calculated with G_τ^{20} =0. A more detailed description of phonons with $\lambda\mu$ =20 is unnecessary as these phonons enter only into two-phonon terms of the wave function (3). The constants $\varkappa_0^{\lambda\mu}$ are fixed for each value of $\lambda\mu$ equally for all nuclei except for $\varkappa_0^{32}$ that changes within 10%; $\varkappa_0^{43}$ in ^{168}Er and $\varkappa_0^{44}$ in ^{178}Hf are taken 10% less than in other nuclei.

The present calculations pretend to a qualitatively correct description of vibrational states. For a detailed description one needs to take the Coriolis interaction into account, to determine more accurately the parameters of the Saxon-Woods potential for each nucleus and to calculate the parameters of the quadrupole and hexadecapole deformation.

We have calculated the energies and wave functions of non-rotational states and the reduced probabilities of E2, E3, and E4 transitions from the ground state $0^+_{g.s.}$ to the excited ones with a fixed value of $I^\pi K$. The reduced probabilities of electromagnetic and isoscalar transitions are written as

$$B(E\lambda, 0^+_{g.s.} \to I^\pi K_\nu) = \langle 0\,0\,\lambda\mu | I K\rangle^2 \Big\{ \sum_{i_0} R^\nu_{i_0} \frac{e}{\sqrt{2}} \Big[(1+e_p^{eff}) \cdot \sum_{q_1 q_2}{}^{p} \rho^{\lambda\mu}(q_1 q_2) U^{(+)}_{q_1 q_2} g^{\lambda\mu i_0}_{q_1 q_2} + e_n^{eff} \sum_{q_1 q_2}{}^{n} \rho^{\lambda\mu}(q_1 q_2) U^{(+)}_{q_1 q_2} g^{\lambda\mu i_0}_{q_1 q_2} \Big] \Big\}^2, \quad (10)$$

$$B(IS E\lambda; 0^+_{g.s.} \to I^\pi K_\nu) = \langle 0\,0\,\lambda\mu | I K\rangle^2 \Big\{ \sum_{i_0} R^\nu_{i_0} \frac{e}{\sqrt{2}} \frac{Z}{A} \cdot \sum_{q_1 q_2} \rho^{\lambda\mu}(q_1 q_2) U^{(+)}_{q_1 q_2} g^{\lambda\mu i_0}_{q_1 q_2} \Big\}^2, \quad (11)$$

where $\rho^{\lambda\mu}(q_1 q_2)$ is the single-particle matrix element of the operator $r^\lambda (Y_{\lambda\mu} + (-1)^\mu Y_{\lambda,-\mu})$; e_τ^{eff} is the proton or neutron effective charge. Our calculations are performed with the radial dependence of multipole interactions in the form of $\frac{\partial V(r)}{\partial r}$ instead of r^λ in [3,4], which leads to a certain decrease in B(Eλ) values. The B(Eλ) calculated with $e_\tau^{eff}=0$ are close to the B(ISEλ) values. More correct are the B(Eλ)-values calculated with e_τ^{eff} =0.2 since we use the truncation of the space of single-particle states. Calculations with single-particle states from the bottom of the well up to +50 MeV with e_τ^{eff}=0 give almost the same B(Eλ) -values as the calculations with single-particle states from the bottom of the well up to +5 MeV and with e_τ^{eff} =0.15÷0.20. In the last case, the B(Eλ) values calculated with e_τ^{eff}=0.2 are almost twice as large as those calculated with e_τ^{eff} =0.

3. Low-Lying Vibrational States with $K^\pi \neq 0^+$ in ^{168}Er, ^{172}Yb and ^{178}Hf

We shall now calculate quadrupole with $K^\pi = 2^+$, octupole with $K^\pi = 0^-$, 1^-, 2^- and 3^- and hexadecapole with $K^\pi = 3^+$ and 4^+ states in ^{168}Er, ^{172}Yb and ^{178}Hf for which there are the most complete experimental data. The results of calculations are listed in Tables 1-4 only for the states for which there are experimental data. The B(Eλ) values are extracted from the Coulomb excitation; B(ISEλ) values are obtained in [26] from the inelastic scattering of α particles and deuterons. Since B(Eλ) and B(ISEλ) values are close to each other, we will

Table I. Vibrational states with $K^{\pi} = 2^+$, 3^+ and 4^+ in ^{168}Er

K^{π}_{ν}	Experiment			Calculation in the QPNM		
	E_x MeV	$B(E\lambda)_{s.p.u.}$	Structure %	E_x MeV	$B(E\lambda)_{s.p.u.}$	Structure %
2^+_1	0.821	B(E2)=4.7	pp 413↑ - 411↑ 50 pp 411↑ + 411↑ 37	0.8	B(E2) = 4.6	221: 96 221 : pp 413↑ - 411↑ 40 pp 411↑ + 411↑ 30 nn 523↑ - 521↑ 20 nn 521↑ + 521↑ 8
2^+_2	0.848			1.7	B(E2) = 0.01	222 : 98 222: mn 512↑ - 521↑ 97 pp 411↑ + 411↑ 2
2^+_3	1.930			1.8	B(E2) = 0.2	223 : 94 223: nn 523↑ - 521↑ 60 pp 411↑ + 411↑ 13
2^+_4	2.193	$(\vec{t}\,\alpha)$: pp 411↑ + 411↑ (20-30)		2.2	B(E2) = 0.06	224: 98 224: nn 521↑ + 521↑ 65 pp 411↑ + 411↑ 28
2^+_5	2.425	$(\vec{t}\,\alpha)$: pp 411↑ + 411↑		2.3	B(E2) = 0.2	225 : 97 225: nn 633↑ - 651↑ 36 nn 521↑ + 521↑ 16 pp 411↑ + 411↑ 15 nn 624↑ - 642↑ 10
3^+_1	1.653	(dd') is large for 4^+3_1		1.5	B(E4) = 0.4	431 : 99 431: nn 512↑ + 521↑ 98
3^+_2	2.186			2.0	B(E4) = 0.003	432 : 98 432: nn 523↑ + 521↑
4^+_1	2.055	B(E4) = 0.6		2.1	B(E4) = 2.0	441 : 88 {201, 441} : 4 441: nn 514↑ + 521↑ 15 nn 512↑ + 512↑ 14 nn 523↑ + 521↑ 10 pp 523↑ + 541↑ 6
4^+_2	2.238			2.5	B(E4) = 0.1	442:53 443: 38 {221, 221} : 1 442: nn 514↑ + 521↑ 62 nn 512↑ + 512↑ 32

