объединенный
институт
ядерных
исследований
дубна

E4-88-886

E.B.Balbutsov, J.Piperova

COLLECTIVE 0^+, 1^+ AND 2^+ EXCITATIONS IN ROTATING NUCLEI

Submitted to "Ядерная физика"

1988

1. Introduction

Nowadays a quite complete experimental information concerning isoscalar quadrupole and monopole resonances exists [1-3]. Similar information about isovector resonances is rapidly accumulating, too [4-8]. Numerous theoretical investigations are devoted to the same subject. These are different microscopic models using RPA [9-13] or macroscopic calculations, based on the energy density functional and the phase space method with "scaling" [14-19], as well as phenomenological models [20,21].

In present work to describe quadrupole and monopole excitations (both isoscalar and isovector) in rotating nuclei we use the method of moments, developed in refs. [22-24]. This approach, based on the phase space method, enable us to describe by the same equations the excited states of the nucleus and its equilibrium shape, which is important in rotating nuclei. In previous works [22-24] the basic equations have been derived in the approximation of a local nucleon-nucleon interaction. Now the possibilities of the method are investigated in the case of nonlocal interaction.

2. Formalism

The starting point for the method of moments [24] is the time-dependent Hartree-Fock equations for the density matrices $\hat{\rho}_q(\vec{r}_1, \vec{r}_2, t)$:

$$i\hbar \frac{\partial \hat{\rho}_q}{\partial t} = [\hat{H}_q, \hat{\rho}_q] \quad , \qquad q = n, p , \tag{1}$$

where $\hat{H}_q$ is the self-consistent single-particle Hamiltonian; the label n, p stands for neutrons and protons, respectively. A Skyrme-type effective interaction called the "modified Skyrme force" or SKM* [25] is used for the nucleon-nucleon interaction. A self-consistent potential with such forces is derived as usual [26] and it can be found in detail in ref. [18].

Following the prescriptions of ref. [24] we transform equations

(1) to the equations for the Wigner functions $f_q(\vec{r},\vec{p},t) =$

$$= \frac{1}{(2\pi\hbar)^3}\int e^{-i\vec{p}\vec{s}/\hbar}\rho_q(\vec{r}+\frac{\vec{s}}{2},\, \vec{r}-\frac{\vec{s}}{2},\, t)\, d\vec{s} \; :$$

$$\frac{\partial f_q}{\partial t} = \frac{2}{\hbar}\sin\left\{\frac{\hbar}{2}\left(\nabla_r^H\cdot\nabla_p^f - \nabla_p^H\cdot\nabla_r^f\right)\right\} H_W f_q \qquad (2)$$

with $H_W(\vec{r},\vec{p}) = \int e^{-i\vec{p}\vec{s}/\hbar}(\vec{r}+\vec{s}/2\,|\hat{H}|\,\vec{r}-\vec{s}/2)\, d\vec{s}$

and $\vec{r} = (\vec{r}_1+\vec{r}_2)/2$, $\vec{s} = \vec{r}_1 - \vec{r}_2$.

Then, integrating (2) over $\vec{p}$ we obtain dynamical equations for the densities, which are just the continuity equations /26/

$$\frac{\partial n_q}{\partial t} + \operatorname{div}(n_q\vec{u}_q) = \beta \operatorname{div} n_q n_{q'}(\vec{u}_q - \vec{u}_{q'}) \qquad (3)$$

with $n_q(\vec{r},t) = \int f_q(\vec{r},\vec{p},t)\,d\vec{p}$ being the nucleon density, $\vec{u}_q(\vec{r},t) = \int f_q(\vec{r},\vec{p},t)\,\vec{p}\,d\vec{p}/n_q m$ is the mean velocity of nucleons; m the nucleon mass; $\beta = m t_+/2\hbar^2$, $t_+ = t_1 + t_2$, t_1 and t_2 are parameters of the Skyrme force (the coefficients before the nonlocal terms); $q' = p$ if $q = n$ and vice versa.

Note, that only the first term of the power expansion for the sine function in eqs. (2) appears in equations (3), the other terms drop out due to integration.

The next step is to integrate (2) over $\vec{p}$ with the weight p_i to find dynamical equations for the new quantities involved in eqs. (3), namely, for the velocities $\vec{u}_q$:

$$n_q\frac{d}{dt}u_{iq} - 2\Omega n_q\sum_{k=1}^{3}\varepsilon_{ik3}u_{kq} - 2\Omega\beta n_q n_{q'}\sum_{k=1}^{3}\varepsilon_{ik3}(u_{kq}-u_{kq'}) -$$

$$-\Omega^2(1-\delta_{i3})x_i n_q + \frac{n_q}{m}\frac{\partial U_q}{\partial x_i} + (P_q + m n_q u_q^2)\frac{\partial C_q}{\partial x_i} + \qquad (4)$$

$$+\sum_{k=1}^{3}\frac{\partial}{\partial x_k}\left[(1/m + 2C_q)P_{ikq}\right] + n_q\vec{u}_q\cdot\frac{\partial}{\partial x_i}\vec{B}_q + \qquad i=1,2,3$$

$$+\beta n_q n_{q'}(\vec{u}_q-\vec{u}_{q'})\cdot\vec{\nabla}u_{iq} - \frac{\hbar^2}{2m}\left(\nabla n_q\cdot\nabla\frac{\partial}{\partial x_i}C_q + \frac{1}{2}\frac{\partial C_q}{\partial x_i}\nabla^2 n_q\right) = 0 \, .$$

In equations (4) other collective variables, the pressure tensors $P_{ijq}(r,t) = m^{-1}\int(p_i - m u_{iq})(p_j - m u_{jq})f_q(\vec{r},\vec{p},t)\,d\vec{p}$ appear and need also dynamical equations to be defined. Here $\frac{d}{dt} = \frac{\partial}{\partial t} + \sum_{k=1}^{3}u_{kq}\frac{\partial}{\partial x_k}$, ε_{ijk} is the Levi-Civita tensor,

$$P_q = \sum_{k=1}^{3}P_{kkq} \; , \quad P = P_n + P_p \; , \quad C_q = \frac{1}{4\hbar^2}\left(t_+ n - \frac{t_-}{2}n_q\right),$$

$$t_- = t_1 - t_2 \; , \quad n = n_p + n_n \; , \quad \vec{B}_q = -\frac{m}{2\hbar^2}\left(t_+ n\vec{u} - \frac{t_-}{2}n_q\vec{u}_q\right),$$

$$n\vec{u} = n_p\vec{u}_p + n_n\vec{u}_n \, , \quad U_q = t_0\left[(1+x_0/2)n - (x_0+1/2)n_q\right] +$$

$$+\frac{1}{16}(3t_2 - 5t_1)\nabla^2 n + \frac{1}{32}(3t_2 + 5t_1)\nabla^2 n_q + \frac{1}{2}\beta\left[P + m(n_q u_q^2 + n_{q'}u_{q'}^2)\right] -$$

$$-\frac{m t_-}{8\hbar^2}(P_q + m n_q u_q^2) + \frac{1}{24}t_3 n^{\sigma-1}\Big\{n^2\left[(1-x_3)\sigma + 2(2+x_3)\right] -$$

$$-2n_q^2\sigma(1+2x_3) + 2n n_q(\sigma-1)(1+2x_3)\Big\} + \delta_{q,p}V_c \; ;$$

$\sigma, t_0, t_3, x_0, x_3$ - are Skyrme force parameters and $V_c = \int n_p(\vec{r}\,')\,e_p^2/|\vec{r}-\vec{r}\,'|\,d\vec{r}\,'$ is the direct part of the Coulomb interaction. Equations (4) are written in a frame of reference rotating with angular velocity $\vec{\Omega}(0,0,\Omega)$.

It is interesting to note that due to the nonlocality of the interaction we use, already two terms of the power expansion of the sine function (2) contribute to equations (4) - a term proportional to $\hbar^2$ has appeared.

As is well known (see, e.g., ref. /27/) equations (2) reduce to Vlasov equations in the limit $\hbar \to 0$. Thus, in our method we are able to study quantum corrections to the Vlasov equations.

Finally, let us write down the dynamical equations for the pressure tensors $P_{ij\varphi}$. We need for that to integrate (2) over $\vec{p}$ with weights $p_i p_j$:

$$\frac{d}{dt}P_{ij\varphi} + P_{ij\varphi}\,\mathrm{div}\,\vec{u}_\varphi + \rho\,\mathrm{div}\left[n_{\varphi'}P_{ij\varphi}(\vec{u}_\varphi - \vec{u}_{\varphi'})\right] +$$

$$+\sum_{k=1}^{3}\frac{\partial}{\partial x_k}\left(\frac{m^2}{m^*}P_{ijk\varphi}\right) + \sum_{k=1}^{3}\Big\{P_{ik\varphi}\Big(\frac{m}{m^*}\frac{\partial}{\partial x_k}u_{j\varphi} +$$

$$+2\frac{m}{m^*}\Omega\varepsilon_{j3k} + 2mu_{k\varphi}\frac{\partial}{\partial x_j}C_\varphi + \frac{\partial}{\partial x_j}B_{k\varphi}\Big) +$$

$$+m^2\frac{\partial C_\varphi}{\partial x_j}P_{ikk\varphi} - \frac{1}{2}\hbar^2 n_\varphi\left(\frac{\partial u_{i\varphi}}{\partial x_k}\frac{\partial^2 C_\varphi}{\partial x_k\partial x_j} + \Omega\varepsilon_{i3k}\frac{\partial^2 C_\varphi}{\partial x_k\partial x_j}\right) -$$

$$-\frac{1}{2}\hbar^2\frac{\partial C_\varphi}{\partial x_j}\left(\frac{\partial u_{i\varphi}}{\partial x_k}\frac{\partial n_\varphi}{\partial x_k} + \frac{1}{2}n_\varphi\frac{\partial^2 u_{i\varphi}}{\partial x_k^2} + \Omega\varepsilon_{i3k}\frac{\partial n_\varphi}{\partial x_k}\right)\Big\} +$$

$$+\sum_{k=1}^{3}\{\,i\leftrightarrow j\,\} = 0 . \tag{5}$$

Here the notations $m^* = m(1+2mC_\varphi)^{-1}$ and

$$P_{ijk\varphi}(\vec{r},t) = m^{-2}\int(p_i - mu_{i\varphi})(p_j - mu_{j\varphi})(p_k - mu_{k\varphi})f_\varphi(\vec{r},\vec{p},t)\,d\vec{p}$$

are introduced.

