объединенный
институт
ядерных
исследований
дубна

E4-89-165

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NEUTRON-PROTON RATIOS OF COLLECTIVE QUADRUPOLE MATRIX ELEMENTS IN EVEN Fe AND Cr ISOTOPES

Submitted to "Modern Physics Letters A"

1989

1. Introduction

In connection with the growing reliability of experimental intermediate-energy hadron inelastic scattering data, one may observe growing interest in the investigation of both neutron and proton components of the low-lying collective - and in a few cases also the isoscalar collective giant resonance - states. Recent M_n/M_p data on low-lying states [1] and on isoscalar giant quadrupole resonances [2] make us publish our calculations of these ratios.

The aim of this letter is to present, at first, the theoretical interpretation of the existing empirical M_n/M_p data on low-lying collective quadrupole transitions, and at second, the theoretical predictions of M_n/M_p for isoscalar giant resonances.

2. Formalism

Calculations in this work are done in the quasiparticle RPA (QRPA) framework with schematic separable residual forces [3,4,5]. The Hamiltonian contains the single-particle , pairing and multipole two-quasiparticle parts. The excited states are in this frame generated through applying the phonon creation operator

$$Q^+_{\lambda\mu,t} = \frac{1}{2} \sum_{JJ'} \left[\psi^{\lambda t}_{JJ'} \, A^+(JJ'\lambda\mu) - (-1)^{\lambda-\mu} \, \phi^{\lambda t}_{JJ'} \, A(JJ'\lambda\mu) \right],$$

where $\psi^{\lambda t}_{JJ'}$ and $\phi^{\lambda t}_{JJ'}$ are the forward-going and backward-going amplitudes, respectively, and $A^+(JJ'\lambda\mu)$ is the two-quasiparticle creation operator. The reduced matrix element of the neutron component of one-phonon state is

$$M_n = M(N\lambda) = \langle Q^+ \| O(N\lambda) \| 0 \rangle$$

$$= \frac{1}{2} \sum_{jj'}^{(n)} \left[\psi^{\lambda i}_{jj'} + \phi^{\lambda i}_{jj'} \right] \mathcal{M}^{(\lambda)}_{jj'} \; u_{jj'},$$

where the reduced matrix element $\mathcal{M}^{(\lambda)}_{jj'}$ is

$$\mathcal{M}^{(\lambda)}_{jj'} = \langle j' \| r^{\lambda} \; Y_{\lambda\mu} \| j \rangle$$

and

$$u_{jj'} = u_j v_{j'} + u_{j'} v_j$$

are coefficients constructed from BCS occupation parameters weighting the two-quasiparticle current density.

To compare the neutron and proton strength it is more instructive to compare not the M_n/M_p ratio, but the neutron-proton ratio reduced to one nucleon

$$\eta = \frac{M_n}{N} \; \frac{Z}{M_p}.$$

The instructiveness of this quantity comes from the fact that in the hydrodynamical model η equals unity. Deviations of η from 1 then characterize degree of the shell and correlation effects on the given excited state.

Input data for this framework are the single-particle basis, the BCS pairing parameters, and the parameters of separable forces. The single-particle basis we used contains all bound and quasibound states generated by the Woods-Saxon potential with the parameters from Ref. 6. The BCS pairing strength parameters are obtained from pairing energies which have been evaluated from the difference of masses in even and odd neighbourhood nuclei. The isoscalar separable force parameters are calculated from the experimental excitation energies of 2^+_1 states. The isovector separable force parameters are obtained from the empirical relation which guarant good experimental-theoretical agreement of isovector giant resonance excitation energies, namely $\kappa_{\lambda=2,\tau=1} = -1.4\ \kappa_{\lambda=2,\tau=0}$ .For a more discussion of the RPA formalism and framework adjustation technique see Refs. 3 and 4.

3. Low-lying collective states

We start discussion of the results from the low-lying collective states. Our results and experimental ones are presented in Table 1 for Fe isotopes and in Table 2 for isotopes of Cr. In all these isotopes there are available experimental values of proton transitional matrix elements. These values shown in rows a) of Tables 1 and 2 have been obtained from Coulomb excitations of Fe isotopes in Ref. 7 and those of Cr isotopes in Ref. 8. Present QRPA results are shown in Tables 1 and 2, rows b). As one can see, the agreement between theoretical and experimental B(*P*2) values is very good, except ^{58}Fe, where about 20% discrepancy is obtained. Since this agreement, as we observe in these two isotopic chains, is usual for QRPA (see Ref. 4 and references therein), we have grounds to believe that this approximation includes the essential physics forming the structure of the one-phonon collective low-lying states.

