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THEORETICAL ASPECTS OF PION PHOTOPRODUCTION OFF LIGHT NUCLEI

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1. Studies of pion photoproduction off nucleons and nuclei are now an extensive branch of the intermediate energy nuclear physics.

The initial step involves knowledge of pion photoproduction on single nucleons, the final step requires understanding of the way pions scatter off nuclei. Nuclear transitions between initial and final states require knowledge of nuclear structure. Each stage is a subject of independent and extensive investigations. A great deal of information is accumulated on each stage. When combined together they seem to succeed in describing the pion photoproduction off nuclei.

How the matter is going on in reality is just the subject of the present report. Recently, the problem of neutral pion photoproduction attracts much attention. This is due to new and precise experimental data which become available both for nucleons and nuclei. The data have raised some problems which will be the main topic of this talk.

2.1. For photon energies to about 400 MeV only S - and P- - pionic waves are important, and the dominant multipoles are the electric dipole (E_{0+}) (S -wave in the πN- system) and two (M_{1+} and M_{1-}) magnetic dipoles (P -wave in the πN -system).

Several versions of a pion photoproduction amplitude are elaborated. Among them the most popular are CGLN[1] and BDW[2] constructed on the dispersive-relations ground. Both the amplitudes describe experimental data for charged pion photoproduction off free proton well enough.

An alternative way to treat the pion photoproduction amplitude is based on its expansion in terms of few relevant diagrams. This method developed in Refs.[3,4] proved to be efficient especially when dealing with pion photoproduction off nuclei.

The $E_{0}+$ multipole dominates even for 200 MeV photon energies in charged pion photoproduction. In the case of neutral pion production the M_1 contribution to the total cross section is equal to that of E_0+ already at 2 MeV above the threshold. Neutral pion photoproduction is a very delicate problem. Effects which are small in charged pion photoproduction become important here.

Schematically the amplitude for pion photoproduction in terms of multipoles can be written as

$\gamma N \to N\pi^{\pm}$: Born $(E_0+;\ M_1+)$ + $\Delta(M_1+)$,

$\gamma N \to N\pi^0$: $\Delta_{33}(M_1+)$+ small nonresonant amplitudes (M_1+, E_0+).

2.2. In contrast with high energies of incoming photons where models have to be used for the pion photoproduction amplitude, at the threshold firm statements for it can be made. The Low Energy Theorems (LET), namely the Kroll - Ruderman one and PCAC give

$$E_{0+} = \frac{eg}{8\pi M_N} \begin{cases} \sqrt{2}\left(1 - \frac{3}{2}\frac{m_\pi}{M_N}\right) & , \text{for } \gamma p \to n\pi^+ \\ \sqrt{2}\left(-1 + \frac{1}{2}\frac{m_\pi}{M_N}\right) & , \text{for } \gamma n \to p\pi^- \\ \left[-\frac{m_\pi}{M_N} + \frac{\mu_p+2}{2}\left(\frac{m_\pi}{M_N}\right)^2\right] & , \text{for } \gamma p \to p\pi^0 \end{cases}$$

T a b l e 1. The electric dipole (E_{0+}) photoproduction amplitude (in units of $10^{-3}/m_{\pi^+}$) at threshold

Channel	Experiments	LET [8]	BL(PV)	BDW[2]	QCM [9]	CBM [10]	Disp. Rel. [11]
$\gamma p \to n\pi^+$	28.3±0.5 [5]	27.6	27.9		29.0	27.7	29.0
$\gamma n \to p\pi^-$	-31.9±0.5 [5]	-32.0	-32.1		-33.3	-33.8	-33.1
$\gamma p \to p\pi^0$	-1.8±0.6 [5]	-2.4	-2.4	-0.13	-2.7		-0.88
	-0.5±0.3 [6]						
	-0.2±0.1 [7]	preliminary					

The numerical values for E_{0+} multipole are given in Table 1. For charged pions the experimental data agree well with the predictions based both on the LET and other versions of the amplitude.

The strong disagreement of the new experimental results with the prediction based on LET occurs in the case of π^0 production. From the total cross-section as well as the angular distribution it follows that the absolute value of $E_{0+}^{p\pi^0}$ is very small and much lower than the LET prediction. At the same time, the BDW and Lebedev Institute [11] versions of the amplitude give the value close to the new experimental data. This could indicate the importance of dispersive corrections to the higher order term in the m_π/M_N development.