According to refs.[22-24] one needs now to integrate equations (4) and (5) over an infinite volume with weights x_j and 1 respectively. In the case of a local interaction all integrals including third-rank tensors $P_{ijk\varphi}$ vanish and thus one obtains "closed" system of coupled equations for the tensors of inertia $\mathcal{J}_{ij\varphi} = m\int n_\varphi x_i x_j\,d\vec{r}$ [24] and the integral pressure tensors $\Pi_{ij\varphi} = \int P_{ij\varphi}\,d\vec{r}$. The situation turns out to be much more complicated when one uses nonlocal

interaction. In that case the integral $m^2\int\partial C_\varphi/\partial x_j\sum_{k=1}^{3}P_{ikk\varphi}\,d\vec{r}$ resulting from eqs. (5) is nonzero and to decouple anyhow the infinite chain of equations for the tensors $\int P_{ij\ldots k\varphi}\,d\vec{r}$ it has to be neglected. Fortunately, we are able to appreciate numerically the accuracy of such an approximation and it will be shown later that the resulting inaccuracy is very small.

The equations we obtain after integrating (4) and (5) over the space are obvious and we don't quote them. In the following they are denoted $I_{4\varphi}$ and $I_{5\varphi}$, respectively.

3. Shape of rotating nuclei

As has been shown in refs.[22,23] the shape of rotating nuclei may be determined using the stationary solution of equations $I_{4\varphi}$ and $I_{5\varphi}$ which describes the secular equilibrium of rotating nucleus. In present paper we consider the simplest case of secular equilibrium when $\vec{u}_\varphi = 0$. We also suppose that at $\Omega = 0$ the nucleus is spherical and due to rotation it becomes an oblate spheroid. With these assumptions eqs. $I_{4\varphi}$ read

$$\frac{1}{m}\int x_j n_\varphi\,\partial U_\varphi/\partial x_i\,d\vec{r} + \int x_j P_\varphi\,\partial C_\varphi/\partial x_i\,d\vec{r} -$$

$$-\frac{\hbar^2}{2m}\int x_j\left(\vec{\nabla}n_\varphi\cdot\vec{\nabla}\frac{\partial C_\varphi}{\partial x_i} + \frac{1}{2}\frac{\partial C_\varphi}{\partial x_i}\vec{\nabla}^2 n_\varphi\right)d\vec{r} - \tag{6}$$

$$-\Omega^2(1-\delta_{i3})\int x_i x_j n_\varphi\,d\vec{r} - \int\left(\frac{1}{m} + 2C_\varphi\right)P_{ij\varphi}\,d\vec{r} = 0.$$

It is easily seen that in the case of three-planar symmetry of the integration volume the first four integrals are nonzero only if $i=j$. Hence, the last integral does not vanish only if $i=j$, i.e. one can put $P_{ij\varphi} = \delta_{ij}P_{ii\varphi}$. In such a case equations $I_{5\varphi}$ yield

$$\Omega\varepsilon_{i3j}\Big\{2m\int\frac{1}{m^*}(P_{ii\varphi} - P_{jj\varphi})\,d\vec{r} + \frac{\hbar^2}{2}\int\Big(n_\varphi\frac{\partial^2 C_\varphi}{\partial x_j^2} - n_\varphi\frac{\partial^2 C_\varphi}{\partial x_i^2} +$$

$$+\frac{\partial C_\varphi}{\partial x_j}\frac{\partial n_\varphi}{\partial x_j} - \frac{\partial C_\varphi}{\partial x_i}\frac{\partial n_\varphi}{\partial x_i}\Big)d\vec{r}\Big\} = 0,$$

the second integral being zero due to three-planar symmetry. Apparently, $i\neq j\neq 3$ and it is possible to assume $P_{11\varphi} = P_{22\varphi} = P_{33\varphi} = P_{0\varphi}$. In the following $P_{0\varphi}$ is approximated by the well-known Thomas-Fermi expression $P_{0\varphi} = \gamma_\varphi n_\varphi^{5/3}$ with $\gamma_\varphi = \frac{\hbar^2}{5m}\left(\frac{3\pi^2 A}{2Z_\varphi}\right)^{2/3} = \gamma\left(\frac{A}{Z_\varphi}\right)^{2/3}$. Assuming the same equilibrium shapes for proton and neutron distributions, we put $n_q = n\,Z_q/A$ with $A = Z_n + Z_p$ ($Z_n (Z_p)$ being the number of neutrons (protons)) and sum up eq.(6) for protons with that for neutrons. Subtracting the sum for $i=j=1$ from the analogous sum with $i=j=3$ one obtains

$$\Omega^2 \mathcal{J}_{11} = \frac{1}{16}\Big[2(t_+ + 2t_-) - \frac{Z_p^2+Z_n^2}{A^2}(2t_+ + t_-)\Big]\cdot\int n\Big(x_3\frac{\partial}{\partial x_3} - x_1\frac{\partial}{\partial x_1}\Big)\vec{\nabla}^2 n\, d\vec{\tau}\ -$$

$$-\frac{1}{8}\Big(t_+ - \frac{Z_p^2+Z_n^2}{2A^2}t_-\Big)\cdot\int\Big[\Big(\frac{\partial n}{\partial x_3}\Big)^2 - \Big(\frac{\partial n}{\partial x_1}\Big)^2\Big]d\vec{\tau}\ - \tag{7}$$

$$-\frac{Z_p}{A}\int n\Big(x_3\frac{\partial}{\partial x_3} - x_1\frac{\partial}{\partial x_1}\Big)V_c\, d\vec{\tau}\ ,$$

where $\mathcal{J}_{ij} = \mathcal{J}_{ijn} + \mathcal{J}_{ijp}$.

In the analogous formula of refs./22-23/ the effect of the nuclear forces has been approximated by the surface tension. Our calculations with more realistic forces (SKM*) show that this approximation works well.

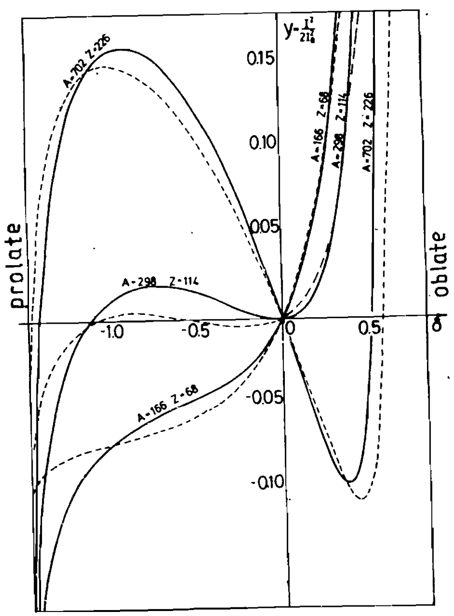

Fig. 1.

Rotational parameter $y=\frac{1}{2}I^2/I_0^2$ as a function of the deformation parameter δ for different Z_p and A ($I_0^2 = 2\beta m A R^4/5\tau_0^2$, β =17 MeV, τ_0 =1.2 fm). The full curve shows the results of our calculations with SKM*, the dashed curve those obtained in ref. /22/ with surface tension. I is angular momentum.

This is not suprising since the Skyrme parameters are chosen to reproduce the Weizsäker mass formula and particularly the surface term $\beta A^{2/3}$. The dependence of Ω^2 on the deformation of the nucleus calculated in both cases with SKM* force and with surface tension, is shown in fig. 1. The details of the calculations and the precise expression for Ω^2 can be found in Appendix A.

4. Small deviations from the equilibrium

Analyzing the reaction of the system to an infinitesimal perturbation one can determine its eigenfrequencies. A small deviation from the state of equilibrium of the system is described by taking the variations of equations I_{4q} , I_{5q} .

Before going on with variations, let us stress out some new points arising here.

Firstly, in refs./22-23/ the volume of integration was finite and one dealt with the Lagrange variations Δn and ΔP_{ij} . In this work we formally integrate over an infinite volume, hence only the Euler variations δn and δP_{ij} survive (see,e.g.,the rules of variation in ref. /28/).

Secondly, the nuclear matter in refs./22-23/ was understood to be incompressible and the whole effect of the nuclear forces was reduced to the surface tension. In this paper we use particular realistic nucleon-nucleon interaction and we are able, moreover, we are obliged to describe also compression modes of excitation (monopole resonances).

The expression for the Euler variations of neutron and proton densities may be easily derived from the continuity equations (3). Multiplying eqs. (3)by dt and defining the infinitesimal displacements of the elements of the neutron and proton liquids

$$\vec{\xi}_q(\vec{\tau},t) = \vec{u}_q(\vec{\tau},t)\cdot dt \tag{8}$$

one obtains

$$\delta n_q = -div(n_q\vec{\xi}_q) - \mathfrak{z}\, div(n_q n_{q'}(\vec{\xi}_q - \vec{\xi}_{q'}))\ . \tag{9}$$

The presence of a term proportional to $\mathfrak{z}$ which is responsible for the exchange effects /26/ makes the last expression different from the classical/28/ one. Obviously the Euler variation of the total density δn does not contain any exchange effects.