The empirical results from dedicated study [1] obtained on the basis of the inelastic resonance $\pi^{\pm}$ scattering for $^{54,56}Fe$ and ^{52}Cr are shown in Tables 1 and 2, rows c). The other dedicated experiment [9] exists for ^{54}Fe. In that work the authors measured and analyzed inelastic scattering of 800 MeV protons and obtained the value of η shown in Table 1, row d).For isotopes of $^{54,56}Fe$,

2_1^+ states of Fe isotopes | Table 1

	^{54}Fe		^{56}Fe		^{58}Fe	
	B(P2)	η	**B(P2)**	η	**B(P2)**	η
a	675 (40)		970 (20)		1234 (36)	
b	643	0.88	1051	0.98	1000	0.97
c		0.92(5)		1.00(3)		
d		0.86(14)				0.99(6)

a – B(*P2*) in [fm^4]; results taken from Ref. 7.

b – present QRPA results.

c – results taken from Ref. 1.

d – the first result taken from Ref. 9;

the second result evaluated from $\beta_{pp'}$ of Ref. 11.

2_1^+ states of Cr isotopes | Table 2

	^{50}Cr		^{52}Cr		^{54}Cr	
	B(P2)	η	**B(P2)**	η	**B(P2)**	η
a	1020(30)		660(30)		850(30)	
b	941	0.99	640	0.82	987	0.89
c				0.77(2)		
d		0.79(5)				0.79(6)

a – B(*P2*) in [fm^4]; results taken from Ref. 8.

b – present QRPA results.

c – results taken from Ref. 1.

d – results evaluated from $\beta_{pp'}$ of Ref. 11.

there are also results obtained from comparative low-energy neutron and proton scattering analyses [10]. Those results are in good agreement with the results shown in Table 1. As is seen, the theoretical results are for all these three isotopes with a very good agreement with both empirical data sets - the η ratios and reduced transitional probabilities.

The other three values of the η ratios for ^{58}Fe and $^{50,54}Cr$ which we evaluated from the dynamical deformation parameters of the proton scattering and from the reduced electric transition probabilities of the Coulomb excitation are shown in Tables 1 and 2, rows d). The dynamical deformation parameters were obtained from analyses of the 35 MeV proton inelastic scattering cross section data on ^{58}Fe ($\beta_{pp'}$=0.26), ^{50}Cr ($\beta_{pp'}$=0.24) and ^{54}Cr ($\beta_{pp'}$=0.20) in Ref. 11 and the reduced electric transition probabilities were obtained in Coulomb excitations [7,8]. The evaluation follows the prescription of the collective model used for analysis of the inelastic proton scattering [11]. In this model [12], the analytic relation between dynamical deformation parameters and transitional matrix elements is

$$M(P2) = (\lambda+2)\ Z\ \beta_p\ R_0\ \langle r^{\lambda-1}\rangle\ /\ (4\pi),$$

where R_0 are the nuclear Woods-Saxon potential radii and $\langle r^{\lambda-1}\rangle$ are the $(\lambda-1)$-momenta of the proton ground state density distributions. These radial momenta we evaluated from the parametrizations of the proton density distributions which were obtained from electron scattering data analysis in Ref. 13. The dynamical deformation parameters of the neutron fields needed for evaluation of the η ratios were obtained from the relation

$$\beta_{pp'} = \frac{V_{pn}\ \beta_n + V_{pp}\ \beta_p}{V_{pn} + V_{pp}},$$

where V_{pn} , V_{pp} are the projectile-neutron and -proton effective interactions [14]. The η ratios obtained from this evaluation procedure have pointed out the uncertainties which are connected with minimal ones of $\beta_{pp'}$, with 0.01 magnitude.

In the case of ^{58}Fe we have a good agreement between the theoretical and empirical η ratio, however, the isoscalar strength is theoretically underestimated by 20% in the reduced transition probability against that from the combination of the low-energy proton and Coulomb excitations. This disagreement would be removed by rising the BCS pairing constants which should have as effect also a small rising of the η ratio because the neutron component in this state tends to be more collective than the proton one.

The isotope ^{50}Cr is the case where the QRPA η ratio does not agree with the empirical one. The agreement of isoscalar reduced transition probabilities is on the level of 15%. This isotope is only one in our set of isotopes where both neutrons and protons have the levels $1f_{7/2}$ as the valence ones. Although from the theoretical point of view the main role is played here by the single-particle energy positions, the big difference of the η ratio from the hydrodynamical limit is unexpected. After the reevaluation of the η ratio using the alternative experimental value of $B(E2)$ = 933 fm^4 obtained from inelastic electron scattering [15], we have the value 0.84(5) for the empirical η ratio which is in better agreement with the QRPA one.

The agreement between the QRPA and empirical η ratio in the isotope ^{54}Cr is better than in ^{50}Cr and the full strength observed in the combination of the low-energy proton and Coulomb excitations is 14% less than the theoretical one.

4. Isoscalar giant quadrupole resonances

The empirical information on relations of neutron and proton components of giant resonances exists only in few cases [2] but not in nuclei studied here. There is however some information on their excitation energies and strengths [16].

