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**GAMOW-TELLER  $\beta^+$  DECAYS  
AND STRENGTH FUNCTIONS  
OF (n,p) TRANSITIONS  
IN SPHERICAL NUCLEI**

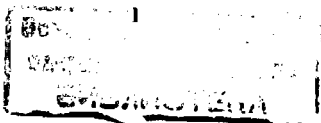
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## 1. Introduction

In describing vibrational states of atomic nuclei the key role is attributed to particle-hole interactions <sup>1)</sup>. They form low- and high-lying (giant resonances) collective vibrational states. Besides particle-hole interactions some authors take into account particle-particle interactions. The interacting boson model <sup>2)</sup> taking account of particle-particle interactions proved to be advantageous in describing some characteristics of collective low-lying states thus reviving interest in the study of the role of particle-particle interactions. A consistent treatment in the Random Phase Approximation (RPA) including both particle-hole (p-h) and particle-particle (p-p) interactions allowed the authors of ref. <sup>3)</sup> to resolve the discrepancy between the experimental and calculated two-neutrino double  $\beta$  decay rates.

The study of the  $\beta^+$  decay of nuclei far from stability provides experimental information on the charge-exchange strength distribution. Thus, the  $\beta^+$  decays from the ground  $0_g^+$  states of doubly even nuclei to the excited  $1^+$  states of doubly odd nuclei provide data on the distribution of the Gamow-Teller (GT) strength. Experiments of this sort have been performed in refs. <sup>4-8)</sup> for some spherical nuclei with  $A=146-152$ . Due to a large energy of the  $\beta^+$  decay of neutron-deficient short-lived isotopes, it is believed that almost the whole GT  $\sigma t_{(+)}$  strength  $S_+$  is observed. This allowed the authors



of refs.<sup>5,9)</sup> to make a conclusion on a strong renormalisation in nuclei of the weak axial-vector constant  $g_A$ .

As is known, progress has been achieved in the experimental study of the giant GT resonance in (p,n) reactions, i.e. of the  $\sigma t_{(-)}$  branch. For the total strength of the GT transitions there is the model independent sum rule

$$S_- - S_+ = 3(N-Z). \quad (1)$$

It connects the total GT strength  $S_-$  related to the  $0^0(p,n)$  cross section and partially to the  $\beta^-$  decay with  $S_+$  related to the  $0^0(n,p)$  cross section and  $\beta^+$  decay. It was concluded from the (p,n) reactions that about (40-50)% of the sum-rule GT strength is missing under the assumption  $S_+ = 0$ .

Since the GT strength is missing, as has been pointed out in ref.<sup>10)</sup>, it is interesting to measure the  $0^0(n,p)$  cross sections. The experiments<sup>11)</sup> for the reaction  $^{54}\text{Fe}(n,p)^{54}\text{Mn}$  allowed one to extract information on  $S_+$ .

The foregoing shows that the  $\beta^+$  decay and strength functions of (n,p) transitions are worthy of further theoretical investigation, for instance, within the quasiparticle-phonon nuclear model (QPNM)<sup>12-14)</sup>. The advantage of the QPNM has been demonstrated in<sup>15)</sup> in describing the fragmentation of the GT resonance. Of some interest is the effect of the p-p interaction on the probabilities of the GT  $\beta^+$  decay and on the strength functions of (n,p) transitions. The QPNM can be generalised to the p-h and p-p interactions in describing charge-exchange states.

The present paper is devoted to the description in the QPNM of the GT  $\beta^+$  decays and strength functions of (n,p) transitions in spherical nuclei in the RPA including both the p-h and p-p interactions.

## 2. The RPA Equations for the GT States Including Particle-Hole and Particle-Particle Interactions

The QPNM Hamiltonian<sup>14,15)</sup> is added by the term describing the GT p-p interaction

$$H_{CS}^{PP} = -2G_1 \sum_{\mu} P_{1\mu}^+ P_{1\mu},$$

$$P_{1\mu}^+ = \sum_{j_p m_p, j_n m_n} (-1)^{j_p - m_p} \langle j_p m_p | \sigma_{\mu} t_{(+)} | j_p m_p \rangle a_{j_n m_n}^+ a_{j_p m_p}^+, \quad (2)$$

$$f(j_p, j_n) \equiv \langle j_p || \sigma t_{(-)} || j_n \rangle.$$

where  $a_{j_p m_p}^+$  and  $a_{j_n m_n}^+$  are the proton and neutron creation operators and  $j_p m_p$  and  $j_n m_n$  are the quantum numbers of single-particle proton and neutron states. The  $\sigma \tau$  interactions are taken into account whereas  $[Y_2, \sigma]_1 \tau$  are not; the role of the latter is insignificant.