Lagrange variation of the mean velocities/28/ can be obtained by calculating the differentials of equations (8):

$$\Delta \vec{u}_{q}(\vec{r},t) = \frac{d \vec{\xi}_{q}(\vec{r},t)}{dt}. \qquad (10)$$

The expressions for their Euler variations $\delta \vec{u}_{q}(\vec{r},t)$ are derived by using the relation /28/: $\Delta = \delta + \sum_{i=1}^{3} \xi_i \frac{\partial}{\partial x_i}$.

Now we are ready to take variations of eqs. I_{4q} and I_{5q} with respect to the equilibrium state with given Ω. Keeping the terms linear in variations, one gets

$$\begin{aligned}
\ddot{V}_{i,jq} &= b_1 V_{ijq} - b_2 V_{ijq'} + b_3 \pi_{ijq} + \\
&+ \delta_{ij}\left(b_4 \tilde{V}_q - b_5 \tilde{V}_{q'} + b_6 \tilde{\tilde{V}}_q - b_7 \tilde{\tilde{V}}_{q'} + b_8 \pi_q + b_9 \pi_{q'}\right) + \\
&+ 2\Omega \sum_{k=1}^{3} \varepsilon_{ik3}\left(b_{10} \dot{V}_{k,jq} - b_{11} \dot{V}_{k,jq'}\right) - \\
&- \delta_{q,p}\, \delta_{ij} \sum_{k=1}^{3} A_{ki}\left(b_{12} V_{kkq} - b_{13} V_{kkq'}\right)
\end{aligned} \qquad (11)$$

$$\begin{aligned}
\dot{\pi}_{ijq} &= - d_1\left(\dot{V}_{i,jq}/\bar{a}_j^2 + \dot{V}_{j,iq}/\bar{a}_i^2\right) + \\
&+ d_2\left(\dot{V}_{i,jq'}/\bar{a}_j^2 + \dot{V}_{j,iq'}/\bar{a}_i^2\right) - \\
&- 2\Omega d_3 \sum_{k=1}^{3}\left(\varepsilon_{j3k} \pi_{ikq} + \varepsilon_{i3k} \pi_{jkq}\right).
\end{aligned}$$

Here and in the following we use the notations:

$$V_{i,jq} = m \int n_q \xi_{iq} x_j \, d\vec{r} \quad , \quad V_{ijq} = V_{i,jq} + V_{j,iq} \, ,$$

$$\tilde{V}_q = \sum_{k=1}^{3} \frac{V_{k,kq}}{\bar{a}_k^2} \quad , \quad \tilde{\tilde{V}}_q = \sum_{k=1}^{3} \frac{V_{k,kq}}{\bar{a}_k^4} \quad , \quad \pi_{ijq} = \delta \Pi_{ijq} \, ,$$

A_{ki} are the so-called index symbols, defined in refs. /22,28/; $\bar{a}_i = a_i/R$ with a_i (i=1,2,3) being the semi-axes of the nuclear spheroid. The coefficients b_n and d_n depend on the SKM* parameters and on the indices i,j,q and can be found in Appendix B.

Deriving equations (11) some natural assumptions have been made. (i) The integrals containing Coulomb interaction have been calculated in the approximation of a sharp suface of the nucleus in which case one can use some formulae from ref. /28/. (ii) Following the prescriptions of ref. /24/ we try to find solutions for displacements and variations of the pressure tensors in the form:

$$\begin{aligned}
\xi_{iq}(\vec{r},t) &= L_{iq}(t) + \sum_{k=1}^{3} L_{i,kq}(t) x_k + \sum_{k,\ell=1}^{3} L_{i,k\ell q} x_k x_\ell \, , \\
\delta P_{ijq}(\vec{r},t) &= n_q(\vec{r})\left[D_{ijq}(t) + \sum_{k=1}^{3} D_{ij,kq}(t) x_k\right].
\end{aligned} \qquad (12)$$

A natural question arises here: are there solutions more complicated than (12)? What will happen if, in particular, one adds in (12) terms with higher powers of coordinates? This problem has been discussed in ref. /24/ and it turns out that such solutions may be constructed; but to do this, the system of equations I_{4q}, I_{5q} must be supplemented with equations for higher-rank tensors $J_{ij\ldots kq}$ and $P_{ij\ldots kq}$, a strict correspondence existing between the number of the terms in equations (12) and the number of the equations for the moments.

Inserting expressions (12) into the definitions of the $V_{i,jq}$ and π_{ijq} we find relation between $V_{i,jq}$ and $L_{i,jq}$:

$$L_{i,jq} = \frac{3A}{4\pi m Z_q \alpha_1^4 \bar{a}_j^2} V_{i,jq} \quad , \quad \alpha_\mu^\nu = \int_0^\infty \rho^\nu n(\rho)\, d\rho$$

and between π_{ijq} and D_{ijq}:

$$\pi_{ijq} = Z_q D_{ijq} \, .$$

Due to three-planar symmetry all the integrals containing L_{iq}, $L_{i,k\ell q}$ or $D_{ij,kq}$ vanish.

We may rearrange equations (11) to obtain equations for the irreducible tensors of ranks λ=2,1,0. These tensors occur to be just the variations of the quadrupole moment $Q_{2\mu q} = \int n_q r^2 Y_{2\mu} d\vec{r}$, angular momentum $I_{iq} = \int n_q \{[\vec{r}\times\vec{u}_q]_i + [\vec{r}\times[\vec{\Omega}\times\vec{r}]]_i\} d\vec{r}$, trace of the inertia tensor $Q_{00q} = \int n_q r^2 Y_{00} d\vec{r}$ (i.e. the mean square radius) and similar combinations of the components of the π_{ijq} tensors. In the present work the relation between the variations $\delta Q_{2\mu q}$, δI_{iq} and the tensors $V_{i,jq}$ is quite more complicated than in ref. /23/ (due to exchange effects).

Inserting the expressions (9) into the formula $\delta Q_{2\mu q}=\int r^2 Y_{2\mu}\,\delta n_q\,d\vec{r}$, one obtains

$$\delta Q_{2,\pm 2q}=\frac{1}{4m}\sqrt{\frac{15}{2\pi}}\left\{\left(1+\bar{\zeta}\frac{Z_{q'}}{A}\right)\left(V_{11q}-V_{22q}\pm 2iV_{12q}\right)-\bar{\zeta}\frac{Z_q}{A}\left(V_{11q'}-V_{22q'}\pm 2iV_{12q'}\right)\right\},$$

$$\delta Q_{2,\pm 1q}=\mp\frac{1}{2m}\sqrt{\frac{15}{2\pi}}\left\{\left(1+\bar{\zeta}\frac{Z_{q'}}{A}\right)\left(V_{13q}\pm iV_{23q}\right)-\bar{\zeta}\frac{Z_q}{A}\left(V_{13q'}\pm iV_{23q'}\right)\right\},$$

$$\delta Q_{2,0q}=-\frac{1}{4m}\sqrt{\frac{5}{\pi}}\left\{\left(1+\bar{\zeta}\frac{Z_{q'}}{A}\right)\left(V_{11q}+V_{22q}-2V_{33q}\right)-\bar{\zeta}\frac{Z_q}{A}\left(V_{11q'}+V_{22q'}-2V_{33q'}\right)\right\}$$

and by analogy for the angular momentum variation:

$$\delta I_{iq}=\int\left\{n_q[\vec{r}\times\delta\vec{u}_q]_i+\delta n_q[\vec{r}\times[\vec{\Omega}\times\vec{r}]]_i\right\}d\vec{r},$$

$$\delta I_{1q}=\dot{V}_{3,2q}-\dot{V}_{2,3q}-\Omega\left\{\left(1+\bar{\zeta}\frac{Z_{q'}}{A}\right)V_{13q}-\bar{\zeta}\frac{Z_q}{A}V_{13q'}\right\},$$

$$\delta I_{2q}=\dot{V}_{1,3q}-\dot{V}_{3,1q}-\Omega\left\{\left(1+\bar{\zeta}\frac{Z_{q'}}{A}\right)V_{23q}-\bar{\zeta}\frac{Z_q}{A}V_{23q'}\right\},$$

$$\delta I_{3q}=\dot{V}_{2,1q}-\dot{V}_{1,2q}+\Omega\left\{\left(1+\bar{\zeta}\frac{Z_{q'}}{A}\right)\left(V_{11q}+V_{22q}\right)-\bar{\zeta}\frac{Z_q}{A}\left(V_{11q'}+V_{22q'}\right)\right\}.$$

For the variation δQ_{00q} one has:

$$\delta Q_{00q}=\frac{1}{\sqrt{4\pi}}\int r^2\delta n_q\,d\vec{r}=\frac{1}{2m\sqrt{\pi}}\left\{\left(1+\bar{\zeta}\frac{Z_{q'}}{A}\right)\sum_{i=1}^{3}V_{iiq}-\bar{\zeta}\frac{Z_q}{A}\sum_{i=1}^{3}V_{iiq'}\right\}$$

The expressions for $\delta\Pi_{2\mu q}$ can be found from those for $\delta Q_{2\mu q}$ replacing V_{ijq} by Π_{ijq} and in a similar way $\delta\Pi_{00q}$ is found from δQ_{00q} . $\bar{\zeta}=\zeta\alpha_2^4/\alpha_1^4$.

5. Eigenfrequencies and transition probabilities

Because of the axial symmetry of the nucleus, the multipole moment projection on the axis of rotation is a good quantum number and the set of equations (11) split into five groups of equations corresponding to $\mu=\pm 2,\pm 1$ and 0.