In Table 3 we present our QRPA results. It should be noted here that formulae used for evaluation of the neutron and proton energy-weighted sum-rule are related [16] as $(N/Z)^2$.

It is a common trend for the QRPA results that giant resonance excitation energies are lower about 2 MeV than the empirical fit $65 \times A^{-1/3}$. As concerns the strength, there are no strong restrictions on magnitudes at present time; but compared to the systematics [16] 36% depletion of IS GQR on ^{52}Cr seems to be on the lower boundary of expected values.

IS GQR states of Fe and Cr isotopes Table 3

	^{50}Cr	^{52}Cr	^{54}Cr	^{54}Fe	^{56}Fe	^{58}Fe
E [MeV]	15.3	14.5	15.3	14.4	14.9	15.0
S_n [%]	77	54	65	87	74	63
S_p [%]	46	36	47	52	52	48
η	1.29	1.23	1.17	1.29	1.20	1.15

Theoretical QRPA predictions of the neutron to proton matrix element ratios η are from 15 to 30% larger than is expected in the hydrodynamical model. One can observe that the QRPA η values are less or equal to the hydrodynamical limit for 2_1^+ states of all the studied isotopes. It is interesting to note that analogical observation was made in Ref. 2 in the case of ^{118}Sn.

5. Conclusion

In conclusion, we may say that the values η of even Fe and Cr isotopes are in the range from 0.8 to 1.0 of the hydrodynamical limit. The theoretical QRPA information on neutron-proton quadrupole matrix element ratios η for 2_1^+ states is in a quite good agreement with that obtained from dedicated experiments, namely for $^{54,56}Fe$ and ^{52}Cr from resonance $\pi^{\pm}$ scattering [1], for ^{54}Fe from 800 MeV inelastic proton scattering [9] and for $^{54,56}Fe$ also from comparative low-energy neutron and proton scattering [10].

Since in the cases of ^{58}Fe, ^{50}Cr and ^{54}Cr there are no dedicated studies of neutron-proton matrix element ratios we evaluated them from the low-energy proton [11] and Coulomb [7,8] excitations. The worst agreement on the level of 20% was obtained in the case of ^{50}Cr between the theoretical and empirical η ratio. However, some problems remain because the isoscalar strength is theoretically underestimated by 20% in the reduced transition probability against that from the combination of the low-energy proton and Coulomb excitations in the case of ^{58}Fe.

In the case of the isoscalar giant quadrupole resonances we have no available empirical information about the neutron-proton matrix element ratios but there is some information about excitation energies and energy-weighted sum rule depletions for isotopes studied here. For IS GQR the model predicts the η ratios from 15 to 30% greater than the hydrodynamical limit is, which is like the case of ^{118}Sn studied experimentally by resonance $\pi^{\pm}$ scattering in Ref. 2 where η=1.4 was obtained. The excitation energies are lower about 2 MeV than the empirical fit and the strengths are comparable with the systematics [16].

Acknowledgments

The author is indebted to Prof. V.G. Soloviev for reading this manuscript.

References

1. D.S.Oakley et al., **Phys.Rev. C 35** (1987) 1392.
2. J.L.Ullmann et al.,**Phys.Rev. C 31** (1985) 177.
3. D.R.Bes, R.A.Broglia and B.S.Nilsson, **Phys.Reports 16** (1975) 1.
4. A.I.Vdovin, V.G.Soloviev, **Sov.J.Part.and Nuclei 14** (1983) 237.
5. Ch.Stoyanov, I.P.Yudin, **Preprint JINR P4-11076**, Dubna, 1977.
6. V.Yu.Ponomarev et al., **Nucl.Phys. A 323** (1979) 446.
7. M.LeVine et al., **Phys.Rev. C 23** (1981) 244.
8. C.W.Towsley et al., **Nucl.Phys. A 250** (1975) 381.
9. G.Adams et al., **Phys.Rev. C 21** (1980) 2485.
10. S.Mellema et al., **Phys.Rev. C 33** (1986) 481; **C 29** (1984) 2385; **C 28** (1983) 2267.
11. E.Fabrici et al., **Phys.Rev. C 21** (1980) 844.
12. A.Bohr and B.R.Mottelson, **Nuclear Structure**, (Benjamin, New York, 1974) Vol 2.
13. H.DeVries et al., **ADNDT 14** (1974) 479; **36** (1987) 495.

14. A.M.Bernstein, V.R.Brown, V.A.Madsen, **Comm.Nucl.Part.Phys.** 11 (1983) 203.

15. J.W.Lightbody et al., **Phys.Rev. C 28** (1983) 113.

16. J.Speth and A.van der Woude, **Rep.Prog.Phys. 44** (1981) 719.

Received by Publishing Department
on March 13, 1989.