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A PHENOMENOLOGICAL UNIFIED DESCRIPTION OF THE GROUND BAND ENERGY LEVELS OF ROTATIONAL NUCLEI

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Almost all of the existing nuclei show some degree of collectivity. As a result a lot of collective models and interpretations have been developed. However, they differ in language and formalism. The purpose of this paper is to unify the description of the rotational low lying levels in the nuclei, by using only the universal nuclear characteristics specific for a given nucleus.

The description should include a large region of nuclei. The latter can be done within the classification scheme proposed in [1] and [2]. According to this scheme all even-even nuclei with $Z \geq 20$, $A\text{-}Z \geq 20$ are systematized in terms of boson representations of the $Sp(24,R)$ — algebra [3]. These nuclei, whose valence nucleons occupy the same major nuclear shell are united in two symplectic multiplets. The symplectic multiplets are defined by $(N_p, N_n \mid N_p', N_n')_\pm$, where N_p, N_n and N_p', N_n' are the numbers of protons and neutrons of the double magic nuclei at the beginning of two consecutive shells. The relevant multiplet is noted by + (-), when the total number of valence bosons is even (odd). Each nucleus corresponds to a definite cell in the symplectic multiplets, given by the irreducible unitary representations (IUR) of the group $U_\pi(6) \oplus U_\nu(6)$ according to the reduction

$$Sp(24,R) \xrightarrow{F_0} U(6,6) \xrightarrow{N} U_\pi(6) \oplus U_\nu(6)\,. \tag{1}$$

It is well known that the last group in (1) is the group of dynamical symmetry of IBM-2 [4]. Its UIT's are given by the proton and neutron boson numbers N_π and N_ν, which can be found by counting the valence proton and neutron pairs. By means of these quantum numbers the quantities N — total boson number and F_0 — third projection of the F — spin are:

$$N = N_\pi + N_\nu\,, \qquad F_0 = (N_\pi - N_\nu)/2\,. \tag{2}$$

Obviously, we have used some group theoretical apparatus in terms of quantities introduced in one of the most popular models IBM-2. But we would like to underline that the physical meaning of these quantities is easily kept in mind, so their direct relation to the nuclear characteristics as number of nucleons of both types N_p, N_n is as simple as that:

$$N = (A - A^{mag})/2\,, \qquad F_0 = (M_T - M_T^{mag})/2\,, \tag{3}$$

where $A = N_p + N_n$ is the mass number and $M_T = (N_p - N_n)/2$ is the third projection of the isospin, A^{mag}, M_T^{mag} are the mass number and third projection of the isospin of the double magic nucleus at the beginning of the shell.

An empirical investigation of the energies of the first excited 2^+ levels of the nuclei from a given multiplet was completed in /1,2/. The curves obtained show a stable periodical structure in the F_0-multiplets of a given shell. Follow up, all the excited states of the ground band present similar behaviour up to the occurrence of the back-banding in the spectrum. This periodical structure is observed in all shells investigated and is especially stable in the case of heavy and superheavy nuclei.

In the most simplified picture the well deformed nuclei, which demonstrate rotational spectra are to be found far from the spherical magic nuclei. In the case of our classification they correspond for each F_o to the lowest almost constant part of the N-dependence curves of the 2^+ energies. In order to specify this we tried to describe all the energies in the ground bands by means of the classical formulae of the type

$$E_{rot} = L(L+1)/2J, \tag{4}$$

where we assume, that the inertion parameter $1/2J$ is a polynomial up to a second degree in the independent combinations of N_π, N_ν, N_p, N_n, N_p', N_n'.

On the other hand the same assumptions are to be reached using the idea of the chain of groups, named general dynamical groups (GDGS), along which one can classify the nuclei. As it was already mentioned we classify the nuclei according to the chain (1), so the coefficient in the Hamiltonian will depend on the invariants of the groups in this chain; in this particular case — N, F_o and the numbers defining the relevent Sp(24, R) — multiplet, namely N_p, N_n, N_p', N_n'.