Now we perform the canonical Bogolubov transformation

$$a_{j m} = u_j d_{j m} + (-1)^{j-m} v_j d_{j, -m}^+$$

and introduce the GT phonon creation operator

$$\Omega_{1\mu i}^+ = \sum_{j_p j_n} \left[ \psi_{j_p j_n}^i A_{j_p j_n}^+(1, \mu) + (-1)^N \varphi_{j_p j_n}^i A_{j_p j_n}^-(1, \mu) \right], \quad (3)$$

where

$$A_{j_p j_n}(\lambda \mu) = \sum_{m_p, m_n} \langle j_p m_p, j_n m_n | \lambda \mu \rangle d_{j_n m_n} d_{j_p m_p}.$$

The corresponding part of the p-p interaction (2) is expressed through the operators  $\Omega_{1\mu i}$  and  $\Omega_{1\mu i}^+$  and added to the p-h interaction (see refs. 14,15). Then, the RPA Hamiltonian is

$$H_{CS\nu} = \sum_{j_p m_p} \varepsilon_{j_p} d_{j_p m_p}^+ d_{j_p m_p} + \sum_{j_n m_n} \varepsilon_{j_n} d_{j_n m_n}^+ d_{j_n m_n} - \frac{1}{3} \sum_{i, \mu} \left\{ x_i^{(+) [(D_+^i)^2 + (D_-^i)^2] + G_i^{(+) [(D_g^i)^2 + (D_w^i)^2] \right\} \Omega_{1\mu i}^+ \Omega_{1\mu i}, \quad (4)$$

where

$$D_+^i = \sum_{j_p j_n} f(j_p j_n) u_{j_p j_n}^{(+)} g_{j_p j_n}^i, \quad D_-^i = \sum_{j_p j_n} f(j_p j_n) u_{j_p j_n}^{(-)} w_{j_p j_n}^i, \\ D_g^i = \sum_{j_p j_n} f(j_p j_n) v_{j_p j_n}^{(+)} g_{j_p j_n}^i, \quad D_w^i = \sum_{j_p j_n} f(j_p j_n) v_{j_p j_n}^{(-)} w_{j_p j_n}^i, \\ u_{j_p j_n}^{(\pm)} = u_{j_p} v_{j_n} \pm v_{j_p} u_{j_n}, \quad v_{j_p j_n}^{(\pm)} = u_{j_p} u_{j_n} \pm v_{j_p} v_{j_n}, \\ g_{j_p j_n}^i = \psi_{j_p j_n}^i + \varphi_{j_p j_n}^i, \quad w_{j_p j_n}^i = \psi_{j_p j_n}^i - \varphi_{j_p j_n}^i,$$

and  $\varepsilon_j$  are single-quasiparticle energies.

Let us find an average value of  $H_{CS\nu}$  over the one-phonon state

$$\Omega_{1\mu i}^+ \Psi_0, \quad (5)$$

where  $\Psi_0$  is the ground state wave function of a doubly even nucleus. To find the RPA equations, we use the variational principle

$$\delta \left\{ \langle \Psi_0^* \Omega_{1\mu i} H_{CS\nu} \Omega_{1\mu i}^+ \Psi_0 \rangle - \langle \Psi_0^* H_{CS\nu} \Psi_0 \rangle - \omega_i \left[ \sum_{j_p j_n} g_{j_p j_n}^i w_{j_p j_n}^i - 1 \right] \right\} = 0, \quad (6)$$

and get

$$\varepsilon_{j_p j_n} g_{j_p j_n}^i - \omega_i w_{j_p j_n}^i - \frac{2}{3} (x_i^{(+)} D_+^i u_{j_p j_n}^{(+)} + G_i^{(+)} D_g^i v_{j_p j_n}^{(+)}) f(j_p j_n) = 0, \\ -\omega_i g_{j_p j_n}^i + \varepsilon_{j_p j_n} w_{j_p j_n}^i - \frac{2}{3} (x_i^{(-)} D_-^i u_{j_p j_n}^{(-)} + G_i^{(-)} D_w^i v_{j_p j_n}^{(-)}) f(j_p j_n) = 0, \quad (7)$$

where  $\varepsilon_{j_p j_n} = \varepsilon_{j_p} + \varepsilon_{j_n}$ . As a result of simple transformations the secular equation for the energies  $\omega_i$  of one-phonon states is