Table 2. Octupole states in ^{168}Er

K^{π}_{ν}	Experiment			Calculation in QPNM		
	E_x MeV	$B(E\lambda)_{s.p.u.}$	Structure %	E_x MeV	$B(E\lambda)_{s.p.u.}$	Structure %
0^-_1	1.786	B(E3) = 1.96		1.9	B(E3)=3.0	301:98 301: nn 642↑ - 512↑ 25 nn 514↓ - 633↑ 7 pp 523↑ - 404↓ 3
1^-_1	1.358	B(E3) = 3.92 (dt),(dp) : nn 633↑ - 512↑ 80		1.4	B(E3)=4.6	311:98 311: nn 633↑ - 512↑ 72 nn 633↑ - 523↓ 4
1^-_2	1.936	$(\vec{t},\alpha)$: is small		1.9	B(E3)=0.35	312:96; {221 311} : 1 312: nn 633↑ - 523↓ 85
2^-_1	1.569	B(E3) = 4.94		1.5	B(E3)=4.6	321:94; {201,321}: 3 321: nn 633↑ - 521↓ 25 nn 642↑ - 521↓ 10 pp 523↑ - 411↑ 29
2^-_2	2.230			2.1	B(E3)=0.2	322:96
3^-_1	1.542	B(E3)=0.25 (dp):nn633↑ - 521↓ 90 $(\vec{t},\alpha)$:pp523↑ -411↓ 4		1.6	B(E3)=0.14	331:98 331: nn 633↑ - 521↓ 95 pp 523↑ - 411↓ 2
3^-_2	1.828	B(E3)=0.60 (dp):nn633↑ - 521↓ 10		2.1	B(E3)=0.60	322:75; 333:20 332: nn 521↓ + 642↑ 80 pp 523↑ - 411↓ 12 nn 633↑ - 521↓ 2
3^-_3	1.999	B(E3)=0.42 $(\vec{t},\alpha)$:pp523↑ -411↓ 75 (dp) :nn633↑ -521↓ 10		2.2	B(E3)=0.3	333:72; 332:22 333: pp 523↑ - 411↓ 76 pp 514↑ - 512↓ 9 nn 633↑ - 521↓ 1
3^-_4	2.262	B(E3)=4.68		2.4	B(E3)=2.0	334:91; 333:2; {221, 311} :2 334: pp 514↑ - 411↑ 51 nn 624↑ - 521↓ 12

Table 3. Vibrational states with $K^{\pi} \neq 0^{+}$ in ^{172}Yb

K^{π}_{ν}	Experiment: $\mathcal{E}_{\nu}$ MeV	Experiment: $B(E\lambda)_{s.p.u}$ Structure %	Calculation in QPNM: $\mathcal{E}_{\nu}$ MeV	Calculation in QPNM: $B(E\lambda)_{s.p.u.}$ Structure %
2^{+}_{1}	1.466	B(E2)=1.4 (dt)	1.4	B(E2)=1.5 221:60; 222:10 {201,221}: 2 {201,222}: 1.5 221: nn 512↑− 521↓ 32 nn 512↑− 510↑ 14
2^{+}_{2}	1.608	B(E2)=0.42 (dp) (dt): nn 512↑− 511↓ is large	1.6	B(E2)=0.7 222:81; 221:10 222: nn 512↑− 521↓ 65
3^{+}_{1}	1.172	B(E4)=6.9 μ : nn 512↑+ 521↓ ~ 70 (pα) : pp 404↓− 411↓ ~ 27	1.16	B(E4)=2.7 431:99 431: nn 512↑+ 521↓ 50 pp 404↓− 411↓ 25
3^{+}_{2}	1.663	(pα): pp 404↓− 411↓ 26	1.65	B(E4)=0.2 432:99 432:nn 512↑+ 521↓ 48 pp 404↓− 411↓ 40
3^{+}_{3}	2.175		1.9	B(E4)=0.03 433:99
4^{+}_{1}	2.073	(pα): pp 404↓+ 411↓ is noticeable	1.9	B(E4)=0.13 411:92 {201,441}:3 {221,222}:1 441: nn 514↓+ 521↓ 80 pp 404↓+ 411↓ 17
4^{+}_{2}	2.344		2.1	B(E4)=0.01 442:93
4^{+}_{3}	2.599		2.3	B(E4)=2.5 443:80; 441:2
0^{-}_{1}	1.600	B(E3)=0.63	1.68	B(E3)=1.1 301:98 301: nn 514↓− 633↑ 40 nn 512↑− 642↑ 4 pp 523↑− 404↓ 1
1^{-}_{1}	1.155 (dt):	B(E3)=1.3 nn 633↑− 512↑ is large	1.2	B(E3)=1.8 311:97 nn 633↑− 512↑ 92

Continuation of Table 3

K^{π}_{ν}	Experiment: $\mathcal{E}_{\nu}$ MeV	Experiment: $B(E\lambda)_{s.p.u.}$ Structure %	Calculation in QPNM: $\mathcal{E}_{\nu}$ MeV	Calculation in QPNM: $B(E\lambda)_{s.p.u.}$ Structure %
2^{-}_{1}	1.757	B(E3)=5.2	1.6	B(E3)=2.5, 321: 97 321: nn 624↑− 512↑ 83 nn 633↑− 521↑ 2 pp 514↑− 402↑ 3 pp 523↑− 411↑ 2
3^{-}_{1}	2.030	B(E3)=4.7	2.0	B(E3)=2.3 321:98 331: nn 633↑− 521↓ 54 nn 615↑− 512↑ 9 nn 624↑− 521↑ 6 pp 514↑− 411↑ 18

not distinguish between them. The experimental and calculated $B(E\lambda)$ values are given in the single-particle units. The experimental data on the structure of states are obtained, as a rule, from the one-nucleon transfer reactions. The contribution of a two-quasiparticle component is determined in some cases given in Tables 1-4 in per cent whereas in other cases it is shown through what two-quasiparticle component the reaction proceeds.

The calculations were performed with the single-particle energies and wave functions of the Saxon-Woods potential of the zones A = 165 for ^{168}Er, A = 173 for ^{172}Yb and A = 181 for ^{178}Hf. The energies and $B(E\lambda)$ values are calculated without taking the Coriolis interaction into account. The calculated structure is given as a contribution (in per cent) of the one-phonon $\lambda\mu i$ and two-phonon $\{\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2\}$ components to normalisation (7) of the wave function (6). In the contribution

Table 4. Vibrational states with $K^{\pi} \neq 0^+$ in ^{178}Hf

K^{π}_{ν}	Experiment			Calculation in QPNM		
	$\mathcal{E}_{\nu}$ MeV	$B(E\lambda)_{s.p.u.}$	Structure %	η_{ν} MeV	$B(E\lambda)_{s.p.u.}$	Structure %
2^+_1	1.174	B(E2)=3.9 (dp): nn 514↓ - 512↓		1.12	B(E2)=4.1	221:94 {221,441} : 1 221: nn 514↓ - 512↓ 29 nn 512↑ - 510↑ 32 nn 512↓ - 510↑ 9 nn 642↑ - 642↑ 2
2^+_2	1.891	(dp): nn 514↓ - 512↓		2.0	B(E2)=0.01	222:92 {202,221} : 3 222: nn 514↓ - 512↓ 56 nn 512↑ - 510↑ 40
3^+_1	1.758	(dp): nn 514↓ - 510↑		1.8	B(E4)=1.7	431:99 431: nn 514↓ - 510↑ 21 nn 514↓ - 521↓ 10 pp 404↓ - 411↓ 37 pp 402↑ - 411↓ 10
3^+_2	(1.942)			1.9	B(E4)=0.4	432:99 432: nn 514↓ - 510↑ 77 pp 404↓ - 411↓ 10
4^+_1	1.554	(dp): nn 514↓ + 510↑		1.5	B(E4)=2.0	441:94; {221,221} 3 441: nn 514↓ + 510↑ 60 nn 512↑ + 512↓ 23
4^+_2	(2.007)			2.0	B(E4)=0.0004	442:99 442: pp 404↓ + 400↑ 98
1^-_1	1.310			1.4	B(E3)=0.5	311:89 331: nn 514↓ - 624↑ 96 pp 404↓ - 514↑ 1
2^-_1	1.260	B(E3)=4.0 (dt): nn 624↑ - 512↑		1.2	B(E3)=2.0	321:98 321: nn 624↑ - 512↑ 86 pp 514↑ - 402↑ 8
2^-_2	1.567			1.9	B(E3)=0.8	322:96 322: pp 514↑ - 402↑ 88 nn 624↑ - 512↑ 10