The numerical values of magnetic multipoles extracted from the same experimental data on the angular distribution are given in Table 2. The M_{1+} amplitude at threshold is close to that given by BL; the M_{1-} amplitude is somewhat less in this version of the amplitude.

Table 2. The magnetic dipole (M_1+ and M_1-) π^0 production amplitudes (in units of $10^{-3}\,kq/m_{\pi^+}^3$) at threshold

Channel		Experiment	BL(PV)[3,4]	BDW[2]	BDW+ρ,ω[12]
$\gamma p \to p\pi^0$	M_1+	8.0±0.3[6]	7.8	3.5	4.8
	M_1-	-2.0±1.5	-4.8	-3.3	-1.4

3.1. Now we start to discuss the problems arising when proceeding to the reaction on complex nuclei.

DWIA is the method extensively used in calculations of the (γ, π) reaction cross-sections for complex nuclei. In this method one neglects any dynamical modifications of the elementary operator by the nuclear medium. Full momentum space techniques are clearly preferred despite the difficulties with the Coulomb interaction, since the full momentum dependence of the basic photopion operator can be included.

There are several techniques for momentum space calculations. In our group the method based on the Lippmann - Shwinger equation is developed. For pion photoproduction amplitude one obtains the following expression[13]:

$$F_{\pi\gamma}(\vec{q}_0,\vec{k}\lambda)=V_{\pi\gamma}(\vec{q}_0,\vec{k}\lambda)-\frac{A-1}{A}\frac{1}{(2\pi)^2}\int\frac{d\vec{q}}{m(q)}F_{\pi\pi}(\vec{q},\vec{q}_0)\frac{1}{\mathcal{E}(q_0)-\mathcal{E}(q)+i0}V_{\pi\gamma}(\vec{q},\vec{k}\lambda) \quad (1)$$

The first term is the plane wave part of the amplitude. The pion-nucleus interaction in the final state is given by the second term through pion-nuclear scattering amplitude $F_{\pi\pi}(\vec{q},\vec{q}_0)$ satisfying the integral equation

$$F'_{\pi\pi}(\vec{q},\vec{q}_0)=U_{opt}(\vec{q},\vec{q}_0)-\frac{1}{(2\pi)^2}\int\frac{d\vec{q}\,'}{m(q')}U_{opt}(\vec{q},\vec{q}\,')\frac{1}{\mathcal{E}(q_0)-\mathcal{E}(q')+i0}F'_{\pi\pi}(\vec{q}\,',\vec{q}_0), \qquad (2)$$

where

$$F'_{\pi\pi}(\vec{q},\vec{q}_0)=\frac{A-1}{A}F_{\pi\pi}(\vec{q},\vec{q}_0)\,. \qquad (3)$$

The details of the optical potential construction are discussed in Refs. [14] and [15] .

The optical potential U_{opt} contains the so-called first order optical potential constructed on the t -matrix of πN -scattering. The second term is added in the optical potential to simulate the effects of true absorption and the second order optical potential effects

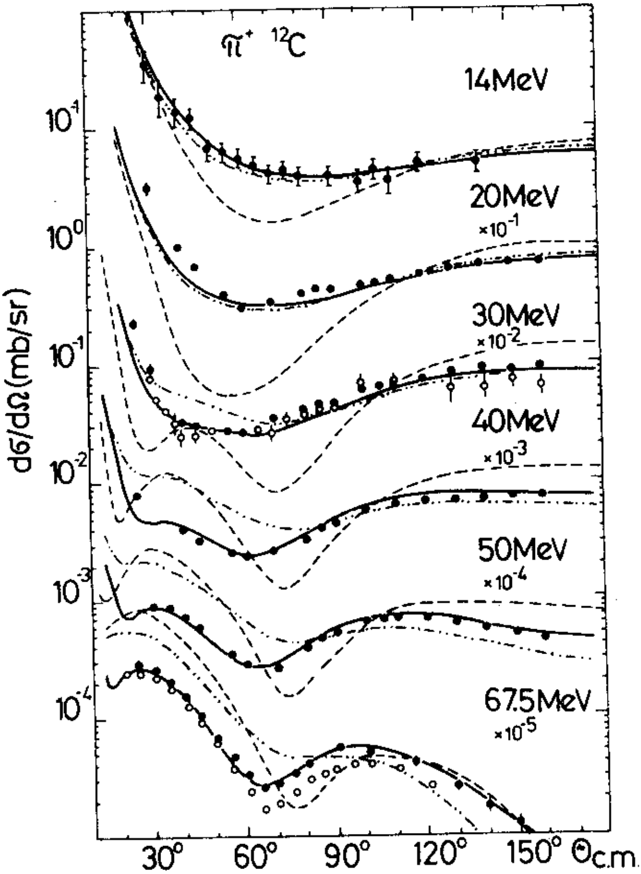
$$U_{opt}(\vec{q},\vec{q}\,')=U_1(\vec{q},\vec{q}\,')+U_2(\vec{q},\vec{q}\,'), \qquad (4)$$