$$\begin{vmatrix} x_1^{(+)} X_{+-}^{i-1} & x_1^{(+)} X_{+-}^i & G_1^{(+)} X_{1+-}^i & G_1^{(+)} X_{1+}^i \\ x_1^{(+)} X_{+-}^i & x_1^{(+)} X_{--}^{i-1} & G_1^{(+)} X_{1-}^i & G_1^{(+)} X_{1-+}^i \\ x_1^{(+)} X_{1+}^i & x_1^{(+)} X_{1-}^i & G_1^{(+)} X_{g-}^{i-1} & G_1^{(+)} X_{gw}^i \\ x_1^{(+)} X_{1+}^i & x_1^{(+)} X_{1-}^i & G_1^{(+)} X_{gw}^i & G_1^{(+)} X_{w-}^{i-1} \end{vmatrix} = 0, \quad (8)$$

where

$$X_{\pm}^i = \frac{2}{3} \sum_{j_p j_n} \frac{(f(j_p j_n) u_{j_p j_n}^{(\pm)})^2 \varepsilon_{j_p j_n}}{\varepsilon_{j_p j_n}^2 - \omega_i^2},$$

$$X_{+-}^i = \frac{2}{3} \sum_{jpn} \frac{[f(jpn)]^2 u_{jpn}^{(+)} u_{jpn}^{(-)} \omega_i}{\varepsilon_{jpn}^2 - \omega_i^2},$$

$$X_{1\pm}^i = \frac{2}{3} \sum_{jpn} \frac{[f(jpn)]^2 u_{jpn}^{(\pm)} v_{jpn}^{(\pm)} \omega_i}{\varepsilon_{jpn}^2 - \omega_i^2},$$

$$X_{1\pm\mp}^i = \frac{2}{3} \sum_{jpn} \frac{[f(jpn)]^2 u_{jpn}^{(\pm)} v_{jpn}^{(\mp)} \varepsilon_{jpn}}{\varepsilon_{jpn}^2 - \omega_i^2},$$

$$X_{gw}^i = \frac{2}{3} \sum_{jpn} \frac{[f(jpn) v_{jpn}^{(\mp)}]^2 \varepsilon_{jpn}}{\varepsilon_{jpn}^2 - \omega_i^2},$$

$$X_{gw}^i = \frac{2}{3} \sum_{jpn} \frac{[f(jpn)]^2 v_{jpn}^{(+)} v_{jpn}^{(-)} \omega_i}{\varepsilon_{jpn}^2 - \omega_i^2}.$$

For the energies found from eq. (8) we calculate the functions  $g_{jpn}^i$  and  $w_{jpn}^i$  using eqs. (7) and the normalisation condition of the wave function (5). At  $G_f^{01} = 0$  eq.(8) turns into equations obtained in ref.<sup>15)</sup>. The wave functions of one-phonon states (5) are a superposition of excitations in the nuclei with  $N-1, Z+1$  and  $N+1, Z-1$ . For each solution  $i$  one can separate a dominating branch. Moreover, for each RPA solution average numbers of neutrons and protons over one-phonon states are calculated in order to determine what branch a given state corresponds to.

### 3. Effect of Particle-Particle Interactions on the $\beta^+$ Decay

Considerable part of the GT strength appears in the  $\beta^+$  decay of the ground state of a doubly even nucleus into the excited  $1^+$  states of a doubly odd nucleus in case of a large energy of the  $\beta^+$  decay. The matrix element of the  $\beta^+$  decay from the ground state of a doubly even nucleus into the one-phonon  $1^+$  state with the wave function (5) is

$$\langle \Psi_0^* \Omega_{1\mu i} H_{\beta}^+ \Psi_0 \rangle = \frac{1}{\sqrt{3}} \sum_{jpn} f(jpn) (v_{jpn} u_{jpn} \psi_{jpn}^i + u_{jpn} v_{jpn} \varphi_{jpn}^i). \quad (9)$$

The matrix element of the GT  $\beta^-$  decay between the ground and one-phonon  $1^+$  states has the form

$$\langle \Psi_0^* \Omega_{1\mu i} H_{\beta}^- \Psi_0 \rangle = \frac{1}{\sqrt{3}} \sum_{jpn} f(jpn) (u_{jpn} v_{jpn} \psi_{jpn}^i + v_{jpn} u_{jpn} \varphi_{jpn}^i). \quad (10)$$

The comparison of the matrix elements shows that due to the neutron excess the terms connected with the direct amplitude  $\psi_{jpn}^i$  for the  $\beta^+$  decay are less than the corresponding terms for the  $\beta^-$  decay. The effect of the p-p interactions is the change of the energy  $\omega_i$  and amplitudes  $\psi_{jpn}^i$  and  $\varphi_{jpn}^i$ .