Continuation of Table 4

K^{π}_{ν}	Experiment			Calculation in QPNM		
	$\mathcal{E}$ MeV	$B(E\lambda)_{s.p.u.}$	Structure %	η_{ν} MeV	$B(E\lambda)_{s.p.u.}$	Structure %
2^-_3	1.857			2.6	B(E3)=0.02	323:98 323: nn 615↑ - 514↓ 98
3^-_1	1.803			1.9	B(E3)=4.0	331:98 331: nn 615↑ - 512↑ 45 nn 624↑ - 521↓ 4 pp 514↑ - 411↑ 24 pp 505↑ - 402↑ 5

of two-phonon components we take into account the factor $\{1 + \mathcal{H}^{K_0}(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)\}$. Then, we list the largest two-quasineutron nn and two-quasiproton pp components of the wave functions of one-phonon states $\lambda\mu i$. To denote single-particle states we use the asymptotic quantum numbers $N n_z \Lambda\uparrow$ at $K=\Lambda+\frac{1}{2}$ and $N n_z \Lambda\downarrow$ at $K=\Lambda-\frac{1}{2}$.

The results of calculations of the vibrational states in ^{168}Er and experimental data [26-28] are listed in Tables 1 and 2. In ^{168}Er five states with $K^{\pi}=2^+$ are observed. The states 2^+_1 and 2^+_4 are excited in the $(\vec{t}\,\alpha)$ reaction. According to the calculations, each wave function of the first five $K^{\pi}=2^+$ states has the dominating one-phonon component. The first 2^+_1 state is collective and the other is weakly collective. The calculations are reasonable in agreement with experimental data on the structure of the first five $K^{\pi}=2^+$ states. Thus, a larger part of the configuration $pp\,411\uparrow+411\downarrow$ belongs to the 2^+_1 and 2^+_4 states; and a smaller part, to the 2^+_3 and 2^+_5 states; however, the 2^+_3 state has not been observed in the $(\vec{t}\,\alpha)$ reaction.

According to [29] the states $I^{\pi}K = 4^+3$, in the Er, Yb and Hf isotopes are strongly excited in the (dd') reactions,

which indicates collectivity of the 3_1^+ states. Description of the $K^\pi=3^+$ and 4^+ states in the QPNM has been made in [17]. In the present calculations we have taken into account the p-p interaction and a better fit of the constants $\varkappa_0^{43}$ and $\varkappa_0^{44}$ was made. The states 3_1^+, 3_2^+, 4_1^+ and 4_2^+ have the dominating one--phonon components; the total contribution of two-phonon components in the 4_1^+ and 4_2^+ states is less than 10%. In comparison with the calculations in [17] the 4_1^+ state energy decreases and becomes close to the experimental one, the difference between the energies of the 4_1^+ and 4_2^+ states becomes larger whereas in [17] they were close to each other. The calculated B(E4) value for the 4_1^+ state turned out to be three times as larger as the experimental one.

The first octupole states 0_1^-, 1_1^- and 2_1^- in ^{168}Er are collective and the B(E3) values for excitation of the $I^\pi K_\nu=3^-K_1$ states are large. According to the calculations the 0_1^-, 1_1^- and 2_1^- states are collective and the 0_2^-, 1_2^- and 2_2^- states are weakly collective. All of them have the dominating one-phonon components. The structure of the 1_1^- state is correctly described. The $K^\pi=3^-$ states in ^{168}Er have an unusual behaviour. The fourth 3_4^- state is collective; the E3 strength concentrated on it is 3.5 times as large as that on the 3_1^-, 3_2^- and 3_3^- states. The 3_1^-, 3_2^- and 3_3^- states are also collective since the B(E3) value is rather large. Their wave functions contain two-quasiparticle components nn 633↑ -521↓ and pp 523↑-411↓ whose distribution is qualitatively correctly reproduced by calculations. A qualitatively correct description of the energies and structure of octupole states is obtained.

According to the experimental data [28] on the (dp) and $(\vec{t}d)$ reactions the 4_1^- and 4_2^- states in ^{168}Er are not pure two--quasiparticle states. The wave function of the 4_1^- state with

the energy 1.094 MeV includes the components nn 633↑+ 521↓~70%, pp 411↓+ 523↑~25%, in 4_2^- with the energy 1.905 MeV pp 411↓+ + 523↑~60% and nn 633↑+ 521↓~30%. Thus, the two-quasineutron and two-quasiproton components are distributed among the 4_1^- and 4_2^- states. To describe this mixing, in [18] the multipole interactions with $\lambda\mu=54$ have been introduced. For ^{168}Er the following results have been obtained: at $\varkappa_0^{54}=0.018$ fm^{2}MeV^{-1} the energies and structure of the 4_1^- and 4_2^- states are equal to

ω_{541} = 1.0 MeV, nn 633↑+ 521↓ 86%, pp 411↓+ 523↑ 12%;

ω_{542} = 1.5 MeV, nn 633↑+ 521↓ 12%, pp 411↓+ 523↑ 87%;

At $\varkappa_0^{54}=0.020$ fm^{2}MeV^{-1} are equal to

ω_{541} = 0.95 MeV, nn 633↑+ 521↓ 81%, pp 411↓+ 523↑ 18%;

ω_{542} = 1.5 MeV, nn 633↑+ 521↓ 18%, pp 411↓+ 523↑ 80%.

Thus, there is mixing of a two-quasineutron with a two-quasiproton state. The energy ω_{542} is less than the experimental one due to the scheme of single-particle states. This example indicates that in some cases multipole interactions of such a high multipolarity as $\lambda=5$ are important.

The calculated energies of two-quasiparticle states in ^{168}Er are close to the energies given in [4] since the inclusion of the quadrupole pairing does not improve considerably their description.

The results of calculations and experimental data [26,30-33] for ^{172}Yb are shown in Table 3. In ^{172}Yb a very rare case takes place when not only the 2_1^+ state but also the 2_2^+ state are collective. Acoording to the calculations [2,4], the B(E2) value for the 2_2^+ state is larger than that for the 2_1^+ state. The p-p interactions improve the description: the B(E2) value for the 2_1^+ state is twice larger than for the 2_2^+ state and the wave function of the 2_2^+ state contains the component nn 512↑ - - 521↓ manifesting itself in the (dp) reaction.