The $\mu=2$ group of equations reads:

$$\delta\ddot{Q}_{2,+2q}=r_1\delta\dot{Q}_{2,+2q}+r_2\delta\dot{Q}_{2,+2q'}+r_3\delta Q_{2,+2q}+r_4\delta Q_{2,+2q'}+r_5\delta\Pi_{2,+2q}+r_6\delta\Pi_{2,+2q}$$

$$\delta\dot{\Pi}_{2,+2q}=r_7\delta\dot{Q}_{2,+2q}+r_8\delta\dot{Q}_{2,+2q'}+r_9\delta\Pi_{2,+2q}. \qquad (13)$$

The $\mu=1$ group is

$$\delta\ddot{Q}_{2,+1q}=s_1\delta\dot{Q}_{2,+1q}+s_2\delta\dot{Q}_{2,+1q'}+s_3\delta Q_{2,+1q}+s_4\delta Q_{2,+1q'}+s_5\delta I_{+q}+s_6\delta I_{+q'}+s_7\delta\Pi_{2,+1q}$$

$$\delta\dot{I}_{+q}=s_8\delta\dot{Q}_{2,+1q}+s_9\delta\dot{Q}_{2,+1q'}+s_{10}\delta Q_{2,+1q}+s_{11}\delta Q_{2,+1q'}+s_{12}\delta I_{+q}+s_{13}\delta I_{+q'} \qquad (14)$$

$$\delta\dot{\Pi}_{2,+1q}=s_{14}\delta\dot{Q}_{2,+1q}+s_{15}\delta\dot{Q}_{2,+1q'}+s_{16}\delta Q_{2,+1q}+s_{17}\delta Q_{2,+1q'}+s_{18}\delta I_{+q}+s_{19}\delta I_{+q'}+s_{20}\delta\Pi_{2,+1q}.$$

Here $\delta I_{+q}=\frac{1}{\sqrt{2}}(\delta I_{1q}+i\,\delta I_{2q})$.

Similar equations with $-\Omega$ in place of Ω in the coefficients r_n and s_n correspond to $\mu=-2$ and $\mu=-1$. The coefficients r_n and s_n are obvious linear combinations of the coefficients b_n and d_n listed in Appendix B.

The most cumbersome group of equations is that for $\mu=0$:

$$\delta\ddot{Q}_{2,0q}=t_1\delta Q_{2,0q}+t_2\delta Q_{2,0q'}+t_3\delta Q_{0,0q}+t_4\delta Q_{0,0q'}+t_5\delta I_{3q}+t_6\delta I_{3q'}+t_7\delta\Pi_{2,0q}$$

$$\delta\ddot{Q}_{0,0q}=t_8\delta Q_{0,0q}+t_9\delta Q_{0,0q'}+t_{10}\delta Q_{2,0q}+t_{11}\delta Q_{2,0q'}+t_{12}\delta I_{3q}+t_{13}\delta I_{3q'}+t_{14}\delta\Pi_{0,0q}+t_{15}\delta\Pi_{0,0q'}$$

$$\delta\dot{I}_{3q}=t_{16}\delta\dot{Q}_{2,0q}+t_{17}\delta\dot{Q}_{2,0q'}+t_{18}\delta\dot{Q}_{0,0q}+t_{19}\delta\dot{Q}_{0,0q'} \qquad (15)$$

$$\delta\dot{\Pi}_{2,0q}=t_{20}\delta\dot{Q}_{2,0q}+t_{21}\delta\dot{Q}_{2,0q'}+t_{22}\delta\dot{Q}_{0,0q}+t_{23}\delta\dot{Q}_{0,0q'}$$

$$\delta\dot{\Pi}_{0,0q}=t_{24}\delta\dot{Q}_{2,0q}+t_{25}\delta\dot{Q}_{2,0q'}+t_{26}\delta\dot{Q}_{0,0q}+t_{27}\delta\dot{Q}_{0,0q'}$$

with t_n being evident combinations of the coefficients b_n and d_n .

Obviously, system (15) contains three integrals of motion, hence it can be simplified by rewriting δI_3, $\delta \Pi_{2,0}$ and $\delta \Pi_{0,0}$ in terms of $\delta Q_{2,0}$ and $\delta Q_{0,0}$. One of these integrals of motion allows a simple physical interpretation, namely $\delta I_{3p} + \delta I_{3n} = const$ means that the projection of the variation of the total angular momentum on the axis of rotation is conserved. Actually, system (14) also contains two integrals of motion δI_1 and δI_2, but these projections are constants only in a laboratory frame of reference. In a rotating frame they rotate with angular velocity Ω. It is easy to obtain from equations (14)

$$\delta \dot{I}_+ = -i\Omega\, \delta I_+$$

and consequently $\delta I_+ = const\; e^{-i\Omega t}$.

The reduced probabilities for electromagnetic transitions may be calculated applying the theory of linear response of a system to an external field perturbation.

$$O(t) = O e^{-i\omega t} + O^{+} e^{i\omega t}. \tag{16}$$

In the case of quadrupole and monopole electric transitions we have $O = e_p \delta_{\varsigma p}\, r^2 Y_{2\mu}$ or $O = e_p \delta_{\varsigma p}\, r^2 Y_{00}$ respectively, and for magnetic dipole transitions

$$O = -i\frac{e\hbar}{2mc}\vec{\nabla}(r Y_{1\mu})\cdot\left[\vec{r}\times(\vec{\nabla}_r + m\Omega\vec{\nabla}_p)\right]\delta_{\varsigma p} \equiv O_1 .$$

A convenient form of the linear response theory is given by Lane /29/. The matrix elements of an operator O obey the relationship

$$|\langle\Psi_a|O|\Psi_0\rangle|^2 = \lim_{\omega\to\omega_a}\hbar(\omega-\omega_a)\overline{\langle\Psi_0'|O e^{-i\omega t}|\Psi_0'\rangle}, \tag{17}$$

where Ψ_0 and Ψ_a are the unperturbed wavefunctions of the stationary states; Ψ_0' is the perturbed ground state wavefunction, $\omega_a = (E_a - E_0)/\hbar$ are the normal frequencies of the system. The bar means averaging over a time interval much greater than $1/\omega$, ω being the frequency of the external field $O(t)$.

Let us consider in more detail the case of quadrupole operator. Its matrix element may be expressed in terms of variables $V_{i,j\varsigma}$:

$$\overline{\langle\Psi_0'|e_p\delta_{\varsigma,p}\, r^2 Y_{2,\mu}|\Psi_0'\rangle e^{-i\omega t}} = e_p\overline{\int n'(\vec{r},t)\delta_{\varsigma,p}\, r^2 Y_{2,\mu}\, d\vec{r}\, e^{-i\omega t}} = \tag{18}$$

$$= e_p\overline{\int n_p(\vec{r})\, r^2 Y_{2,\mu}\, d\vec{r}\, e^{-i\omega t}} + e_p\overline{\int \delta n_p(\vec{r},t)\, r^2 Y_{2,\mu}\, d\vec{r}\, e^{-i\omega t}} =$$

$$= e_p\,\overline{\delta Q_{2,\mu p}(t)\, e^{-i\omega t}}.$$

To find the absolute value of $\delta Q_{2,\mu\varsigma}(t)$ we need to add potential (16) to the Hamiltonian in equations (1). The right-hand side (r.h.s) of equations (2) are then modified by the terms

$$\frac{2}{\hbar}\sin\left\{\frac{\hbar}{2}\left(\vec{\nabla}_r^{\,o}\cdot\vec{\nabla}_p^{\,f} - \vec{\nabla}_p^{\,o}\vec{\nabla}_r^{\,f}\right)\right\}\left\{O_W(\vec{r},\vec{p})e^{-i\omega t} + O_W^{*}(\vec{r},\vec{p})e^{i\omega t}\right\}f_\varsigma(\vec{r},\vec{p},t). \tag{19}$$

Proceeding in the same way as before, one obtains equations for all the moments of the Wigner function needed to calculate $\delta Q_{2,\mu\varsigma}(t)$. The only new element now is the presence of the terms (19) that make the equations for the moments inhomogeneous. Thus, the perturbation by a field proportional to $Y_{2,2}$ does not affect equations (14) and (15), but makes the first equation of the system (13) (for protons only) inhomogeneous:

$$\delta\ddot{Q}_{2,2p} = \cdots - 10\,\frac{e_p Z_p}{mA}\alpha_1^4\,\bar{a}_1^2\, e^{i\omega t}\cdot\left(1+\bar{\zeta}\,\frac{Z_n}{A}\right).$$

The perturbation with a field proportional to $Y_{2,1}$ makes inhomogeneous the first proton equation of system (14):

$$\delta\ddot{Q}_{2,1p} = \cdots - 10\,\frac{e_p Z_p}{mA}\alpha_1^4\,\frac{1}{2}(\bar{a}_1^2 + \bar{a}_3^2)\, e^{i\omega t}\cdot\left(1+\bar{\zeta}\,\frac{Z_n}{A}\right).$$

Finally, the perturbation with a field proportional to $Y_{2,0}$ makes inhomogeneous the first equation (for protons) of system (15):

$$\delta\ddot{Q}_{2,0p} = \cdots - 10\,\frac{e_p Z_p}{mA}\alpha_1^4\,\frac{1}{3}(\bar{a}_1^2 + 2\bar{a}_3^2)\cos\omega t\cdot\left(1+\bar{\zeta}\,\frac{Z_n}{A}\right).$$

According to the time dependence of the inhomogeneity we suppose that the variations $\delta Q_{2,\mu\varsigma}$, $\delta\Pi_{2,\mu\varsigma}$, etc. must depend on time like $e^{i\omega t}$, $\cos\omega t$ or $\sin\omega t$. For such type of t-dependence equations (13)-(15) become a system of linear algebraic equations and the problem of calculating $\delta Q_{2,\mu\varsigma}$ reduces to the calculation of determinants.

