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Our recent investigations aim to construct such a variant of semi-microscopic theory of atomic nucleus which could be a basis for a detailed description of the low-lying states. It should give a general structure of wave functions for the states of intermediate and high excitation energies. On the basis of this structure some"averaged" description of these states has been developed in the language of different strength functions. In ref./1/ it is shown that we have all grounds to work out a unified description of low-lying, intermediate, and highly excited states of atomic nuclei.

In the present talk we give a part of the results obtained in the framework of the above mentioned programme concerning the fragmentation of single-particle states over many nuclear levels and concerning the calculation of neutron strength functions.

2. In the study of the structure of states of intermediate and high excitation energy in atomic nuclei of much importance is the single-particle fragmentation, that is the distribution of the single particle strength over many nuclear levels. In the independent-particle and quasiparticle models/2/ the single-particle strength is concentrated on a single level. In the extreme statistical model it is chaotically distributed over all nuclear levels. In the earlier period of the study of resonance nuclear reactions and the construction of neutron strength functions Lane, Thomas and Wigner/3/ introduced a model of intermediate coupling for describing fragmentation. Since that time the frag-

mentation and neutron strength functions are represented in the Breit-Wigner form[4]. Phenomenological description of neutron strength functions in the framework of optical model has serious difficulties, for instance, due to absence of coupling with the quasiparticle and phonon branches of nuclear excitation. It does not describe the dependence of strength functions on the excitation energy and gives no accurate values of their minima.

To describe fragmentation, in refs.[5,6] the mechanism of interaction of quasiparticles with phonons was suggested. The quasiparticle-phonon interaction is very important in the calculation of the energy and of the structure of low-lying non-rotational states of atomic nuclei (see refs.[1,7,8]). The idea of the model based on the account of the quasiparticle-phonon interaction was formulated in ref.[9], and in ref.[10] an approximate method of solving its equations was developed. In ref.[11] was formulated a variant of the model which was applied to spherical nuclei.

With increasing excitation energy the density of levels increases and their structure becomes complicated. The coupling of intrinsic and collective motions described as the quasiparticle-phonon interaction is, to a considerable extent, responsible for the complicating state structure with increasing excitation energy. Qualitatively general picture of the complicating state structure and the regularities of fragmentation of single-particle and many-particle states were discussed in refs.[6,12].

The calculations of the single-particle state fragmentation are performed in the framework of the suggested model. It has a number of advantages making it applicable for describing the structure of highly excited states in complex nuclei. In the model the phonon and quasiparticle excitations are treated in a common way. The two-quasiparticle operators are presented in the form of phonon operators of the multipolarity λ in that limiting case when the root of the secular equation tends to the corresponding pole. The phonons of the multipole and spin-multipole type are introduced and a large number of phonons of each type is considered. For instance in ref.[13] the phonons of 15 multipolarities and 10-70 roots for the corresponding secular equations for each multipolarity have been used. The low-lying collective quadrupole and octupole phonons and many weak-collective phonons are considered, as well as the high-lying phonons of the giant resonances type. The configurational space of the model is large, and the wave functions of highly-excited states have millions of different components. Thus, in the framework of the model the state complexity usually treated as a compound state, may be reflected. The calculations of the state density[14] evidence the completeness of the configurational space.