The 3_1^+ state in ^{172}Yb has earlier been treated [1,4] as a two-quasineutron state. Experiments on the $(d\alpha')$, (dd') and $(p\alpha)$ reactions and measurement of the magnetic moment have shown that the state is collective with a complex structure represented in Table 3. The calculations correctly reproduce the collectivity and structure of the 3_1^+ state and weak collectivity of the 3_2^+ and 3_3^+ states. One should remember that the structure of the 2_1^+, 2_2^+, 3_1^+ and 3_2^+ states in ^{172}Yb is strongly complicated by the Coriolis interaction disregarded by us.

There are three $K^\pi=4^+$ states in ^{172}Yb which are not however observed in the $(d\alpha')$ reaction [26]. The calculated energies of the 4^+ states are 0.15 - 0.30 MeV as less as the experimental ones. According to our calculations, the 4_1^+ and 4_2^+ states are weakly collective and the 4_3^+ state is collective. It would be interesting to measure experimentally the B(E4) value for the 4_3^+ state. The structure of the 4_1^+ state we have calculated differs from the one in [17], which indicates the dependence of the structure on p-p interactions. The wave functions of the four $K^\pi= 3^+$ and 4^+ states have one dominating one-phonon component.

In ^{172}Yb all four octupole states with $K_\nu^\pi=0_1^-$, 1_1^-, 2_1^- and 3_1^- have been observed; they turned out to be collective. The B(E3) values shown in Table 3 are taken from [26] where the B(IS E3) values have been determined in the $(d\alpha')$ reaction; they somewhat differ from the B(E3) values obtained in [31] from the Coulomb excitation. The B(E3) values calculated by us are between the values obtained in [26,31] and more close to the values in [31]. The wave functions of the first four octupole states with $K^\pi= 0_1^-$, 1_1^-, 2_1^- and 3_1^- have one dominating one-phonon component.

Vibrational states in ^{172}Yb should be studied in the (dp) and (dt) reactions.

We shall list the two-quasiparticle state energies in ^{172}Yb calculated without quadrupole pairing $G_\tau^{2Q}=0$ and with quadrupole pairing with the constant $G_\tau^{2Q}=0.5\,\varkappa_0$ and taking account of the blocking effect. The energy of $K_\nu^\pi=6_1^-$ nn 633↑+ + 512↓ state is equal to 1.5 MeV at $G_n^{2Q}=0$ and 1.4 MeV at $G_n^{2Q}= 0.5\,\varkappa_0^{2Q}$. The energy of 4_1^- nn 633↑+ 521↓ state in both the cases equals 1.7 MeV. The energy of the 5_1^- nn 642↑+ 512↑ state equals 2.2 MeV at $G_n^{2Q}=0$ and 2.0 MeV at $G_n^{2Q}=0.5\,\varkappa_0^{2Q}$. The state 8^+ pp 523↑+ 514↑ has the energy 2.7 and 2.4 MeV, whereas the energy of the 8^- nn 633↑+ 624↑ state is 1.9 MeV at $G_n^{2Q}=0$ and 1.8 MeV at $G_n^{2Q} = 0.5\,\varkappa_0^{2Q}$.

The results of calculations and experimental data [34-36] for ^{178}Hf are listed in Table 4. The first 2_1^+ state is collective, the two-quasineutron configuration nn 514↓ - 512↓ enters into the wave functions of the 2_1^+ and 2_2^+ states. Therefore, they are well excited in the (dp) reactions. These properties of the 2_1^+ and 2_2^+ states are well reproduced in the calculations. At the same time, the calculated 2_2^+ energy equals 2 MeV, which is due to the absence of two-quasiparticle poles with an energy less than 2.2 MeV. If the $I^\pi = 2^+$ state with the energy 1.561 MeV has K=2, then there is descrepancy with the calculated results. It can be due to the hexadecapole deformation $\beta_4=-0.16$ given in [25] whereas our calculations have been performed at $\beta_4=-0.03$.

According to the calculations, the 3_1^+ state in ^{178}Hf is collective and it should be well excited in the (dp) reaction; the second 3_2^+ state is weakly collective. The states 3_3^+, 3_4^+ and 3_5^+ have the energies 2.2, 2.3 and 2.4 MeV. The wave functions of these states have one dominating one-phonon component.

The 4_1^+ state in ^{178}Hf is collective with the dominating component $\lambda\mu i = 441$ in the wave function; the contribution of the two-phonon component $\{221,221\}$ equals 3%. According to experimental data [36] there is no evidence for E2 collectivity of the transition from the 4_1^+ state to the 2_1^+ one. The contribution of the two-phonon component $\{221,221\}$ to the 4_2^+, 4_3^+ and 4_4^+ states with the energies 2.0, 2.3 and 2.4 MeV does not exceed 2%. The energy and structure of the $K^\pi = 2^+$, 3^+ and 4^+ states in ^{178}Hf are influenced by the Coriolis interaction.

The experimental data on the octupole states in ^{178}Hf are scarce. Thus, the $K_\nu^\pi = 0_1^-$ state was not detected. Among the 1_1^-, 2_1^- and 3_1^- states only the B(E3) value for excitation of the 3^-2_1 state was measured. The calculated B(E3)-values for the 1_1^-, 2_1^- and 2_3^- states are small; therefore, it is not surprising that they were not measured experimentally. The calculated energies of the 2_2^- and 2_3^- states lie above the experimental ones.

The energies of two-quasiparticle states calculated with the monopole and quadrupole pairing are close to the data from [4]. The energies of two states with $K^\pi = 6^+$ and 5^- are overestimated as compared with the experimental ones. Here we again observe shortcomings of the scheme of single-particle levels.

It would be desirable to add rich experimental data on ^{178}Hf by new measurements of the (dp) and (dt) reactions as in [28] and $(d\alpha')$ reactions as in [26].

The calculations performed with the p-h and p-p interactions are shown to give a reasonable description of the energies and structure of vibrational states.

The structure of states depends rather strongly on the energies of single-particle states and blocking effect. The B(Eλ) values depend on the constant $G_\tau^{\lambda\mu}$; with increasing $G_\tau^{\lambda\mu}$ the B(Eλ) values decrease.

4. Distribution of the Eλ -Strength Among Low-Lying States

The first quadrupole β - and γ states are assumed to be collective and there are no other collective states up to collective states forming giant isoscalar and isovector quadrupole resonances. In the case of octupoles, apart from the first octupole states collective are the states forming the low-lying (LEOR) and high-lying (HEOR) isoscalar and isovector octupole giant resonances. In some nuclei, the low-lying collective hexadecapole states were observed. The collectivity of the first quadrupole and octupole states and absence of higher-lying collective states up to the giant resonances underlie phenomenological models including the interacting boson model (IBM).

Let us study the Eλ-strength distribution among low-lying states with an energy up to 3 MeV. The coupling between the vibrational and rotational motion will be neglected. Experiments [26] on inelastic scattering of α particles made it possible to study the isoscalar Eλ -strength distribution.