$$U_1(\vec{q},\vec{q}\,')=(A-1)\sqrt{m(q')m(q)}\,\langle\vec{q}_f|t(z_0)|\vec{q}_i\rangle F_0(\vec{q}-\vec{q}\,')/2\pi, \qquad (5)$$

$$U_2(\vec{q},\vec{q}\,')=(A-1)^2\sqrt{m(q')m(q)}\,(B_0+C_0\vec{q}\,'\vec{q})\frac{g_{\pi N}(q')g_{\pi N}(q)}{g^2_{\pi N}(q_0)}G_0(\vec{q}\,'-\vec{q})/m_\pi, \qquad (6)$$

$$g_{\pi N}(q)=(1+0.224\,q^2)^{-2}. \qquad (7)$$

3.2. The problem of pion photoproduction contains the pion elastic scattering as an independent ingredient. Now we shall discuss briefly how the experimental data on pion scattering are described in the framework of our method. The differential cross sections of positive pion elastic scattering at low pion energies taken from Ref. [15] are given in Fig. 1. The microscope first-order optical potential U_1 fails to reproduce the data for the differential cross-section (dashed line). The term U_2 brings the results of the calculations very close to the experimental data.



F i g. 1. Differential cross-sections for π^+ scattering on C^{12} calculated with $\mathcal{U}_1$ (dashed line) and ($\mathcal{U}_1 + \mathcal{U}_2$). The parameters of $\mathcal{U}_2$ have been chosen according to Ref.[15] (solid line) and according to mesoatomic data (dash-dotted line). The experimental data as quoted in Ref.[15],the results of the calculations - from Ref.[15] .

3.3. According to the impulse approximation the plane wave part can be expressed through the elementary (on the nucleon) $t_{\pi\gamma}$ matrix and nuclear transition density $\rho(\vec{p},\vec{p}')$

$$V_{\pi\gamma}(\vec{q},\vec{k}\lambda)=\int \rho(\vec{p},\vec{p}')\langle\vec{q},\vec{p}'|t^{\lambda}_{\pi\gamma}(\omega)|\vec{k},\vec{p}\rangle d\vec{p}\, d\vec{p}', \qquad (8)$$

where ω is the reaction energy and is equal to the full energy of the πN system in the c.m. frame for the free nucleon

$$\omega = E_{\pi}(\vec{q}_{c.m.}) + E_{N}(\vec{q}_{c.m.}) \quad . \qquad (9)$$

7.

For the photoproduction off nucleus one needs already to know the off-shell behaviour of $t_{\pi\gamma}$. Very often it is given by the relation

$$t_{\ell}(\omega)= t_{\ell}(z)\, g^{(\ell)}_{\pi N}(q_{c.m.})/g^{(\ell)}_{\pi N}(q_{z}), \qquad (10)$$

where q_z and $q_{c.m.}$ are the pion momenta in the c.m. frame which correspond to the total energy z and ω respectively, and

$$g^{(\ell)}_{\pi N}(q)= q^{\ell}/(1+0.224\, q^{2})^{2} . \qquad (11)$$

Using the extrapolation given by (10) we get a new free parameter $z(q_z)$. It is not clear yet how this parameter is coupled to the pion-nuclear energy. There are different prescriptions for fixing z

$$z_0 \equiv \omega_i = [(E_{\gamma}+E_N(\vec{p}))^2-(\vec{k}+\vec{p})^2]^{1/2} , \qquad (12)$$

$$z_1 = [m_{\pi}^2+M_N^2+2E_{\pi}(\vec{q}_0)E_N(\vec{p}')-2\vec{q}\cdot\vec{p}']^{1/2}, \qquad (13)$$