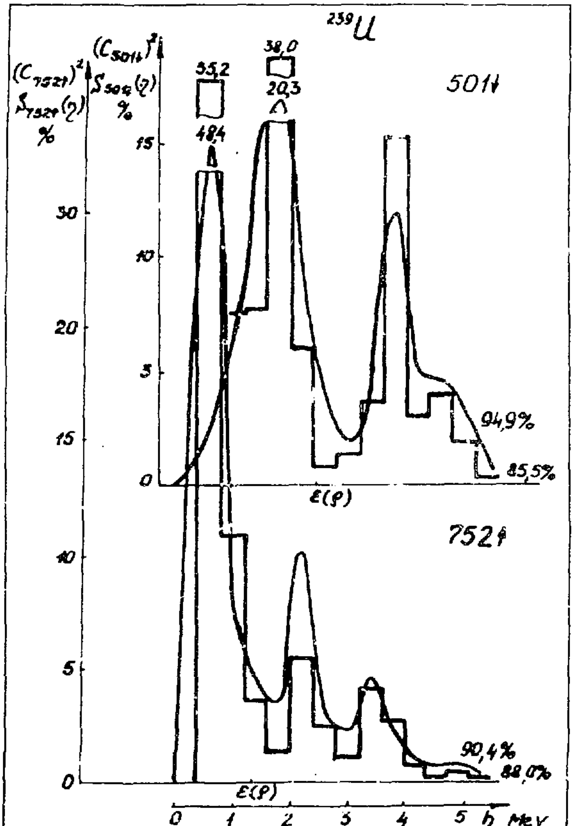
An important feature is that in the model there are no free parameters. The parameters of the Saxon-Woods potential describing an average field of nucleus, the superconducting pairing correlation constants and the multipole-multipole interaction constants were fixed earlier in describing the low-lying states. The interaction constants connected with the giant resonances are determined when calculating the energies of these resonances. To obtain numerical results, presented in this talk, we have used the Saxon-Woods potential parameters and the interaction constants from refs.[7,8,15].

3. To describe fragmentation of single-particle states we use the the equations of the model relating to odd-A nuclei.

In ref.[13] it was shown that in order to obtain, in general, the fragmentation of single-particle states one can use a simplified variant of the model when the wave function has one-quasiparticle and quasiparticle plus phonon terms . Even in this case one has to find several hundreds of solutions of the corresponding secular equations in order to obtain the strength distribution of single-particle states over the levels of odd-mass deformed nucleus.

The basic features of the fragmentation of the single-particle states in deformed nuclei were clarified in ref.[13]. The calculations were performed for ^{239}U and ^{169}Er . We studied the strength distribution as a function of the position of the single-particle level with respect to the Fermi level and the shape of the distribution.

To illustrate general features of the fragmentation, the histogram (fig.1) exhibits the strength distribution of the 752↑ and 501↓ single-particle states in ^{239}U . The quantities (C_ρ^i)^2, defining the contribution of one-quasiparticle component to the normalization condition of the corresponding wave function, are represented as a sum over the states in the energy interval ΔE =0,4 MeV. These are denoted as $(C_\rho)^2 = \sum_{\Delta E} (C_\rho^i)^2$ and are given in percents. On the abscissa axis common for both the states are the excitation energies reckoned from the ground state energy which for ^{239}U is $\eta_{622\uparrow}$ =0,4 MeV. The compact representation of the data has given no possibility of following firmly the scale for some values which are marked by numbers above them. The figure gives the quasiparticle energies $\varepsilon(\rho)$ and the total contribution of the (C_ρ)^2 values up to the energy 5.6 MeV.



^{239}U
501↓
752↑
$(C_{752↑})^2$
$S_{752↑}(\eta)$
%
$(C_{501↓})^2$
$S_{501↓}(\eta)$
%
35,2
38,0
20,3
48,4
94,9%
85,5%
90,4%
88,0%
E(ρ)
η, MeV

Fig. 1

The 752↓ and 501↑ single-particle states lie lower the Fermi surface energy by 1,34 and 2,53 MeV. Their strength is distributed in a wide energy interval and the distribution function has a complex form. The examples of fragmentation of several single-particle states in ^{235}U and ^{249}Cm are given in /13/. According to /13/ the strength distribution for the single-particle states in deformed nuclei displays the following particular features:

i) at high excitation energies, in addition to the first maximum, there appears a second one,

ii) the distribution function is nonsymmetric with respect to its largest value due to its slower fall in favour of high energies,

iii) the shape of the distribution function is mainly defined by the position of the single-particle level with respect to the Fermi surface, it depends on the wave function of the single-particle state,

iv) the strength distribution has a long tail which, even for the single-particle states lying near the Fermi surface, expands essentially farther than the neutron binding energy.