Let us consider the quadrupole strength distribution. Experimental data and results of calculations in the QPNM and IBM [26] are listed in Table 5. In ^{168}Er and ^{178}Hf a standard case takes place - the main part of the E2 strength is concentrated on the γ vibrational state. According to our calculations there are about 30% in ^{168}Er and 15% in ^{178}Hf of the E2 strength of the γ vibrational state in other 2^+ states.

The E2 strength distribution in ^{172}Yb differs qualitatively from ^{168}Er, ^{178}Hf and other nuclei. The E2 strength distribution in ^{172}Yb is specified by that both the first 2_1^+ and second 2_2^+ states are collective and a considerable part of the E2 strength is concentrated in the energy interval 2 ÷ 3 MeV. It is very difficult to describe phenomenologically

Table 5. E2-Strength Distribution

Nucleus	K_ν^π	$\mathcal{E}$, MeV $\Delta\mathcal{E}$, MeV	$B(E2)_{s.p.u.}$ exp. ref.	calc. QPNM	calc. IBM [26]
^{168}Er	2_1^+	0.821	4.7 [26]	4.6	4.7
	0^+, 2^+	1.2 - 2.5	-	1.5	0.02
^{172}Yb	2_1^+	1.47	1.4 [26,31]	1.5	I.0
	2_2^+	1.61	0.42 [26,31]	0.7	0.007
	0^+	1.0 - 2.0	0.005 [26]	0.9	0.2
			0.66 [31]		
	0^+, 2^+,	2.0 - 3.0	2.4 [26]	1.0	0.2
^{178}Hf	2_1^+	1.174	3.9 [35]	4.1	-
	2^+	3.0 - 3.2	-	0.6	

tne cases when both the first 2_1^+ and the second 2_2^+ states are collective. Thus, according to the IBM calculations [26], in ^{172}Yb the E2 strength concentrated on the 2_2^+ state is 100 times as less as on the 2_1^+ state, i.e. there is a sharp discrepancy with experiment. Our calculations provide reasonable agreement with experimental data. According to the experimental data [26], in ^{172}Yb the E2 strength in the energy interval from 2 to 3 MeV is 1.3 times larger than in the first two collective states and 1.7 times larger than in the 2_1^+ state. This is a new and very important result which shows a marked discrepancy with the generally accepted E2 strength distribution. This distribution of the E2 strength cannot be described in the IBM. According to our calculations about 2/3 of the E2 strength on the 2_1^+ state is concentrated in the interval 2÷3 MeV.

Now let us consider the isoscalar E3 strength distribution in ^{168}Er. According to [26], there are first collective states with 0_1^-, 1_1^- and 2_1^- ans six collective $K^\pi=3^-$ states. On the first three 3_1^-, 3_2^- and 3_3^- states there are 1.3 s.p.u.; and on the fourth 3_4^- state, 4.68 s.p.u.. In the interval from 2.25 to 2.50 MeV 7.9 s.p.u. are concentrated. This distribution of the E3 strength sharply differs from the standard one.

The E3 strength distribution in ^{168}Er is shown in Fig. 1. These are experimental data from [26], our calculations within the QPNM, the IBM calculations in [26] and the calculations [37] within the IBM -1 + f boson model. In the calculations [26] the B(E3) values were normalized to the experimental value of the 3^-1_1 state. As a result, for the first 3^-3_1 state the calculated B(E3) value turned out to diverge by a factor of 500 from the experimental one. If most of the Eλ strength is concentrated not on the first K_1^π state, it is practically impossible to describe it within the IBM. This is confirmed by the calculations in [37] in the IBM-1 + f boson model in which the firat three $K^\pi= 3^-$ levels are omitted. The main part of the E3 strength is concentrated on the 3_4^- state which is considered by them as a first collective $K^\pi=3^-$ state. It should be noted that in [37] weakly collective $K^\pi=3^-$ states with energies larger than the 3_4^- state energies are listed for ^{168}Er. The first three $K^\pi= 3^-$ states in ^{168}Er have been omitted in [37] as two-quasiparticle ones and the whole E3 strength is concentrated on one $K_\nu^\pi=3_4^-$ state. According to the experimental data [26] on inelastic scattering of α particles, the 3_1^-, 3_2^- and 3_3^- states are collective enough, and according to the experimental data [28] on the (dp) and ($\vec{t}\alpha$) reactions, their wave functions contain the sum of two-quasiproton and two-quasineutron terms. If the 3_1^-, 3_2^- and 3_3^- states

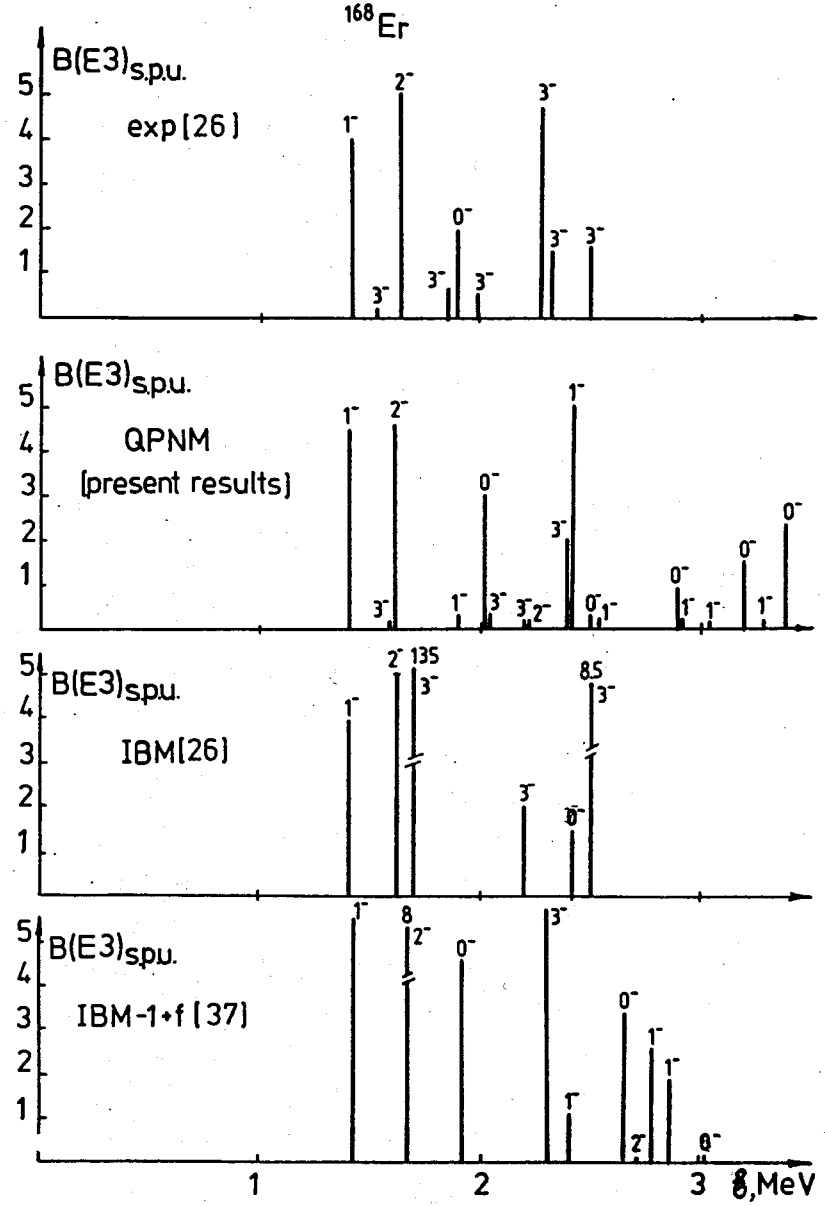

Fig. 1. E3 strength distribution in ^{168}Er. K^{π}-values equal to 0^-, 1^-, 2^- and 3^- are given above the virtical lines.