The classification chain will lead to the group of dynamical symmetry (DG) whose reduction will generate the spectrum of the already specified nucleus. The terms of the Hamiltonian itself are to be the first and second order Casimir operators of a subchain of DG. As we are trying to describe the well deformed nuclei, an obvious candidate is the chain

$$U(6) \supset SU(3) \supset SO(3),$$

which describes their spectrum in most algebraic models. On the states of the ground band, defined as: $|[\lambda]_6, [\lambda, 0]_3, L\rangle$ this type of Hamiltonian has eigenvalues

$$E_{rot} = F(N, F_o, N_p, N_n, N_p', N_n')\, L(L+1). \tag{5}$$

In order to describe the rotational energies using (5) it was necessary to introduce two new quantum numbers. These are N_π' and N_ν', defining the numbers of valence proton and neutron boson holes for the relevant nucleus. For convenience the following system of the independent quantum numbers was chosen:

$$\begin{aligned} N &= (N_\pi + N_\nu), \quad F_o = (N_\pi - N_\nu)/2, \quad A_P = N_p + N_p' \\ N' &= (N_\pi' + N_\nu'), \quad F_o' = (N_\pi' - N_\nu')/2, \quad A_N = N_n + N_n'. \end{aligned} \tag{6}$$

Thus the mathematical expression of the energy is

$$\begin{aligned} E_{rot} = L(L+1)\{&C_o + C_1 N + C_2 N' + C_3 F_o + C_4 F_o' + C_5 A_P + C_6 A_N + \\ &+ C_7 N^2 + C_8 N'^2 + C_9 F_o^2 + C_{10} F_o'^2 + C_{11} N F_o + \\ &+ C_{12} N' F_o' + C_{13} A_P^2 + C_{14} A_N^2 + C_{15} A_P A_N\}. \end{aligned} \tag{7}$$

All the rotational nuclei belonging to the multiplets $(50, 50|\, 82, 82)_\pm$; $(50, 82|\, 82, 126)_\pm$; $(82, 126|\, 126, \ldots)_\pm$ have been considered. It is obvious that we are looking for the solution of the overdetermined system of equations:

$$E_{rot}(L, N, N', F_o, F', A_P, A_N, (C_i)) = E_{rot}^{exp}(L). \tag{8}$$

$E_{rot}^{exp}(L)$ are the experimental values of the energies /5/ of the ground bands for $L = 2^+, 4^+, \ldots$ up to the occurrence of back-banding.

Using an autoregularized iterational method of Gauss — Newton type /6/ the coefficients C_i are evaluated for 328 excited states of 70 nuclei. As a result for the rotational energies $E_{rot}(L)$ the following simple formulae were obtained:

$$\begin{aligned} E_{rot}(L) = L(L+1)\{&D_1(N+N') + D_2(F_o + F_o') + \\ &+ D_3(N^2 + N'^2/2) + D_4(F_o^2 - F_o'^2/2) + D_5 A_N\}. \end{aligned} \tag{9}$$

The values of the parameters D_i $(i=1, \ldots, 5)$ are given in Table 1 together with their uncertainties $\pm\ \Delta D_i$.

Table 1

i	D_i	$\pm \Delta D_i$
1	16.44	0.37
2	-36.82	3.06
3	1.92	0.06
4	-9.67	0.57
5	-3.38	0.04

Obviously, the values of $(N+N')$, $(F_0 + F_0')$ and A_N are constants for the nuclei belonging to a given Sp(24, R)-multiplet. The necessity of introducing the quantum numbers N', F_0' reveals the asymmetry of the transition from the spherical nuclei to the well deformed ones and vice versa. Hence, for the manifestation of the nuclear collectivity it is important not only how many valence nucleons the relevant nucleus has but as well how more you need to fill up the shell. The values of the coefficients in front of the nuclear characteristics show the different contribution of each one to the parameter of inertia J.

In order to investigate this, it is more convenient to transform (9), reduced for the nuclei only from a given shell, in the following way:

$$E^{shell}_{rot} = \{(D_3 + D_4/4)(N_\pi^2 + N_\nu^2) + (D_3 - D_4/4)(N_\pi'^2 + N_\nu'^2) +$$

$$+ 2(D_3 - D_4/4)N_\pi N_\nu + 2N_\pi' N_\nu'(D_3 + D_4/4)\}L(L+1);$$

or using the coefficients from Table 1 for the rotational parameter in a given shell one has

$$1/2J = 8.675 N_\pi N_\nu - 0.995 N_\pi' N_\nu' - 0.4985(N_\pi^2 + N_\nu^2) + 4.3375(N_\pi'^2 + N_\nu'^2).$$

Obviously, the most important part in the rotational motion plays the interaction between the proton and neutron systems, depending on the product $N_\pi N_\nu$ [7]. And as a new result one can consider the rather big coefficient in front of the term $(N_\pi'^2 + N_\nu'^2)$ depending on the number of boson holes, or which is the same, on the number of valence nucleons which are needed to fill up the shell.