It should be noted that in our calculations of fragmentation there is a strong strength fluctuation from one energy interval to another and, especially, from level to level. Strong fluctuations are, to a large extent, due, firstly, to the use of the one-phonon approximation and, secondly, to the roughness of the model in the present formulation wich disregards some collective excitation branches.

4. To obtain the strength distribution of the single-particle state one has to calculate the energies and the wave functions of a large number of states and then summarize the quantities $(C_\rho^i)^2$ in the fixed energy interval. For instance when constructing the histogram (fig.1) for the 752↑ state in [illegible]U, it was found 755 solutions of the secular equation. The components of the wave function were calculated for each solution and the quantities $(C_\rho^i)^2$ were determined from the wave function normalization condition. As a result, a very small part of information is used.

There are methods of direct calculation of average characteristics without a detailed calculation of each state. Together with Malov we have made use of one of them and constructed the function

$$S_\rho(\eta) = \sum_i (C_\rho^i)^2 \rho(\eta_i - \eta) , \qquad (1)$$

where according to /4/ we take

$$\rho(\eta_i - \eta) = \frac{1}{2\pi} \frac{\Delta}{(\eta - \eta_i)^2 + (\Delta/2)^2} , \qquad (2)$$

where Δ represents the energy interval around η_i over which the averages are taken, Δ is the free parameter. The secular equation for defining the energies can be written in the form

$$P(\eta_i) = 0 , \qquad (3)$$

$(C_\rho^i)^{-2} = \left.\frac{\partial P(\eta)}{\partial \eta}\right|_{\eta=\eta_i}$ and then (1) can be written as follows:

$$S_\rho(\eta) = \sum_i \left(\frac{\partial P(\eta_i)}{\partial \eta_i} \right)^{-1} \rho(\eta_i - \eta) . \qquad (4)$$

The expression (4) may be written as a contour integral with the contour which has as poles the roots of equation (3). We pass to integrals encircling two poles $z = \eta \pm i\Delta/2$ and calculate them. As a result we obtain:

$$S_\rho(\eta) = \frac{\Delta}{2\pi} \frac{\Gamma(\eta)}{(E(\eta) - \delta(\eta) - \eta)^2 + (\Delta/2)^2\,\Gamma^2(\eta)} \qquad (5)$$

i.e., the function $S_\rho(\eta)$ is represented in the Breit-Wigner form. We stress that the functions $\Gamma(\eta)$ and $\delta(\eta)$ depend on η. To describe the neutron strength functions one usually uses an expression of the type (5) with the values Γ and δ independent of η.

The functions $S_\rho(\eta)$ calculated according to (5) with Δ =0.2 MeV for the 752↑ and 501↓ states in ^{235}U are represented in fig.1 as curves. It is seen from fig.1 that the general Breit-Wigner form $S_\rho(\eta)$ is strongly distorted due to the dependence of Γ and δ on η. The function $S_\rho(\eta)$ is similar with $(C_\rho)^2$ in all its properties slightly smoothing out fluctuation. It is seen from fig.1 that $S_\rho(\eta)$ exhausts the strength of single-particle state up to the energy 5,6 MeV a little more than $(C_\rho)^2$. Comparing the behaviours of $S_\rho(\eta)$ and $(C_\rho)^2$ one can make the following conclusion. In the study of fragmentation of single-particle states in odd-A deformed nuclei the corresponding calculations can be performed by (5).

5. The obtained results for the single-particle state fragmentation made it possible to formulate a fundamentally new semimicroscopic method of calculation of the neutron strength functions. The neutron strength function is defined as:

$$S_0 = \frac{\langle \Gamma_n^{0} \rangle}{\langle D \rangle}, \tag{6}$$

where Γ_n^{0} is the reduced neutron width, D is the spacing between the levels with given I^{π}. Using the wave functions of neutron resonances we get the following expression for the s -wave strength function for a deformed nucleus

$$S_0 = \frac{5\,(keV)}{\Delta E\,(keV)} A^{2/3} \sum_{i\,\Delta E} \Big| \sum_{\rho} \mathcal{U}^{\rho}_{c\,1/2}\, u_{\rho}\, C^{i}_{\rho} \Big|^2 , \tag{7}$$