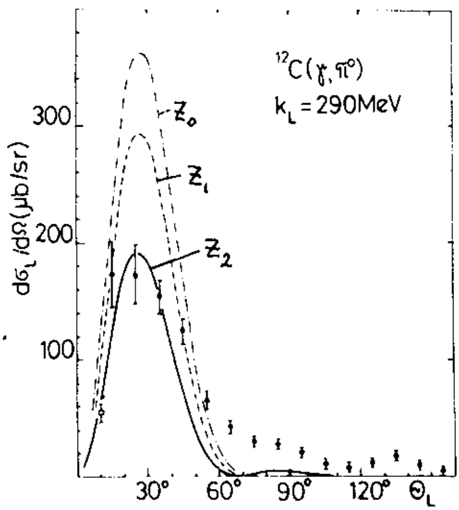
$$z_2 \equiv \omega_f = [(E_{\pi}(\vec{q})+E_N(\vec{p}'))^2-(\vec{q}+\vec{p}')^2]^{1/2} . \qquad (14)$$

A large effect in the cross section arises due to this ambiguous situation when t_ℓ is strongly energy dependent. This is a case for the M_{1+} multipole in the resonance region.

Different possibilities of choosing z have been investigated for the pion elastic scattering (Ref.[17]) and for (γ, π^0) reaction (Ref.[16]). The best agreement with experimental data has been achieved when

$$z = z_2 = \omega_f(\vec{q}\,) , \tag{15}$$

(the so-called half-of-shell extrapolation). To illustrate the situation in Fig. 2 the differential cross-section for ^{12}C is given at E_γ = 290 MeV. In this region the Δ -resonance term dominates in the amplitude. So no ambiguity arises.



F i g. 2. Angular distribution for the coherent $^{12}C(\gamma,\pi^0)$ reaction at E_γ^{LAB}=290 MeV. DWIA calculations[16] have been done with the BD amplitude[20] and different reaction energy, experimental data - from Ref.[21].

In the relativistic potential theory z is considered as eigenvalue of the relativistic Hamiltonian for the free πN system. With such a definition of z we have no need to introduce the formfactor $g_{\pi N}^{(\ell)}$. Further, in this case one gets the most simple relation between t -matrix in an arbitrary frame with the elementary amplitude in the πN c.m. frame (the so-called half-off-shell connection) [18] :

$$\langle\vec{q},\vec{p}'|t(\omega)|\vec{k},\vec{p}\rangle=-2\pi\,\delta(\vec{q}+\vec{p}'-\vec{k}-\vec{p})\sqrt{\frac{W_i W_f}{E_\pi E_\gamma E_N E_{N'}}}\,\langle\tilde{\vec{q}}\,|f_{\pi\gamma}(t_2)|\tilde{\vec{k}}\rangle, \quad (15)$$

Here, the pion and photon momenta $\tilde{\vec{q}}$ and $\tilde{\vec{k}}$ (in the πN c.m. frame) are coupled with momenta $\vec{q}$, $\vec{p}'$ and $\vec{k}$, $\vec{p}$ by the Lorentz transformations.

Due to a strong energy dependence of the multipoles in the Δ_{33} - region, the correct inclusion of the Fermi-motion of nucleons becomes of great importance. This motion has been taken into account by the factorization approximation: the nucleonic momenta in the elementary t -matrix are substituted by their effective values according to

$$\vec{p}=-\frac{\vec{k}}{A}-\frac{A-1}{2A}(\vec{k}-\vec{q}) \quad (16a)$$

and

$$\vec{p}'=-\frac{\vec{q}}{A}+\frac{A-1}{2A}(\vec{k}-\vec{q})\,. \quad (16b)$$

The corresponding DWIA method is usually called the "local DWIA". In general, this approximation works well enough [19] .

4. We have discussed all ingredients of the theory and in some cases have checked them by comparing with experimental data. Now we will apply the full theory to coherent π^0 photoproduction.

4.1. In the threshold region the total coherent cross-section is measured for four nuclei with $J_i = 0$, namely ^{4}He, ^{12}C, ^{40}Ca and ^{208}Pb. Three of them have an equal number of protons and neutrons. In this case the coherent process singles out the isoscalar spin-independent part from the full amplitude:

$$\langle \tilde{\vec{q}}_{c.m.} | f_{\pi\gamma}(\omega) | \tilde{\vec{k}}_{c.m.} \rangle = \tilde{f}_2(\omega) [\tilde{\vec{q}}_{c.m.} \times \tilde{\vec{k}}_{c.m.}] \cdot \vec{\epsilon} \ .$$

Only magnetic multipoles contribute to this part of the amplitude.