are close to the two-quasiparticle ones, the B(E3) values are equal to 0.005, 0.02 and 0.005 s.p.u. Hence, it is seen that the B(E3) values for the 3^-_1, 3^-_2 and 3^-_3 states listed in Table 2 are 30-50 times larger than the B(E3) values for the corresponding two-quasiparticle states. All this shows that these states are not two-quasiparticle ones.

Our calculations, as is seen from Fig. 1, describe correctly the E3 strength distribution in ^{168}Er. The calculated B(E3) values for the first 0^-_1, 1^-_1, 2^-_1 and 3^-_1 reasonably agree with the experimental ones. The fourth 3^-_4 state among the $K^{\pi}=3^-$ states has the largest B(E3) value. According to the calculations, at the energy 2.4 MeV there is a state 3^-1_3 with B(E3)= =4.9 s.p.u. It is probable that the calculated B(E3) values for the 3^-3_4 and 3^-1_3 states can describe the experimentally observed states with $I^{\pi}=3^-$ and energies 2.324 and 2.486 MeV. According to the calculations, the E3 strength equal to about 5 s.p.u. is concentrated on the $K^{\pi}=0^-$ states in the interval 2.8 - 3.4 MeV. The total octupole E3 strength concentrated on the states with an energy up to 2.5 MeV equals 20 s.p.u., according to the experimental data [26], and 20.3 s.p.u., according to our calculations.

All the first octupole states that turned out to be collective were observed in ^{172}Yb [26]. Their total E3 strength equals 12 s.p.u. According to our calculations, all the first octupole states are collective; their total E3 strength equals 8 s.p.u. The total E3 strength equal to 11 s.p.u. is concentrated in the interval 2 + 3 MeV. According to the calculations [37] in the IBM-1 + f boson model the E3 strength is concentrated on the first octupole states; and only about 3%, on the 3^-1_2 state. It would be interesting to check this discrepancy in the distribution of the E3 strength at 2-3 MeV experimentally.

Experimental data on the E3 strength distribution in ^{178}Hf are scarce. According to our calculations for K=2 and 3, the E3 strength is concentrated on the first states. The 0_1^- and 0_2^- states have the energies 2.0 and 2.4 MeV and B(E3) values equal to 2 and 4 s.p.u.; in the interval 1.5 - 3.0 MeV the E3 strength is equal to 0.5 s.p.u. The E3 strength distribution on the $K^\pi=1^-$ states is the following: on the first two states there are 0.8 s.p.u.; and in the interval 2-3 MeV, 7.5 s.p.u., i.e. the main part of the E3 strength is above the first two 1_1^- and 1_2^- states. In this case, there is a strong discrepancy with the results of calculations [37].

Note that in the calculations of LEOR in deformed nuclei in [38] the E3 strength was observed to be somewhat higher than the first octupole states.

Data on the E4 strength distribution are scarce. It follows from the analysis of the experimental data [29] that in the isotopes of Er, Yb and Hf the first $K_\gamma^\pi=3_1^+$ states and in the isotopes of Os the first $K_\gamma^\pi=4_1^+$ states are collective. This collectivity of the 3_1^+ and 4_1^+ states is correctly described in [17] and confirmed by the present calculations.

According to our calculations, in ^{168}Er for the $I^\pi K_\gamma=4^+3_1$ state B(E4) =0.4 s.p.u., and according to the calculations [39] in the sdg IBM B(E4) = 50.8 s.p.u. Such a large discrepancy should be verified experimentally. According to our calculations in ^{168}Er for the $K^\pi=3^+$ states with the energy 2-3 MeV the E4 strength equals 1 s.p.u. and the B(E4) value for the 4_1^+ states is overestimated. In ^{172}Yb the most part of the E4 strength is concentrated on the 3_1^+ state, and on all other $K^\pi=3^+$ states up to 3 MeV it equals 1 s.p.u. As concerns the $K^\pi=4^+$ states in ^{172}Yb, the most part of the E4 strength is concentrated on the 4_3^+ state with the energy 2.3 MeV. In ^{178}Hf

the most part of the E4 strength is concentrated on the 3_1^+, 3_2^+ and 4_1^+ states; on the other levels up to 3 MeV it equals 0.7 s.p.u.

It should be noted that according to the experimental data [40] a large part of the hexadecapole strength with $\lambda\mu=42$ is concentrated on γ-vibrational states of some rare-earth nuclei. A satisfactory description of these experimental data has been obtained in [41] in the RPA calculations with a simultaneous inclusion of quadrupole $\lambda\mu=22$ and hexadecapole $\lambda\mu=42$ interactions.

It should be emphasised that p-p interactions considerably influence the $E\lambda$ strength distribution among the low-lying states. With increasing $G_\tau^{\lambda\mu}$ part of the $E\lambda$ strength is shifted towards first and second states with fixed value of K^π.

5. On Energy Centroids of Two-Phonon Collective States

Based on the calculations in the QPNM we have concluded in [14,16] that collective two-phonon states cannot exist in deformed nuclei. If the contribution of the two-phonon component to the wave function normalisation exceeds 50%, this state is thought to be two-phonon one. The existence of two-phonon states is still being discussed in a number of papers, for instance, in [39,42-44]. Yet, there are no experimental data on collective two-phonon states in deformed nuclei. The fragmentstion of two-phonon states cannot be calculated in the QPNM since for these calculations the wave function (5) should be supplemented by three-phonon terms. Therefore, we shall discuss only the energy centroids of two-phonon states.

Increase in the energy centroids of two-phonon states $\{\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2\}$ with respect to the sum of energies of the vibrational states with the dominating one-phonon components

of their wave functions is caused by two reasons. The first is anharmonicity of vibrations since the energies of one-phonon states are larger than the energies of the states with the dominating one-phonon component described by the wave function (6). The second is the shift of the two-phonon pole $\Delta\omega(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)$ after taking the Pauli principle into account in the two-phonon terms of the wave function (6). In the calculations [16,20], a very large shift of $\Delta\omega$ was obtained for strongly collective states; in some cases this shift exceeded 2-3 MeV. Since the shift $\Delta\omega$ is due to going beyong the boson approximation for phonons, then a very large shift $\Delta\omega$ makes the applicability of the RPA doubtful. A large shift of $\Delta\omega(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)$ is due to a strong collectivity of the first $i_1=1, i_2=1$ states in the sum over i' and to a large value of $|\mathcal{H}^{K_0}(\lambda_1\mu_1 i_1=1, \lambda_2\mu_2 i_2=1)|$.