The characteristic equation of the homogeneous set of equations yields the eigenfrequencies ω_a.

A formula analogous to (18) is derived for the matrix element of the operator responsible for monopole excitations

$$\overline{\langle\Psi_0'|e_p\delta_{\varsigma,p}\, r^2 Y_{00}|\Psi_0'\rangle e^{-i\omega t}} = e_p\,\overline{\delta Q_{0,0p}(t)e^{-i\omega t}}.$$

The perturbation with a field $O = e_p \delta_{q,p} r^2 Y_{0,0}$ affects the second equation of system (15) (for $q = p$):

$$\delta \ddot{Q}_{0,0p} = \dots + 8\,\frac{e_p Z_p}{mA}\,\mathcal{L}_1^4\,\frac{1}{3}(\bar{a}_3^2 + 2\bar{a}_1^2)\left(1+\bar{\xi}\,\frac{Z_n}{A}\right)\cos\omega t.$$

For the matrix element of the magnetic dipole operator one gets

$$\overline{\langle \Psi_0' | O_1 | \Psi_0' \rangle e^{-i\omega t}} = \overline{\delta m_{1,\mu p}\, e^{-i\omega t}}$$

with $\delta m_{1,\mu p}$ being the variation of the proton magnetic dipole moment

$$m_{1,\mu p} = \frac{e_p}{2c}\int n_p \vec{\nabla}(r Y_{1,\mu})\cdot\left[\vec{r}\times(\vec{u}_p + [\vec{\Omega}\times\vec{r}])\right] d\vec{r}.$$

Obviously, the components of $m_{1,\mu}$ are proportional to the angular momenta components, i.e. $m_{1,0} = \frac{e_p}{2c}\sqrt{\frac{3}{4\pi}}\, I_3$, $m_{1,1} = -\frac{e_p}{2c}\sqrt{\frac{3}{4\pi}}\, I_+$.

The perturbation with a field O_1 makes inhomogeneous the second equation of system (14) with $q = p$:

$$\delta \dot{I}_{+p} = \dots - i\Omega\sqrt{\frac{\pi}{3}}\,\frac{e_p Z_p}{cA}\,\mathcal{L}_1^4(\bar{a}_1^2 + \bar{a}_3^2)\, e^{i\omega t}.$$

Thus, we have constructed everything we need to calculate the energies and transition probabilities of 2^+, 1^+ and 0^+ states.

6. Numerical results

6.1. Ω =0 case

In figs. 2-5, the calculated eigenfrequencies of β -stable non-rotating nuclei are compared with the empirical data.

For Ω =0 equations (13-15) yield two five-fold degenerated 2^+ states (isoscalar (IS) and isovector (IV) ones) and two 0^+ states (IS and IV respectively). Having in mind the physical meaning of the dynamical variables (in these equations), it is natural to identify the above states with the giant quadrupole and monopole resonances of both types.

The calculations have been performed with two parametrizations (see Appendix A) of the equilibrium density n : of Bernstein[30] and of Bohr and Mottelson[31] (BM). Besides we check the accuracy of the widely used approximation in which protons and neutrons vibrate in or out of phase with $V_{ijn}/Z_n = \pm V_{ijp}/Z_p$.

The results presented in **figs.2,3** demonstrate that the energies of the isoscalar quadrupole resonance (ISQR) in medium-mass and heavy nuclei are closer to the data when calculated with parametrization of Bernstein but in light nuclei the parameters of BM are to be pre-

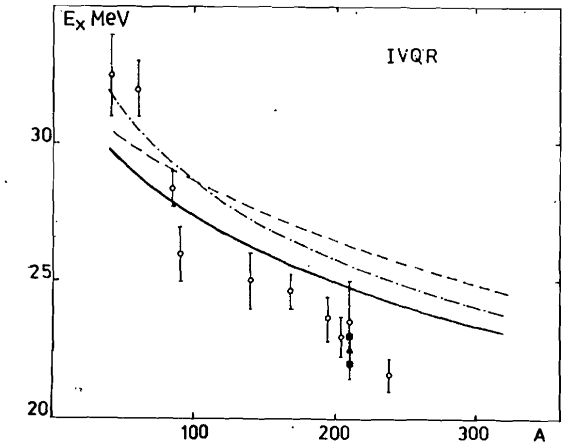

Fig. 2. The energies of the **isovector** quadrupole resonance calculated with the equilibrium density parameters by Bernstein /30/ (full curve), by BM /31/ (dashed-dotted curve) and in the approximation (see text) of equal neutron and proton amplitudes (dashed line). The experimental data are from ref. /4/ .

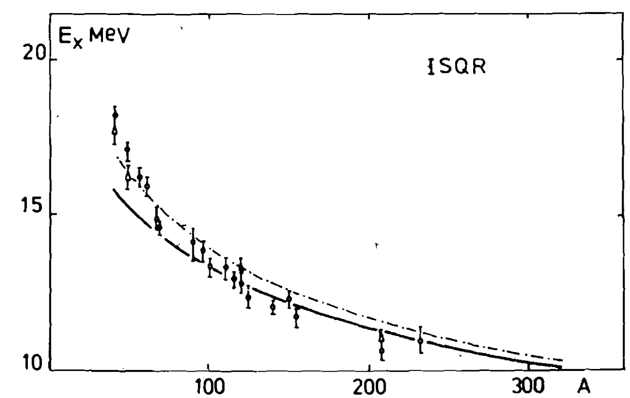

Fig. 3. The same as in fig. 2 for the **isoscalar** quadrupole resonance. The experimental data are from ref. /1/ .

ferred. The solution with $V_{ijn}/Z_n = V_{ijp}/Z_p$ for the isoscalar quadrupole resonance (ISQR) practically coincides with the exact one. The approximate solution for the IVQR ($V_{ijn}/Z_n = -V_{ijp}/Z_p$) deviates significantly both from the data and the exact result.

Comparing the energies of the ISQR plotted in fig. 3 with the analogous results of refs. /15,22/ one sees that in the region of light nuclei our more realistic calculations describe the experimental data worse than the simplified calculations with a sharp shape of the nuclear density and Fermi-step approximation for the momentum distribution. Obviously, the Thomas-Fermi approximation underestimates the mean values $\langle p^2\rangle \sim P_0$, which occur to be very important in calculating the giant resonance energies. In heavy nuclei the smoothness of the shape is smaller and the above effect is not so prominent.

In calculating the energies of the monopole resonances presented in figs. 4,5 the parametrization of Bernstein seems to be preferable. The approximate solution for the ISMR practically coincides with the exact one.

The empirical information on IVMR is very scarce. Strictly speaking, there is only one experiment /7,8/ (fig. 5) to compare with.

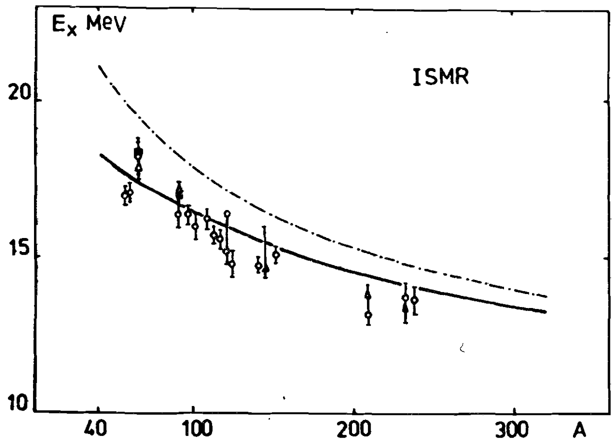

Fig. 4. The same as in fig. 2 for the isoscalar monopole resonance. The experimental data are from ref. /3/.

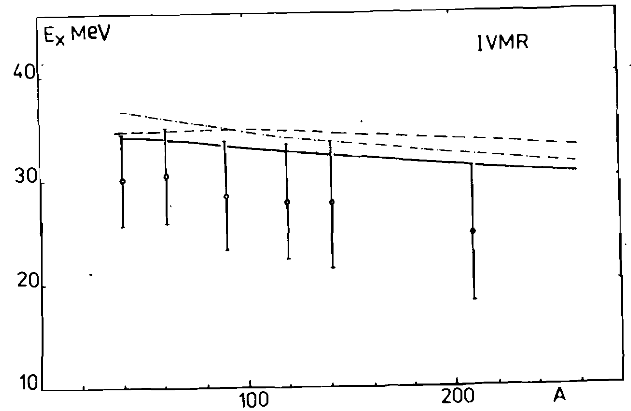

Fig. 5. The same as in fig. 2 for the isovector monopole resonance. The experimental data are from ref. /7/.

The approximate solution like in the case of IVQR deviates from the exact one, the discrepancy being of about 10% in heavy nuclei.

The E2- and E0- transition probabilities have been calculated as well as their contributions to the electro-magnetic energy-weighted sum rule /32/:

$$S(E2) = \frac{25}{4\pi} Z_p e_p^2 \frac{\hbar^2}{m}\left[\langle r^2\rangle + \langle n r^2\rangle \frac{m}{2\hbar^2}\left(t_+ - \frac{t_-}{2}\frac{Z_p}{A}\right)\right] = \tag{20}$$

$$= \frac{25}{2} S(E0) .$$

Here $A\langle x\rangle = \int n x \, d\vec{r}$. The correction to the classical sum rule (the term with t_+ , t_-) results from the nonlocal part of Skyrme interaction. Our calculations demonstrate that both 2^+ excitations give nearly the same contribution to the sum rule (20) and completely exhaust it. The same is true concerning the two 0^+ excitations (see the Table).It is interesting to note that the contribution from the isoscalar 2^+ excitation goes down when the atomic mass increases and the contribution from the isovector 2^+ excitation decreases.