The strength of the GT transitions to the  $1^+$  state is defined for the (p,n) and  $\beta^-$  decay and (n,p) and  $\beta^+$  decay by

$$B^{(1^+, i)} = \left| \sum_{jpn} f(jpn) (u_{jpn} v_{jpn} \psi_{jpn}^i + v_{jpn} u_{jpn} \varphi_{jpn}^i) \right|^2, \quad (11)$$

Table 1. GT  $\beta^+$  transitions  $0_{g.s.}^+ \rightarrow 1^+$

$\beta^+$ transition	exp.	$\log \tilde{ft}$				Part of total strength	
		calculation				$ g_A/g_V =1$	$ g_A/g_V =1.25$
		$x_1^{01}=0$ $G_1^{01}=0$	$x_1^{01}A=-23$ $G_1^{01}=0$	$x_1^{01}A=-23$ $G_1^{01}A=7.5$	$x_1^{01}A=-23$ $G_1^{01}A=7.5$		
$152_{Yb} \rightarrow 152_{Tm}$	3.4	2.8	3.0	3.4	0.60	0.50	
$152_{Er} \rightarrow 152_{Ho}$	3.9	2.9	3.2	3.5	0.80	0.77	
$150_{Er} \rightarrow 150_{Ho}$	3.6	2.9	3.1	3.5	0.55	0.50	
$150_{Dy} \rightarrow 150_{Tb}$	4.1	3.0	3.3	3.6	0.80	0.80	
$148_{Dy} \rightarrow 148_{Tb}$	3.9	3.0	3.3	3.8	0.50	0.43	
$146_{Dy} \rightarrow 146_{Tb}$	3.8	3.0	3.3	4.1	0.30	0.20	
$126_{Ba} \rightarrow 126_{Cs}$	4.4	3.4	4.0	4.4 <sup>*)</sup>	0.20 <sup>*)</sup>		
$120_{Xe} \rightarrow 120_{I}$	4.4	3.3	4.0	4.5 <sup>*)</sup>	0.14 <sup>*)</sup>		
$96_{Pd} \rightarrow 96_{Rh}$	3.3	2.7	3.1	3.4	0.35	0.32	

\*) Calculations with  $G_1^{01} = \frac{6.0}{A}$  MeV.

Consider now the GT transitions in the  $\beta^+$  decay of  $146, 148_{Dy}$ . In comparison with the independent quasiparticle model, the p-h interaction decreases twice the integral strength of the  $\beta^+$  transitions. The inclusion of the p-p interaction provides an additional suppression by 3-5 times. The low-lying states used in the calculations of  $\log \tilde{ft}$  are separated from the high-lying ones by a gap of 4-5 MeV. With increasing constant from  $G_1^{01}=7.5/A$  MeV up to  $G_1^{01}=7.7/A$  MeV for  $148_{Dy}$ , the gap diminishes by 0.5 MeV. For the  $\beta^+$  decay of  $146_{Dy}$  at  $G_1^{01} =$

$7.3/A$  MeV we have  $\log \tilde{ft}=3.9$  MeV and 50% of the total strength is exhausted in the  $\beta^+$  decay; the calculations at  $|g_A/g_V|=1.25$  were performed with  $G_1^{01}=7.7/A$  MeV. In this case, the RPA is inapplicable at  $G_1^{01} \geq 7.9/A$  MeV.

For the  $\beta^+$  decay of  $96_{Pd} \rightarrow 96_{Rh}$  at  $x_1^{01}=-23/A$  MeV and  $G_1^{01}=0$   $\log \tilde{ft}=3.0$  and the high-lying levels are separated by a gap of 3 MeV. At  $G_1^{01}=7.1/A$  MeV we have  $\log \tilde{ft}=3.3$  equal to that measured in ref. <sup>16</sup>). With increasing  $G_1^{01}$  the integral strength, appearing in the  $\beta^+$  decay, decreases gradually thus increasing  $\log \tilde{ft}$ .