The present calculations taking account of p-h and p-p interactions provide the energies and B(Eλ) values for the 2_1^+ and first octupole states which are in reasonable agreement with the experimental data. With p-p interactions included the collectivity of the first 2_1^+ and octupole states decreases thus diminishing the shift of $\Delta\omega$. The results of calculations of the shifts $\Delta\omega(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)$, quantities $1+\mathcal{H}^{K_0}(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)$ and energy centroids of two-phonon states are shown in Table 6. The function $1+\mathcal{H}^{K_0}(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)$ entering into normalisation (7) differs from unity due to the Pauli principle. It is seen from the Table that shifts $\Delta\omega$ are from 0.1 to 1.5 MeV. As a rule, $\Delta\omega$ increases with decreasing $1+\mathcal{H}^{K_0}$. This is responsible for a large shift $\Delta\omega(221,221)$ for the $K^\pi=4^+$ state in comparison with the $K^\pi=0^+$ state. The largest shifts appear for the $K^\pi=4^+\{221,221\}$ states; for other states they do not exceed 1 MeV.

Table 6. Centroid Energies of Two-Phonon States

Nucleus	K^π	$\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2$	$\Delta\omega(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)$ MeV	$1+\mathcal{H}^{K_0}(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)$	Centroid energy, MeV
^{168}Er	0^+	221, 221	0.9	0.90	2.7
	4^+	221, 221	1.5	0.75	3.3
	2^-	221, 30I	0.4	0.95	3.2
	3^-	221, 311	0.4	0.95	2.7
^{172}Yb	0^+	221, 221	0.5	0.94	3.4
	0^+	221, 222	0.2	0.97	3.3
	0^+	321, 321	0.1	0.98	3.3
	2^+	221, 441	0.6	0.42	4.1
	4^+	221, 222	0.6	0.47	3.6
	4^+	221, 221	1.1	0.60	4.0
	4^+	222, 222	0.4	0.62	3.7
	1^-	221, 311	1.0	0.45	3.7
	2^-	221, 30I	0.3	0.95	3.4
	3^-	221, 311	0.2	0.96	2.7
^{178}Hf	0^+	221, 221	I.0	0.90	3.5
	0^+	321, 321	0.1	0.99	2.5
	2^+	30I, 321	0.1	0.97	3.1
	4^+	221, 221	1.5	0.78	4.0
	4^+	221, 222	0.4	0.88	3.9
	0^-	221, 321	1.3	0.65	3.9
	2^-	221, 30I	0.6	0.88	3.8
	3^-	221, 311	0.2	0.95	2.8
	3^-	431, 30I	0.2	0.93	4.0

The energy centroid of the $0^+\{221,221\}$ state in ^{168}Er equals 2.7 MeV; according to the calculations in [43] it equals 2.9 MeV and in [44] it equals 2.8 MeV. The energy centroids of the $0^+\{221,221\}$ states calculated in the QPNM are approximately the same as those calculated in [44] by the multiphonon method. The discrepancy takes place for the $4^+\{221,221\}$ states. It is unclear from this discrepancy what is more im-

portant: a large number of degrees of freedom as a large number of one- and two-phonon states in the QPNM or one degree of freedom as a γ-vibrational phonon and the wave function with multiphonon configurations in the multiphonon method. In [16] the wave function (6) has been added by three-phonon terms and the shifts of three-phonon poles have been calculated. It is shown that these shifts may exceed three or more times the shifts of two-phonon poles. It has been shown in [15] that the energy shift of η_1 of the 2_1^+ states with respect to the one-phonon energies ω_{221} is approximately twice smaller when the Pauli principle is taken into account in comparison with the case when it is neglected. The multiphonon terms of the wave function does not usually lead to a strong shift of the root from the corresponding pole. If nevertheless the root η_ν is strongly lowered with respect to the pole $\omega_{\lambda_1\mu_1 i_1} + \omega_{\lambda_2\mu_2 i_2} + \Delta\omega(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)$ the two-phonon state turns out to be strongly fragmented. Therefore, additional large shifts of the energy centroids of two-phonon states can hardly be expected without strong fragmentation when multiphonon terms are added in the wave function (6).

According to our calculations, the shifts $\Delta\omega(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)$ are as a rule considerably less than the sum $\omega_{\lambda_1\mu_1 i_1} + \omega_{\lambda_2\mu_2 i_2}$. The shifts are caused by two reasons: first, deviation of the phonon operators from the ideal bosons, and second, collectivity of phonons. In case of large shifts, decrease of the B(Eλ) value by (10-20)% leads to decrease of the shift $\Delta\omega$ by 1/3. We can state that in well deformed nuclei of the rare-earth region the RPA can undoubtedly be applied to describe the QPNM phonon basis.

As is seen from Table 6, the energy centroids of collective two-phonon states equal 2.5 - 4.0 MeV. At these energies the two-phonon strength should be distributed over many levels.

The present calculations confirm the conclusion we have made in [14,16] that collective two-phonon states cannot exist in well deformed nuclei.

6. Conclusion

The study of vibrational states with $K^\pi \neq 0^+$ in well deformed doubly even nuclei has shown that the energy and structure of each state are determined mainly by the single-particle energies and wave functions of the Saxon-Woods potential, monopole pairing and p-h isoscalar multipole interaction. The multipole p-h isovector interaction, quadrupole pairing and multipole p-p interaction are of minor importance. Inclusion of the p-p interaction improves the description of vibrational states. Moreover, it justifies the applicability of RPA to describe states with an energy less than 1 MeV.

Nonrotational states with $K^\pi = 0^-$, 1^-, $2^\pm$, $3^\pm$ and 4^+ with energies up to 2.5 MeV have dominating one-phonon components. For the states with an energy up to 2 MeV the dominating one-phonon component contributes more than 90% and the two-phonon components not more than (3-5)% to normalisation of the wave function. Taking into account the fact that in our calculations we disregarded the Coriolis interaction and used the single-particle energies and wave functions of the Saxon-Woods potential with the parameters fixed in 1968-1970 we can state that the experimental data on the energies, B(Eλ) values and structure of vibrational states in ^{168}Er, ^{172}Yb and ^{178}Hf are described correctly.

A qualitatively new result became evident in the picture of low-lying vibrational states: the Eλ strength distribution differs in some cases from the generally accepted one. In particular, there are cases when collective is not the first but a higher lying state with a given K^π, or the largest part

of the $E\lambda$ strength is concentrated not on the first states but in the energy interval 2 ÷ 3 MeV. This distribution of the $E\lambda$ strength can hardly be described within phenomenological models even in such as the sdg IBM or sdf IBM.