In Table 2 the experimental [5] and theoretical values of the energies for each L are compared, for the nucleus from each multiplet with the longest rotational band. It must be mentioned that for the unknown values of N_p' and N_n' for the third multiplet we use those values that the shell model predicts, i.e. $N_p' = 126$ and $N_n' = 184$.

The empirical formulae (9) obtained on the basis of the large investigation of the energies of the nuclei classified in a simple symplectic scheme can be used as a tool for evaluating the degree of the deformation or the validity of the rotational description of the nuclear spectra.

Table 2

$(50,50|82,82)_+$

$^{128}_{60}Nd_{68}$

L	E_{exp}	E_{th}
2	0.134	0.120
4	0.426	0.401
6	0.850	0.842

$(50,50|82,82)_-$

$^{130}_{60}Nd_{70}$

L	E_{exp}	E_{th}
2	0.159	0.132
4	0.485	0.441
6	0.940	0.927

$(50,82|82,126)$

$^{156}_{62}Sm_{94}$

L	E_{exp}	E_{th}
2	0.076	0.067
4	0.250	0.225
6	0.518	0.471
8	0.878	0.808

$^{152}_{60}Nd_{92}$

L	E_{exp}	E_{th}
2	0.076	0.069
4	0.241	0.228
6	0.488	0.480
8	0.810	0.822

$^{156}_{64}Gd_{92}$

L	E_{exp}	E_{th}
2	0.089	0.076
4	0.288	0.253
6	0.585	0.532
8	0.965	0.912
10	1.416	1.394
12	1.924	1.977

$^{164}_{68}Er_{96}$

L	E_{exp}	E_{th}
2	0.091	0.078
4	0.299	0.260
6	0.614	0.545
8	1.025	0.935
10	1.518	1.428
12	2.083	2.025

$^{180}_{74}W_{106}$

L	E_{exp}	E_{th}
2	0.104	0.08
4	0.338	0.29
6	0.688	0.62
8	1.138	1.07
10	1.660	1.64

$^{184}_{76}Os_{108}$

L	E_{exp}	E_{th}
2	0.120	0.098
4	0.384	0.327
6	0.774	0.686
8	1.275	1.176
10	1.870	1.797

Table 2 (contin.)

$^{172}_{66}Dy_{96}$

L	E_{exp}	E_{th}
2	0.081	0.072
4	0.266	0.240
6	0.549	0.504
8	0.921	0.864
10	1.375	1.320
12	1.903	1.871
14	2.494	2.519

$^{176}_{70}Yb_{106}$

L	E_{exp}	E_{th}
2	0.082	0.078
4	0.272	0.261
6	0.565	0.548
8	0.954	0.940
10	1.431	1.436
12	1.985	2.036
14	2.602	2.741

$^{178}_{72}Hf_{106}$

L	E_{exp}	E_{th}
2	0.093	0.083
4	0.307	0.277
6	0.632	0.582
8	1.059	0.998
10	1.571	1.524
12	2.151	2.162
14	2.778	2.910

(82, 126 | 126, 184)

$^{232}_{90}Th_{142}$

L	E_{exp}	E_{th}
2	0.049	0.043
4	0.162	0.143
6	0.333	0.300
8	0.557	0.515
10	0.827	0.787
12	1.137	1.116
14	1.483	1.502
16	1.859	1.946

$^{236}_{92}U_{146}$

L	E_{exp}	E_{th}
2	0.045	0.040
4	0.148	0.133
6	0.307	0.279
8	0.518	0.479
10	0.776	0.731
12	1.077	1.037
14	1.415	1.396
16	1.788	1.808

$^{244}_{94}Pu_{150}$

L	E_{exp}	E_{th}
2	0.046	0.041
4	0.156	0.137
6	0.318	0.288
8	0.535	0.493
10	0.801	0.754
12	1.114	1.069
14	1.467	1.439
16	1.858	1.864

$^{260}_{98}Cf_{162}$

L	E_{exp}	E_{th}
2	0.043	0.042
4	0.142	0.140
6	0.296	0.294
8	0.509	0.505

$^{266}_{100}Fm_{166}$

L	E_{exp}	E_{th}
2	0.046	0.049
4	0.160	0.165
6	0.332	0.346
8	0.564	0.593

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