Now let us show the decrease of the  $\beta^+$  transition rates with increasing  $G_1^{01}$  taking as an example the  $\beta^+$  decay of the ground state of  $146_{Dy}$  into the  $1^+$  state of  $146_{Tb}$  described by the wave function  $\Omega_{1\nu i_0}^+ \Psi_0$  with the dominating component  $p1h_{11/2} n1h_{9/2}$ . We shall write out the components of the wave function  $\Omega_{1\nu i_0}^+ \Psi_0$  which have the largest  $\psi_{j_p j_n}^{i_0}$ ,  $\varphi_{j_p j_n}^{i_0}$  and the largest  $\beta^+$  transition amplitudes

$$B(i_0, j_p j_n) = f(j_p j_n) (\nu_{j_p}^+ \nu_{j_n} \psi_{j_p j_n}^{i_0} + \nu_{j_p} \nu_{j_n}^+ \varphi_{j_p j_n}^{i_0}). \quad (16)$$

At  $x_1^{01}=-23/A$  MeV and  $G_1^{01}=0$  the state  $i_0$  has the energy  $\omega_{i_0}=8.042$  MeV and the total transition strength, defined by eq.(12), equal to  $B(i_0, j_p j_n)=3.431$  which is smaller than  $|B(i_0, p1h_{11/2} n1h_{9/2})|^2=6.47$ . The configuration  $\{p1h_{11/2} n1h_{9/2}\}$  gives a 99.8% contribution to the normalisation of the wave function  $\Omega_{1\nu i_0}^+ \Psi_0$ . However, the transition strength decreases 1.9 times in comparison with 6.47.

At  $x_1^{01}=-23/A$  MeV and  $G_1^{01}=6/A$  MeV the total transition strength to the state  $i_0$  decreases 1.5 times in comparison with  $G_1^{01}=0$ . The contribution of the configuration  $\{p1h_{11/2} n1h_{9/2}\}$



been studied in ref.<sup>19</sup>). The constant  $G_1^{01}$  was chosen from the energies of low-lying states so that the  $\beta^+/EC$  transition was not considerably weakened. The smallest  $\tilde{ft}$ -values were used to compare the results of calculations<sup>19</sup>) with experimental data. If for the available experimental data one uses  $\tilde{ft}$  values, the difference from the calculation reduces.

We have calculated the  $\beta^+$  decays of  $^{126}\text{Ba}$  and  $^{120}\text{Xe}$  which have rather large decay energies;  $\log \tilde{ft}$  values are shown in Table 1. The  $\log \tilde{ft}$  values calculated at  $x_1^{01} = G_1^{01} = 0$  and  $x_1^{01} = -23/A$  MeV,  $G_1^{01} = 0$  are in agreement with the calculation<sup>19</sup>) and the  $\log \tilde{ft}$  values calculated with  $x_1^{01} = -23/A$  MeV,  $G_1^{01} = 6/A$  MeV are in agreement with the experimental data. It is to be noted that our calculations for a small  $\beta^+$  decay energy are rough. The quasiparticle-phonon interactions can change the amplitudes (16) for the low-lying states out of which only a part appears in the  $\beta^+$  decay with small energy.

The RPA with both the p-h and p-p interactions enables one to describe  $\log \tilde{ft}$  for the  $\beta^+$  transitions in even spherical nuclei. Fast  $\beta^+$  transitions are observed in odd-A nuclei (4,9,20); the most thoroughly studied is the  $\beta^+$  decay of  $^{147m}\text{Dy}$  for which  $\log \tilde{ft} = 3.67$ . The question now arises of whether the  $\beta^+$  transitions in odd-A nuclei can be described correctly.

Since we have to extract the most important degrees of freedom, we write down the wave function, for instance, of an odd  $\lambda$  nucleus as

$$\Psi_{\nu_p}(j_p, M_p) = C_{\nu_p \nu_p} \left\{ d_{j_p M_p}^+ + \sum_{i, j_n} D_{j_n}^{i1}(j_p, \nu_p) \sum_{m_n, \mu} \langle j_n m_n \mu | j_p M_p \rangle d_{j_n m}^+ \Omega_{\mu i}^+ \right\} \Psi_0 \quad (17)$$

where  $\Psi_0$  is the ground state wave function of a doubly even nucleus with  $\lambda = 0$ . One can easily derive, as in refs.<sup>1,14</sup>), the system of equations for the energies and coefficients  $C_{\nu_p \nu_p}$  and  $D_{j_n}^{i1}(j_p, \nu_p)$  of the wave function (17). Since we do not solve equations of this sort, we do not give them but make some estimations.