where ΔE is the energy interval inside which a summation of C^{i}_{ρ} over the excited states is performed; u_{ρ} is the Bogolubov canonical transformation coefficient calculated with the correlation function and the chemical potential for the ground state of the target-nucleus; a summation over ρ is performed over the single-particle states with $K^{\pi}=1/2^{+}$. According to [8], the single-particle wave function φ_{ρ} is represented as an expansion in the spherical basis

$$\varphi_{\rho} = \sum_{n\ell_j} \mathcal{U}^{\rho}_{n\ell_j}\, \varphi_{n\ell_j} , \quad \mathcal{U}^{\rho}_{\ell_j} = \sum_{n} \mathcal{U}^{\rho}_{n\ell_j} \tag{8}$$

In the case when one single-quasiparticle state gives the main contribution into the strength function, the function S_0 has a more simple form

$$S_0 = \frac{15\,(keV)}{\Delta E\,(keV)} A^{2/3} \left(\mathcal{U}^{\rho}_{c\,1/2}\right)^2 u_{\rho}^2 \sum_{i\,\Delta E} (C^{i}_{\rho})^2 \tag{9}$$

The expression for the p-wave strength function consists of three terms

$$S_1 = S_1(1/2-1/2) + S_1(3/2-1/2) + S_1(3/2-3/2), \tag{10}$$

where the first term is relative to the $I^\pi K = \frac{1}{2}^- \frac{1}{2}$ states, the second one describes the contribution of the corresponding rotational components for which $I^\pi K = \frac{3}{2}^- \frac{1}{2}$, and the third term gives the contribution from the $K^\pi = 3/2^-$ states. They are

$$S_1\left(\tfrac{1}{2}^- \tfrac{1}{2}\right) = \frac{15\,(keV)}{\Delta E\,(keV)} A^{2/3} \sum_{i\in\Delta E} \Big|\sum_\rho u^\rho_{1/2}\, u_\rho\, C^i_\rho\Big|^2 , \tag{11}$$

$$S_1\left(\tfrac{3}{2}^- \tfrac{1}{2}\right) = \frac{30\,(keV)}{\Delta E\,(keV)} A^{2/3} \sum_{i\in\Delta E} \Big|\sum_\rho u^\rho_{1/2}\, u_\rho\, C^i_\rho\Big|^2 , \tag{12}$$

$$S_1\left(\tfrac{3}{2}^- \tfrac{3}{2}\right) = \frac{30\,(keV)}{\Delta E\,(keV)} A^{2/3} \sum_{i\in\Delta E} \Big|\sum_\sigma u^\sigma_{3/2}\, u_\sigma\, C^i_\sigma\Big|^2 \tag{13}$$

where ρ describes the $K^\pi = 1/2^-$ states, σ describes the $K^\pi = 3/2^-$ states. If the main contribution comes from the single states then the expressions (11),(12) and (13) can be written in the form (9).

In the case of spherical nuclei the s - and p -wave strength functions (on the even-even targets) have a more simple form, namely:

$$S_0 = \frac{15\,(keV)}{\Delta E\,(keV)} A^{2/3} u^2_{j_0} \sum_{i\in\Delta E} \left(C^i_{j_0}\right)^2 , \tag{14}$$

$$S_1 = \frac{15\,(keV)}{\Delta E\,(keV)} A^{2/3} \left\{ u^2_{j_1} \sum_{i\in\Delta E} \left(C^i_{j_1}\right)^2 + 2\, u^2_{j_2} \sum_{i\in\Delta E} \left(C^i_{j_2}\right)^2 \right\} , \tag{15}$$

where $j_0 = s_{1/2}$, $j_1 = p_{1/2}$, $j_2 = p_{3/2}$.