The overall agreement between theory and experiment is quite good, Fig. 3. Let us now discuss the effects of different ingredients of the calculations having ^{12}C as an example. Some results are given in Table 3:

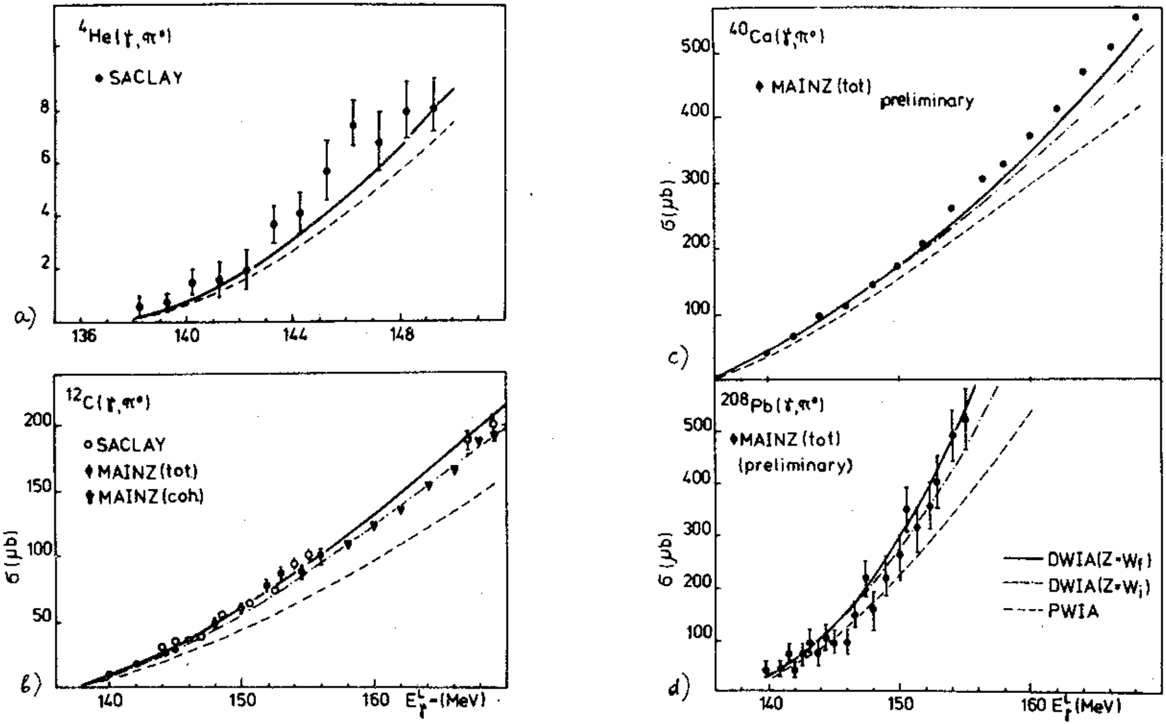
i) final state interaction increases the cross-section by about 15% as compared with PWIA. Our present result in PWIA slightly differs from that given in Ref.[23] due to some modification of the amplitude: here, the unitarized version of Bosted - Laget's amplitude with the momentum dependent width of the Δ -isobar is used. This enhancement takes place for all the nuclei up to E_γ = 200 MeV;

ii) at two energies of incoming photons we have switched off the U_2 term in the optical potential and the cross-section became larger exceeding the experimental one significantly.

The coherent π^0 photoproduction seems to be a good instrument to get information on a neutral pion interactions with nuclei. This information is important for the theory of pion-nucleon scattering as well as for many others, π^0 production in heavy ion collisions for example.