Table. Percentage contribution of the different resonances to the energy-weighted sum rule. IS-isoscalar, IV-isovector, QR and MR-quadrupole and monopole resonances

	ISQR	IVQR	ISMR	IVMR
^{40}Ca	46.9	53	44	56
^{144}Sm	40.7	59.2	48.3	51.6
^{208}Pb	37.8	62.1	50	49.9

As for the 0^+ excitations the tendency is just the opposite. The experimental data are by tradition normalized to the isoscalar or isovector sum rule. The relation between these two sum rules is given by the formula /32/:

$$S(\tau=0,\lambda) = S(\tau=1,\mu_\tau=0,\lambda) \approx \frac{A}{Z_p e_p^2} S(E\lambda) .$$

Our calculations show that one of the two 2^+ excitations approximately exhausts the isoscalar EWSR and the other the isovector EWSR. The same is true for the corresponding 0^+ excitations. The results of our calculations along the data are shown in fig. 6.

The overexhausting of the isovector EWSR for IVQR results just from the impossibility to decouple exactly the isoscalar and isovector modes.

The large discrepancy between the theoretical and experimental results for ISMR in light nuclei is probably connected with the incorrect treating of the empirical data.

In calculating the sum rule we can answer the question about the significance of the integrals including the third-rank tensors $P_{ijk\gamma}$ we have neglected in our equations. Indeed, the equations of motion are written directly in terms of multipole moments and therefore the described excitations must exhaust the corresponding multipole sum rule. All the energies and probabilities are calculated neglecting $P_{ijk\gamma}$ terms and hence the l.h.s. of the sum rule

$$\sum_i (E_i-E_0)|\langle i|F|0\rangle|^2 = \frac{1}{2}\langle 0|[F,[H,F]]|0\rangle \qquad (21)$$

contains some uncertainty. As to the commutators in the r.h.s. of (21), they are calculated with exact Hamiltonian without any approximation. That's why the difference between the r.h.s. and the l.h.s. of eq.(21) serves as a criteria for the importance of the neglected terms.

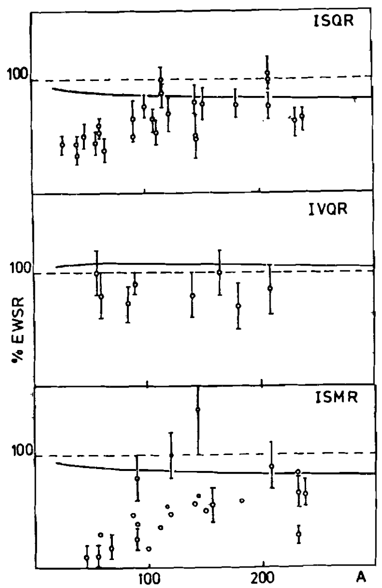

Fig. 6.

Percentage exhaustion of the isoscalar energy weighted sum rule (EWSR) by the IS-quadrupole resonance (upper part) and by the IS-monopole resonance (lower part). In the middle part - percentage exhaustion of the isovector EWSR by the isovector quadrupole resonance. The experimental data are from ref. /1-6/.

Our calculations show that for all nuclei the proportion l.h.s./ /r.h.s. is more than 99,9%, i.e. the role of $P_{ijk\gamma}$ terms is insignificant when describing excitations up to quadrupole.

6.2. $\Omega \neq 0$ case

In fig. 7 we show calculated energies of 2^+ and 0^+ excitations in ^{154}Er as a function of the velocity of rotation (or more precisely of the eccentricity $e^2=1-a_3^2/a_1^2$ and the angular momentum I related one-to-one with Ω). Due to deformation of the nucleus and the Coriolis forces each of the two 2^+ states (ISQR and IVQR) split into five branches corresponding to $\mu=\pm 2$ (the so-called γ-mode), $\mu=\pm 1$ (denoted as α-mode) and $\mu=0$ (β-mode). The two β-modes are always coupled with the volume (monopole) excitations. The picture of the ISQR splitting is very similar to that obtained in ref. /22/ with an average field on the nuclear surface approximated by a surface tension.

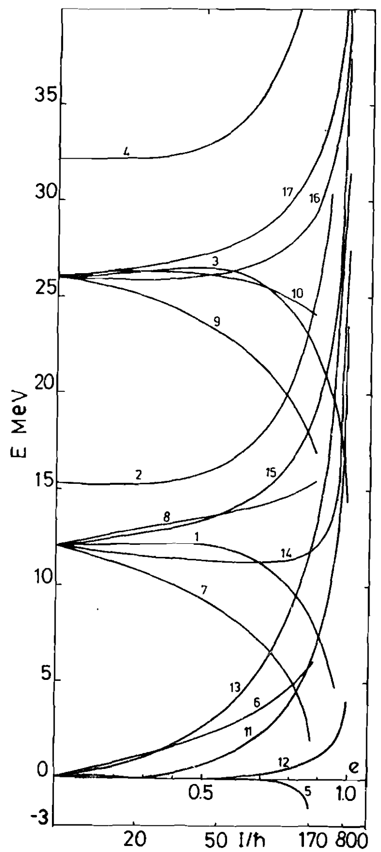

Fig. 7.

Energies of the 0^+, 1^+ and 2^+ excitations as a function of the eccentricity e and the angular momentum I. The levels are numbered as follows:
isoscalar - 1(0), 2(0), 5(-2), 7(+2), 8(-2), 13(-1), 14(+1), 15(-1);
isovector - 3(0), 4(0), 6(-2), 9(+2), 10(-2), 16(+1), 17(-1).
The corresponding multipole moment projections on the axis of rotation are indicated in the brackets. The isoscalar level № 12 and the isovector level № 11 correspond to the projection μ =-1 before the point in which their energies are zero and μ =+1 after this point.

When $\Omega \neq 0$ five low-lying modes appear (figs. 7,8). Two of them, the isoscalar γ- and α-modes (modes №5 and 12) have been described in refs./22,23/. In general, their behaviour doesn't change very much, only the critical angular momenta are slightly shifted. So, the bifurcation point goes down from $I \approx 70\hbar$ to $I \approx 52\hbar$ and the spheroidal shape stability point from $I \approx 190\hbar$ to $I \approx 170\hbar$. As for the point in which the α-mode goes to zero, it doesn't change ($I_c \approx 25\hbar$). In the present work these two isoscalar modes supply with isovector analogues-the curves №6 and 11. The isovector low-lying γ-mode has no any peculiarities, but the isovector α-mode goes to zero at $I_c \approx 19\hbar$.

The fifth low-lying mode (the curve №13) corresponds to the vibration of the proton angular momentum towards the neutron one (like scissors). This last mode has no isoscalar analogue as the total (neutron plus proton) angular momentum is conserved. Evidently, it can be classified as an isovector 1^+ excitation. In nuclei with static deformation such a mode is known as a scissors mode or angular resonance /33/. In ref./33/ another mode corresponding to the rotation of the proton angular momentum with respect to neutron one has been predicted. The mode №11 in the fig.7,8 seems to be just this mode.

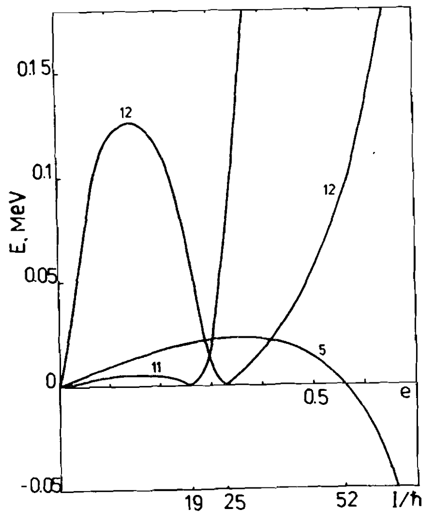

Fig. 8.

Energies of the low lying excitations as a function of the eccentricity e and the angular momentum I. The legend is the same as in fig. 7.

In fact, as has been shown in ref./22/ the isoscalar analogue of mode №11, i.e. the mode №12, is similar to precession mode. Hence, the corresponding isovector mode (№11) should describe the "precession" of the proton matter with respect to the neutron one.

If one forgets for a moment that in present work the nuclei may deforme only due to the rotation and compare the calculated 1^+ energy for the same deformation δ =0.258 (i.e. for $e \simeq 0.66$, $I \simeq 69\hbar$ in fig. 7) with the experimental value /34/ $E_{1^+}^{exp}$= 3.1. MeV in ^{156}Gd, one finds that the theoretical results E_{1^+}(№13) $\simeq$ 4.9 MeV, E_{1^+}(№11) $\simeq$ 1.9 MeV seem to be quite reasonable. As for the theoretical B(M1) factors of the above mentioned two levels they are practically the same ($\simeq 1.6\mu_N^2$) and agree nicely with the experimental B(M1)↑ $= 1.3 \pm 0.2\, \mu_N^2$.

It should be noted, that particular calculations are needed for nuclei with static deformation. The results of such investigation will be presented in a next paper.

In fig. 9 the angular momentum dependence of the calculated B(E2)-factors is shown. For moderate I the behaviour of the curves NN 1,5,7,8,12,14 and 15 corresponding to E2-transitions from the isoscalar excited states to the ground state is similar to that of the analogous curves calculated in ref./23/ with surface tension. Noticeable differences appear at $I \simeq 40\hbar$.