6. Now we give the calculation results of the s - and p-wave strength functions in deformed and spherical nuclei.

In ref./13/ the s- and p -wave strength functions at the energies near B_n in ^{235}U and ^{169}Er were calculated. The experimental data and the calculation results performed by formula

(9) are represented in the Table. It is seen from the Table that the calculated strength function S_0^{cal} for ^{239}U is in good agreement with experiment. The main contribution in S_0^{cal} comes from the fragmentation of the 600↑ and 880↑states, a noticeable contribution is given by the 611↑ state. If the single-particle state fragmentation is calculated by (5) then we obtain S_0^{cal}=1,9.10^{-4}, i.e., we obtain somewhat larger value for the s -wave strength function.

The situation of the S_0^{cal} calculations in ^{169}Er is more complicated and interesting. In the single-particle wave functions, in ref. /8/, there is no contribution from the $4s_{1/2}$ subshell which is in the continuous spectrum. Therefore, the related calculations yield S_0^{cal}=0.05 . 10^{-4}. The main contribution comes from the fragmentation of the 400↓ state which is weakened due to the factor U_ρ^2=0.02. Employing the single-particle wave functions /16/ in which the contribution of the $4s_{1/2}$ subshell is taken into account, then we get S_0^{cal}= 1.10^{-4} which is in agreement with experiment. The main contribution comes from the fragmentation of the 640↓state.

The S_0^{cal} calculations with two sets of the single-particle wave functions are of great methodic interest. Inclusion of the $4s_{1/2}$ subshell which seems to be a transition from the minimum to the maximum of the strength function, results in a 20 times increase of S_0^{cal}.

The calculation of the p-wave strength function in ^{239}U has been performed for two sets of the single- particle wave function which include /16/ or do not include /9/ the contribution of the $4p_{1/2}$ and $4p_{3/2}$ subshells. The calculations with the wave functions containing the $4p_{1/2}$ and $4p_{3/2}$ subshells give the value

$S_1^{cal} = 2.7 \cdot 10^{-4}$, which is in agreement with experiment. The values of three terms in (10) are the following: $S_1(1/2^- 1/2) = 0.5 \cdot 10^{-4}$, $S_1(3/2^- 1/2) = 2 \cdot 10^{-4}$, $S_1(3/2^- 3/2) = 0.2 \cdot 10^{-4}$.

The calculated values of the p-wave strength functions in ^{169}Er are also in a sufficiently good agreement with experiment The value $S_1^{cal} = 1.2 \cdot 10^{-4}$ includes $S_1(1/2^- 1/2) = 0.2 \cdot 10^{-4}$, $S_1(3/2^- 1/2) = 0.2 \cdot 10^{-4}$, and $S_1(3/2^- 3/2) = 0.8 \cdot 10^{-4}$. The largest contribution comes from the fragmentation of the 501↓ state.

Thus, the results of semi-microscopic calculations of the s - and p -wave neutron strength functions in ^{239}U and in ^{169}Er give a good description of the experimental data/17/. The results of calculations of S_0 and S_1 depend on the averaging interval ΔE. An essential expansion of ΔE results in an increase of S_0 and S_1 by a factor up to 1.5-2.0.

T a b l e I.

The neutron strength functions

Compound nucleus	$S_0 \cdot 10^4$ Experiment	$S_0 \cdot 10^4$ Calculations	$S_1 \cdot 10^4$ Experiment	$S_1 \cdot 10^4$ Calculat.
^{239}U	1.05±0.1	1.2	2.2±0.6	2.7
^{169}Er	1.5±0.3	1	0.7±0.2	1.2
^{123}Sn	0.4±0.25	0.1	-	5
^{121}Sn	0.09±0.05	0.1	3.7±1.8	7
^{119}Sn	0.35±0.20	0.2	4.5	6
^{117}Sn	0.37±0.15	0.3	1.4	5

The study of single-particle state fragmentation in spherical nuclei and the calculations of the neutron strength functions in the range of their minima are of great interest. In ref./18/

the preliminary results of calculation of the s -wave strength functions for the Sn isotopes are given. The calculations of fragmentation of the $3s_{1/2}$, $3p_{1/2}$ and $3p_{3/2}$ subshells are performed by the formula of the type (5). It is seen from the Table that in the framework of semi-microscopic method one has obtained a satisfactory description of neutron strength functions in Sn isotopes. The calculations for several spherical and deformed nuclei are in progress.

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