With the inclusion of p-p interactions the pole shifts of two-phonon collective states decrease in comparison with those when p-p interactions are dis-regarded. They equal 0.1-1.5 MeV. The calculated energy centroids of the lowest two-phonon states equal 2.5 ÷ 4.0 MeV. The conclusion we have made earlier about the absence of collective two-phonon states in deformed nuclei is valid.

We can assert that in the framework of the QPNM we have constructed the basis for describing the structure of states of deformed nuclei. If necessary, the methematical apparatus of the QPNM can be generalised, as in [22], to the finite rank separable interactions that can reproduce complex effective interactions between quasiparticles. In further calculations one should take into account the coupling between vibrational and rotational motion, introduce additional zones with respect to A and determine more accurately the parameters of the Saxon-Woods potential. Maybe the form of the potential describing the average field of a nucleus will need to be complicated.

We hope that the present calculations will be useful in experiments at a new generation of accelerators. Of great interest is experimental study of excited states of deformed nuclei at 2-3 MeV.

Acknowledgements

We are grateful to V.O.Nesterenko and A.V.Sushkov for help and discussions.

References

1. Gallagher C.J., Soloviev V.G.: Mat.Fys.Skr.Dan.Vid. Selsk. I, No 11 (1962).
2. Soloviev V.G.: Nucl.Phys. 69, I (1965).
3. Soloviev V.G.: Theory of complex nuclei, Moscow: Nauka, 1971. (Engliash translation: Theory of complex nuclei Oxford: Pergamon Press 1976).
4. Grigoriev E.P., Soloviev V.G.: Structure of Even Deformed Nuclei, Moscow: Nauka, 1974.
5. Vogel P., Zirnbauer M.R.: Phys.Rev.Lett., 57, 3148 (1986). Civitarese P., Faessler A., Tomoda T.: Phys.Lett., B194, II (1987). Muto K., Klapdor H.V.: Phys.Lett., B201, 420 (1988).
6. Kuzmin V.A., Soloviev V.G.: Pisma JETP 47, 68 (1988).
7. Kuzmin V.A., Soloviev V.G.: Nucl.Phys., A486, 118 (1988); Mod.Phys.Lett., A3, 1533 (1988).
8. Suhonen J., Faessler, Taigel T., Tomoda T.: Phys.Lett., B202, 174 (1988). Suhonen J., Taigel T., Faessler A.: Nucl.Phys., A486 91 (1988).
9. Soloviev V.G., Sushkov A.V.: Phys.Lett., B216, 259 (1989).
10. Soloviev V.G.: Particle and Nuclei 9, 580 (1978) (Transl. Sov. J.Part. Nucl. 9, 343 (1978)). Malov L.A., Soloviev V.G.: idem 11, 301 (1980) (Transl. Sov. J.Part. Nucl. II, 111 1980).
11. Soloviev V.G.: Teor.Mat.Fiz. 53, 399 (1988).
12. Soloviev V.G.: Prog.Part.Nuol.Phys. 19, 107 (1987).
13. Soloviev V.G.: Theory of Atomic Nuclei. Quasiparticle and Phonons, Moscow: Energoatomizdat 1989.
14. Soloviev V.G., Shirikova N.Yu.: Z.Phys. A. - Atoms and Nuclei 301, 263 (1981).

15. Soloviev V.G., Shirikova N.Yu. e.a.: Izv.Akad.Nauk SSSR Ser.Fyz. 45, 1834 (1981).
16. Soloviev V.G., Shirikova N.Yu.: Yad.Fyz. 36, 1376 (1982).
17. Nesterenko V.O., Soloviev V.G., Sushkov A.V., Shirikova N.Yu.: Yad.Fyz. 44, 1449 (1986).
18. Karadjov D., Soloviev V.G., Sushkov A.V.: Preprint JINR E4-89-55, Dubna, 1989.
19. Soloviev V.G., Shirikova N.Yu.: Izv.Akad.Nauk SSSR Ser.Fyz. 52, 2095 (1988).
20. Soloviev V.G.: Pisma JETP, 40, 398 (1984); Yad.Fyz. 47, 332 (1988).
Soloviev V.G.: Z.Phys. A - Atomic Nuclei 324, 393 (1986).
J.Phys. G. - Nucl.Phys. 14 Supp. S. 39 (1988).
21. Soloviev V.G.: Preprint JINR P4-88-845,Dubna,1988,Z.Phys.A.
22. Soloviev V.G.: Preprint JINR, E4-88-737, Dubna, 1988.
23. Malov L.A., Meliev F.M., Soloviev V.G.: Z.Phys. A - Atoms and Nuclei, 320, 521 (1985).
24. Soloviev V.G.: Theory of Atomic Nuclei. Nuclear Models, Moscow: Energoiddat 1981.
25. Nettles W.G., Ramayya A.V. e.a.: J.Phys. G.: Nucl.Phys., 14, L223 (1988).
26. Govil I.M., Fulbright H.W, e.a.: Phys.Rev., C33, 793 (1986); C36, I442 (1987).
27. Davidson W.F., Dixon W.R., Storey R.S.: Can. J. Phys. 62, 1538 (1984).
28. Burke D.G., Maddock B.L.W. e.a.: Nucl.Phys., A442, 424 (1985).
Burke D.G., Davidson W.F. e.a.: Nucl.Phys., A445, 70 (1985).
29. Walker P.M., Carvalho J.L.S., Bernthat F.M.: Phys.Lett., 116B, 393 (1981).
30. Gonging W.: Nuclear Data Sheets 51, 577 (1987).
31. Cresswell J.R., Forsyth P.D., Martin D.G., Morgen R.C.: J.Phys. G.: Nucl.Phys. 7, 235 (1981).

32. Burke D.G., Blezins J.W.: Can. J. Phys. 60, 1751 (1982).
33. Walker P.M., Faber S.R., e.a.: Nucl.Phys., A343, 45 (1980).
34. Browne E.: Nuclear Data Sheets 54, 199 (1988).
35. Ronningen R.M., Hamilton J.H., Ramayya A.V.: Phys.Rev., C15, 1671 (1977).
36. Hegue A.M., Casten R.F. e.a.: Nucl.Phys., A455,231 (1986).
37. Barfield A.F., Barrett B.R., Wood J.L., Scholten O.: Annals Phys., 182, 344 (1988).
38. Malov L.A., Nesterenko V.O., Soloviev V.G.: J.Phys. G.: Nucl.Phys., 3, L219 (1977).
39. Yoshinaga N., Akiyama Y., Arima A.: Phys.Rev., C38, 419 (1988).
40. Ichihara T., Sakaguchi H. et al.: Phys.Lett., 149B, 55 (1984);
Phys.Rev., C36, 1754 (1987).
41. Nesterenko V.O., Kukhtina I.N. e.a.: J.Phys., G: Nucl.Phys. 14, 725 (1988).
42. Soloviev V.G.: Rev.Roum.Phys., 33, 947 (1988).
43. Matsuo M., Matsuyanagi K.: Prog.Theor.Phys., 74, 1227 (1985); 76, 93 (1986).
44. Piepenbring R., Jammari M.K.: Nucl.Phys., A481, 81 (1988), A487, 77 (1988).

Received by Publishing Department
on March 29, 1989.