The following pecularity is nicely seen - the probability for the transition from isoscalar levels is greater than that from the isovector ones. Only at extremely large I the picture may be just the opposite.

As has been noted in Sect. 5, each level can be distinguished by a quantum number μ (the multipole moment projection on the axis of rotation). The levels NN12 and 11 have the following peculiarity:

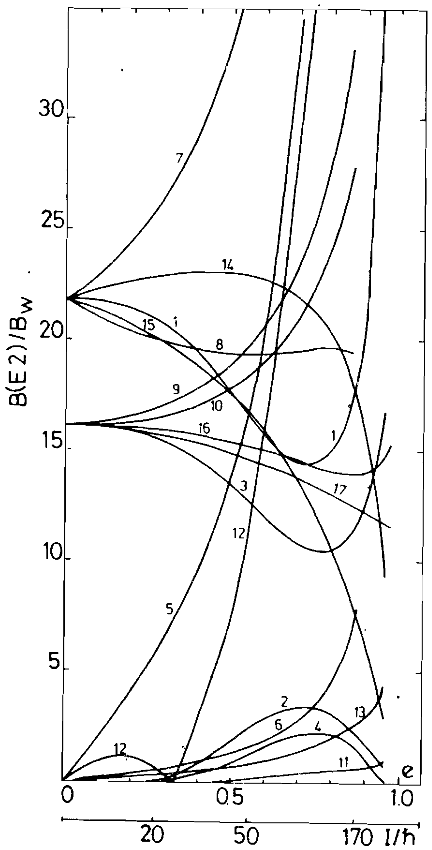

Fig. 9. B(E2)-factors (in Weisskopf units B_W) of the transitions from the excited to the ground state. The curves are numbered in correspondence with the levels (fig. 7) from which the transitions go.

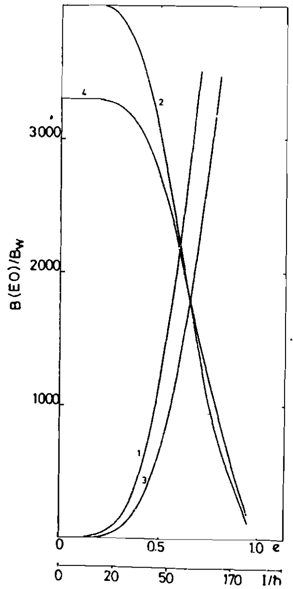

Fig. 10. B(E0)-factors for the transitions from the excited to the ground state. The curves are numbered as in fig. 9.

in the points in which they are zero μ changes from -1 to +1. This means that the corresponding modes are excited with decreasing the spin if $I<I_c$ and with increasing it if $I>I_c$. The mechanical interpretation of such a behaviour seems to be also interesting. As the mode №12 is the precession mode, one can say that in the point I_c the vector of the precession changes its direction.

In fig. 10 the calculated B(E0)-factors are shown as a function of e and I. As is seen, in the region $e\simeq 0.6$-0.7 ($\delta\simeq 0.3$) B(E0)-factors for compression mode and for β-mode have the same value. This is a demonstration of a strong coupling of the monopole and quadrupole excitations in deformed nuclei. Experimentally such coupling is manifested in the "splitting" of the giant monopole resonance in deformed nuclei /3/. It is interesting to note that the last result has been obtained in the simple schematic model /13/.

6. Conclusion

The results obtained in this work quite convincingly demonstrate the virtues of the method of moments. Indeed, isoscalar as isovector 0^+, 1^+ and 2^+ excitations in rotating nuclei are described in a united approach. Simultaneously with the giant resonances low lying modes appear. In spite of the quite complicated interaction the calculations are relatively simple: one needs to find roots of polynoms of order ≤ 8, the coefficients being simple integrals. The nonlocality of the interaction doesn't lead to any additional difficulties.

Of course, the possibilities of the method are not exhausted by present work. The area of its applications seems to be very vast, but we enumerate only some of the "next day" problems such as investigations of nuclei with static deformation, description of the excitations with $\lambda>2$, the inclusion of spin degrees of freedom, etc.

The authors would like to thank Professor I.N.Mikhailov for fruitful discussions and for his constant interest in our work.

Appendix A

Equidensity shapes are approximated by ellipsoids and hence the shape with $n(\vec{r}) = \frac{1}{2}n(0)$ obeys:

$$x_1^2/a_1^2 + x_2^2/a_2^2 + x_3^2/a_3^2 = 1 .$$

By changing the variables $\xi_i^2 = R^2 x_i^2/a_i^2$ we go to a reference system in which the foregoing shape becomes sphere with radius R, $\rho^2 \equiv \xi_1^2 + \xi_2^2 + \xi_3^2 = R^2$. In this system we assume a Fermi distribution for the density $n(\rho) = n_0[1+\exp((\rho-R)/a)]^{-1}$ with a being the diffuseness parameter.

Integrating by parts

$$\int n x_3 \frac{\partial}{\partial x_3} \vec{\nabla}^2 n \, d\vec{r} = \int \left(\frac{\partial n}{\partial x_3}\right)^2 d\vec{r} + \frac{1}{2}\int (\vec{\nabla} n)^2 d\vec{r}$$

and iserting into eq.(7) we finally obtain

$$\Omega^2 = \frac{m}{4}\left(\frac{t_0}{2}\frac{Z_p^2+Z_n^2}{A^2} - t_-\right)\frac{R^4}{a_1^2}\left(\frac{1}{a_1^2} - \frac{1}{a_3^2}\right)\frac{S_2^2}{\mathcal{L}_1^4} - 2\pi m n_0\left(\frac{e_p Z_p}{A}\right)^2\left(A_1 - \frac{a_3^2}{a_1^2}A_3\right),$$

where A_i are the index symbols of ref. /28/, e_p - the proton charge,

$$S_2^2 = \int_0^\infty \rho^2 \left(\frac{\partial n}{\partial \rho}\right)^2 d\rho , \quad \mathcal{L}_1^4 = \int_0^\infty n \rho^4 d\rho .$$

The deformation parameter δ is defined as previously in ref.[22], $a_1^2 = a_2^2 = a_0^2(1+\frac{2}{3}\delta)$, $a_3^2 = a_0^2(1-\frac{4}{3}\delta)$ and a_0 is fixed by the conservation of the nuclear volume $a_1 a_2 a_3 = R^3$. We use two sets of parameters of the Fermi distribution. The former is from the Bernstein's work [30]: $R = 1.115A^{1/3} - 0.53A^{-1/3}$ fm, $a = 0.568$ fm for all nuclei with $A \geqslant 20$. The latter is from the book of Bohr and Mottelson [32]: $R = 1.12A^{1/3} - 0.86A^{-1/3}$ fm, $a = 0.54$ fm. In both cases the third parameter n_0 is fixed by the condition $4\pi\int_0^\infty n(\rho)\rho^2 d\rho = A$.

Appendix B

$$b_1 = \Omega^2(1-\delta_{i3})(1+\bar{\zeta}z') + \frac{z'\mathcal{A}}{m\mathcal{L}_1^4\bar{a}_i^2} + \frac{\gamma t_+\mathcal{E}_{q'}}{4\hbar^2\mathcal{L}_1^4\bar{a}_i^2} + \delta_{q,p}\varphi(1+\zeta n_0 z')2B_{ij} +$$

$$+\frac{2T_{3q}}{\bar{a}_i^2\bar{a}_j^2} + z'G, \qquad b_3 = 1 + \frac{2\pi m\mathcal{L}_2^2}{\hbar^2 A}(t_+ - zt_-/2),$$

$$b_2 = \Omega^2(1-\delta_{i3})\bar{\zeta}z + \frac{z\mathcal{A}}{m\mathcal{L}_1^4\bar{a}_i^2} + \frac{\gamma t_+\mathcal{E}_q}{4\hbar^2\mathcal{L}_1^4\bar{a}_i^2} + \delta_{q,p}\varphi\zeta n_0 z 2B_{ij} + zG,$$

$$b_4 = \frac{1}{m\mathcal{L}_1^4}\left\{z'\mathcal{B}_q + \frac{3}{2}t_0\mathcal{L}_2^2\left[(x_0+\tfrac{1}{2})z - 1 - x_0/2\right] - zF\right\} + \frac{\gamma}{4\hbar^2\mathcal{L}_1^4}(z'\mathcal{C}_q + \mathcal{D}_q) +$$

$$+\frac{15\gamma z}{2\hbar^2\mathcal{L}_1^4}\left[\frac{1}{8}\mathcal{L}_{8/3}^2(t_-/2 - t_+) + \frac{t_-}{22}\zeta\mathcal{L}_{11/3}^2 z'\right] + z'K + \left(\sum_{k=1}^{3}\frac{1}{\bar{a}_k^2} + \frac{2}{\bar{a}_i^2}\right)T_{3q},$$

$$b_5 = \frac{z}{m\mathcal{L}_1^4}(\mathcal{B}_q + F) + \frac{\gamma z}{4\hbar^2\mathcal{L}_1^4}\left(\mathcal{C}_q + t_+\mathcal{R}^{2/3} + \frac{15}{4}t_+\mathcal{L}_{8/3}^2 + \frac{15}{11}t_-\zeta z\mathcal{L}_{11/3}^2\right) + zK,$$

$$b_6 = 2z'(T_1 + T_{2q}) + 2T_{3q}, \qquad b_7 = 2z(T_1 + T_{2q}), \qquad b_9 = \frac{m\pi\mathcal{L}_2^2}{2\hbar^2 A}zt_+,$$

$$b_8 = \frac{m\pi\mathcal{L}_2^2}{\hbar^2 A}\left[t_+(1-\frac{z'}{2}) - z\frac{t_-}{2}\right], \quad b_{10} = 1 + \bar{\zeta}z', \quad b_{11} = \bar{\zeta}z,$$

$$b_{12} = \varphi a_i^2(1+\zeta n_0 z'), \quad b_{13} = \varphi a_i^2\zeta n_0 z, \quad d_1 = \frac{3\gamma}{m\mathcal{L}_1^4}\left(\mathcal{L}_{5/3}^2 + \frac{5}{8}\zeta z'\mathcal{L}_{8/3}^2\right),$$

$$d_2 = \frac{15}{8}\gamma\zeta z\frac{\mathcal{L}_{8/3}^2}{m\mathcal{L}_1^4}, \qquad d_3 = 1 + \frac{2m}{A}\int n\,C_q\,d\vec{r}.$$