Now we shall discuss the contribution of different diagrams to the coherent process, again having ^{12}C as an example. Fixing the energy of incoming photons (E_γ^{LAB} = 160 MeV) and the angle of outgoing pions ($\Theta_\pi^{c.m.} = 60^o$), we get the results given in Table 4 (for notation see Ref.[23]) where

$$f_2 = (t'_{BORN} + t'_\Delta + t'_\omega) \frac{M_N}{4\pi\omega} \ .$$



F i g. 3. The total coherent cross section in the threshold region; a) - ^{4}He, experimental data - from Ref.[22], b), c), d) - ^{12}C, ^{40}Ca and ^{208}Pb, experimental data - from Ref.[7]. DWIA calculations are based on the BL[4] amplitude; U_2 term is taken into account; two versions of the reaction energies are given.

The version D differs from the version C$'$ having $\Gamma_\Delta \neq const$ and $e^{i\Psi_1} \neq 1$. We want to note that Δ -terms are normalized to the charged pion mass in contrast with Ref.[23] where the neutral pion mass was used.

4.2. The angular distribution of outgoing pions is more sensitive to different versions of ingredients of the theory.

In the case of ^{40}Ca the final state interaction changes the form of the curve in region of large angles.

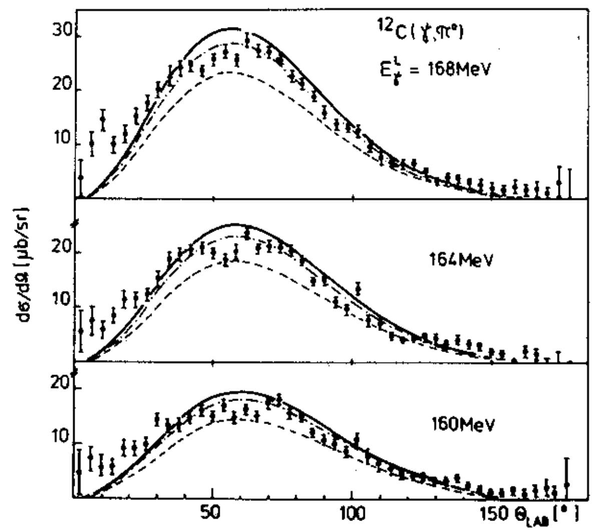
T a b l e 3. Comparison of PWIA and DWIA calculations (in μb) of coherent π^0 -production on ^{12}C. Two versions of the reaction energy are given. The total cross sections calculated for $U_2 = 0$ are given in brackets

E_γ, MeV	PWIA	DWIA($z=\omega_i$)	DWIA($z=\omega_f$)
138	2.90	3.34	3.80
142	13.4	16.8	18.2
150	45.4	58.8	61.1
160	98.2	125(155)	133(199)
175	194	230(279)	257(358)

T a b l e 4. Numerical values for different components of the BL amplitude (in fm^3); $E_\gamma^{LAB} = 160$ MeV, $\Theta_\pi^{c.m.} = 60^\circ$

	$t'_{BORN} \times 10^3$	$Re\, t'_\Delta$	$Im\, t'_\Delta$	t'_ω
C$'$	5.92	-0.256	-0.114	-0.119
D	5.92	-0.305	-0.228	-0.119

For lighter nuclei (^{12}C) the final state interaction does not change a shape of the curve in the same energy region, increasing only the absolute value. Different choice of the reaction energy changes the absolute value of the differential cross sections too ; and the effect becomes larger the larger the energy of incoming photons. At small angles some disagreement takes place (see Fig.4).



F i g. 4. Angular distribution for the coherent $^{12}C(\gamma,\pi^0)$ reaction at different photon energies. Dashed-line - the PWIA calculation, the rest - the DWIA one with the following choice of the reaction energy; $z = \omega_i$ - dash-dotten line, $z = \omega_f$ - solid line. Experimental data - from Ref.[7] . Bosted - Laget's amplitude is used.

5. Closing the discussion one can conclude that :

- For the neutral pion photoproduction at the threshold the experimental data contradict the LET prediction. In some versions of the amplitude the dispersive corrections are taken into account. Their extrapolation to the threshold region gives the result close to the recent experimental data. So one needs more detailed theoretical examination of this region.

- The coherent pion photoproduction off double magic nuclei at the threshold is described well enough. Some ingredients of the theory have been checked in an independent way. Having no free parameters theory suceeds to describe quantitatively both the total cross-section and the angular distribution. So some self-consistent description of the reaction has been achieved.

- At the threshold, in coherent π^0 production Δ -term playes an important role, too. So one has a possibility of studying the low-energy Δ behaviour in nuclei.

- The Δ resonance region can also be explained by the theory having the same ingredients.

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