Here the following notations are used

$$z = Z_q/A, \quad z' = Z_{q'}/A, \quad \bar{\zeta} = \zeta\mathcal{L}_2^4/\mathcal{L}_1^4, \quad \varphi = \pi n_0\frac{e_p^2}{m}\frac{Z_p}{A},$$

$$\mathcal{A} = \frac{t_0}{5}(1+\frac{x_0}{2})(S_2^4 + 2\zeta\mathcal{R}^1) + \frac{t_3}{120}\left\{(\sigma+1)\left[\sigma(1-x_3) + 2(2+x_3)\right] +\right.$$

$$\left.+2\sigma(\sigma-1)(1+2x_3)zz'\right\}\cdot(\mathcal{R}^\sigma + 2\zeta\mathcal{R}^{\sigma+1}),$$

$$\mathcal{B}_q = \mathcal{A} - \frac{t_3}{8}\left\{(\sigma+1)\left[(\sigma+2)(1-x_3) + 2(1+2x_3)(z'-\sigma z)\right] +\right.$$

$$\left.+2(1+2x_3)\sigma(3\sigma+1)zz'\right\}\cdot\left(\frac{\mathcal{L}_{\sigma+2}^2}{\sigma+2} + \zeta\frac{\mathcal{L}_{\sigma+3}^2}{\sigma+3}\right) + \zeta t_0\mathcal{L}_3^2\left\{z(x_0+\tfrac{1}{2}) - 1 - \frac{x_0}{2}\right\},$$

$$\mathcal{E}_q = z(\mathcal{R}^{2/3} + 2\zeta\mathcal{R}^{5/3}), \qquad \mathcal{C}_q = \zeta\left[\frac{45}{11}\mathcal{L}_{11/3}^2(\frac{t_-}{2}z - t_+) + 2t_+\mathcal{R}^{5/3}\right],$$

$$\mathcal{D}_q = \frac{45}{8}z\mathcal{L}_{8/3}^2(\frac{t_-}{2} - t_+) + t_+z'(\mathcal{R}^{2/3} - \frac{45}{8}\mathcal{L}_{8/3}^2),$$

$$G = \frac{1}{\bar{a}_i^2}\left[T_1\left(\frac{2}{\bar{a}_i^2} + \frac{2}{\bar{a}_j^2} + \sum_{k=1}^{3}\frac{1}{\bar{a}_k^2}\right) + \frac{2}{\bar{a}_j^2}T_{2q}\right], \quad K = (T_1 + T_{2q})\left(\frac{2}{\bar{a}_i^2} + \sum_{k=1}^{3}\frac{1}{\bar{a}_k^2}\right),$$

$$F = \frac{t_3}{8}\frac{(\sigma+1)^2}{\sigma+2}\left[(\sigma+2)(1-x_3) + 2(1+2x_3)z'(1+\sigma z)\right]\mathcal{L}_{\sigma+2}^2,$$

$$T_1 = \frac{\hbar^2}{4m\mathcal{L}_1^4}\frac{t_-}{35}\left[4S_2^2 + S_4 + 2\zeta(4Q_2 + Q_4 - \frac{4}{3}S_3)\right],$$

$$T_{3q} = \frac{\hbar^2 S_2^2}{8m\mathcal{L}_1^4}(z\frac{t_+}{2} - t_-), \qquad T_{2q} = \frac{\hbar^2}{4m\mathcal{L}_1^4}\zeta(z\frac{t_+}{2} - t_-)(Q_2 + \frac{S_3}{5}),$$

$$\mathcal{L}_\mu^\nu = \int_0^\infty n^\mu(\rho)\rho^\nu d\rho, \qquad \mathcal{R}^\mu = \int_0^\infty n^\mu(\rho)\rho^4\left(\frac{\partial n}{\partial\rho}\right)^2 d\rho,$$

$$S_2^{\mu} = \int_0^\infty \rho^{\mu}\left(\frac{\partial n}{\partial \rho}\right)^2 d\rho, \quad S_3 = \int_0^\infty \rho^3\left(\frac{\partial n}{\partial \rho}\right)^3 d\rho, \quad S_4 = \int_0^\infty \rho^4\left(\frac{\partial^2 n}{\partial \rho^2}\right)^2 d\rho,$$

$$Q_2 = \int_0^\infty n(\rho)\rho^2\left(\frac{\partial n}{\partial \rho}\right)^2 d\rho, \quad Q_4 = \int_0^\infty n(\rho)\rho^4\left(\frac{\partial^2 n}{\partial \rho^2}\right)^2 d\rho .$$

B_{ij} - the index symbols defined in /22,28/.

References

1. Satchler G.R. - Proceedings of the International School of Physics "Enrico Fermi", Varenna, Italy, 1976, Course LXIX, Elementary Modes of Excitation in Nuclei (Societa Italiana di Fisica, Bologna, 1977), p. 271.
2. Bertrand F.E., Satchler G.R., Horen D.J. et al. - Phys.Rev. 1980, v. C22, p. 1832.
3. Buenerd M. - J. de Physique 1984 v. 45, p. C4-115.
4. Pitthan R. - Proceedings of the Giant Multipole Resonance Topical Conference held at the American Museum of Science and Energy, Oak Ridge, Tennessee, 1979, ed. by F.E.Bertrand, p. 161.
5. Zorro R., Bergquist I. et al. - Nucl.Phys. 1987 v. A472, p. 125.
6. Murakami T., Halpern I., Storm D.W. et al. - Phys.Rev. 1987, v. C35, p. 479.
7. Erell A., Alster J., Lichtenstadt J. et al. - Phys.Rev. 1986, v. C34, p. 1822.
8. Van der Woude A. - Preprint KVI-583, Gröningen, 1986.
9. Liu K.F., Brown G.E. - Nucl.Phys. 1976, v. A265, p. 385.
10. Malov L.A., Soloviev V.G. - Particles and Nuclei, 1980, v. 11, i. 2, p. 301.
11. Auerbach N., Klein A. - Nucl.Phys. 1983, v. A395, p. 77.
12. Civitarese O., Furui S., Ploszajczak M., Faessler A. - Nucl. Phys. 1983, v. A408, p. 61.
13. Åberg S. - Nucl.Phys. 1987, v. A473, p. 1.
14. Krivine H., Treiner J., Bohigas O. - Nucl.Phys. 1980 v. A336, p. 155.
15. Nix J.R., Sierk A.J. - Phys.Rev. 1980, v. C21, p. 396.
16. Holzwarth G. - Proceedings of the Winter College on Fundamental Nuclear Physics (ICTP, Trieste, Italy) 1984, ed. by K.Dietrich, M. Di Toro, H.J.Mang, v. 2, p. 967.
17. Da Providência J.P., Holzwarth G. - Nucl.Phys. 1985, v. A439, p. 477.
18. Di Toro M., Lombardo V., Russo G. - Nucl.Phys. 1985, v. A435, p. 173.
19. Nishizaki S., Andô K. - Prog.Theor.Phys. 1984, v. 71, p. 1263.
20. Jang S., Szymanski Z. - Nucl.Phys. 1985, v. A436, p. 397.
21. Jang S., Bouyssy A. - Phys.Rev. 1987, v. C36, p. 1587.
22. Balbutsev E.B., Vaishvila Z., Mikhailov I.N. - Yad. Fiz. 1982, v. 35, p. 836; v. 36, p. 1109.
23. Balbutsev E.B., Mikhailov I.N., Vaishvila Z. - Nucl.Phys. 1986, v. A457, p. 222.
24. Balbutsev E.B., Mikhailov I.N. - J.Phys. G: Nucl.Phys. 1988, v. 14, p. 545.
25. Bartel J., Quentin P., Brack M., Håkansson H.B. - Nucl.Phys. 1982, v. A386, p. 79.
26. Engel Y.M., Brink D.M.,Goeke K., Vautherin D. - Nucl.Phys. 1975, v. A249, p. 215.
27. Bertsch G.F. - Nuclear Physics with Heavy Ions and Mesons 1977 Les Houches Lectures, ed. R.Balian et al. (Amsterdam: North Holland) 1978, vol. 1, p. 175.
28. Chandrasekhar S. - Ellipsoidal figures of equilibrium . Yale University Press, 1969.
29. Lane A.M. - Nuclear Theory, Ed. Benjamin, New York 1964.
30. Bernstein A.M. - Adv.Nucl.Phys. 1969, v. 3, p. 325.
31. Bohr A., Mottelson B.R. Nuclear Structure, Ed. W.A.Benjamin, New York, Amsterdam, 1969, v. 1.
32. Bohr A., Mottelson B.R. Nuclear Structure, Ed. W.A.Benjamin, New York, Amsterdam, 1974, v. 2.
33. De Franceschi G., Palumbo F., Lo Iudice N. - Phys.Rev. 1984, v. C29, p. 1496.
34. Bohle D., Richter A. et al. - Phys.Lett. 1984, v. B137, p. 27.

Received by Publishing Department
on December 22, 1988.