Our estimations are illustrated by the GT  $\beta^+$  decay of  $^{147m}\text{Dy}$  into the levels of  $^{147}\text{Tb}$ . Following ref.<sup>20</sup>), we consider the level  $\frac{11}{2}^-$  of  $^{147}\text{Dy}$  to be close to the neutron one-quasiparticle state  $n1h_{11/2}$  and write its wave function as

$$\Psi_{\nu_n}(\frac{11}{2}^-, M_n) = C_{11/2, \nu_n} d_{1h_{11/2}, M_n}^+ \Psi_0 \quad (18)$$

where  $\Psi_0$  is the ground state wave function of  $^{146}\text{Dy}$ . Assume that the  $\beta^+$  decay proceeds to the components  $n1h_{11/2} \otimes \Omega_{\mu i}^+ \Psi_0'$ , and therefore, the wave function  $^{147}\text{Tb}$  becomes

$$\Psi_{\nu_p}(j_p, M_p) = C_{\nu_p \nu_p} \sum_i D_{n1h_{11/2}}^{i1} \sum_{m_n} \langle \frac{11}{2} m_n \mu | j_p M_p \rangle d_{1h_{11/2}, m_n}^+ \Omega_{\mu i}^+ \Psi_0' \quad (19)$$

$\Psi_0'$  is the ground state wave function of  $^{146}\text{Gd}$ . The matrix element of the  $\beta^+$  decay of  $^{147m}\text{Dy}$  from the one-quasiparticle state  $n1h_{11/2}$  with the wave function (18) into the states  $n1h_{11/2} \otimes \Omega_{\mu i}^+ \Psi_0'$  of  $^{147}\text{Tb}$  described by the wave function (19) is

$$\langle \Psi_{\nu_p}(j_p, M_p) | H_{\beta} | \Psi_{\nu_n}(\frac{11}{2}^-, M_n) \rangle = C_{\nu_p \nu_p} C_{11/2, \nu_n} \sum_{i, \mu, M_n} \langle \frac{11}{2} M_n \mu | j_p M_p \rangle \cdot D_{n1h_{11/2}}^{i1} \sum_{j_p, j_n} \frac{f(j_p, j_n)}{\sqrt{3}} (v_{j_p}^+ v_{j_n} \psi_{j_p j_n}^i + v_{j_p} v_{j_n}^+ \varphi_{j_p j_n}^i) \quad (20)$$

Here the sum over  $j_p, j_n$  coincides with the matrix element (9) describing the  $\beta^+$  decay of  $^{146}\text{Dy}$  into the  $1^+$  states of  $^{146}\text{Tb}$ . Assuming that the  $\beta^+$  decay proceeds from the one-quasiparticle



state  $n1h_{11/2}$  of  $^{147}\text{Dy}$  to the  $n1h_{11/2} \otimes \Omega_{1\mu}^+ \Psi_0'$  state of  $^{147}\text{Tb}$ , we get  $\log ft$  equal to  $\log \tilde{f}t$  for the  $\beta^+$  decay of  $^{146}\text{Dy}$  to the  $1^+$  state of  $^{146}\text{Tb}$ , i.e.  $\log \tilde{f}t = 3.9$ . Due to  $C_{11/2, \nu_n} < 1$  and  $C_{p,p} D_{nh_{11/2}}^{1i} < 1$  we get  $\log \tilde{f}t > 3.9$  for the  $\beta^+$  decay of  $^{147m}\text{Dy}$  on  $^{147}\text{Tb}$ . Estimates of this type are tabulated in Table 3.

The analysis shows that fast  $\beta^+$  decays of odd-A spherical nuclei are in agreement with the calculations within the RPA with both the p-h and p-p interactions.

Table 3. GT  $\beta^+$  transitions in odd-A nuclei

$\beta^+$ transition	$\log \tilde{f}t_{\text{exp.}}$	$\log \tilde{f}t_{\text{calc.}}$
$^{149}\text{Ho} \rightarrow ^{149}\text{Dy}$	4.2	> 3.8
$^{147}\text{Dy} \rightarrow ^{147}\text{Tb}$	3.9	> 4.1
$^{147}\text{Tb} \rightarrow ^{147}\text{Gd}$	4.2	> 4.1

We now turn our attention to  $\beta^-$  decays for comparison. A small part of the total GT strength is observed in  $\beta^-$  decays. As an example, we consider  $\beta^-$  transitions from the ground state of  $^{80}\text{Zn}$  to the  $1^+$  levels in  $^{80}\text{Ga}$  with an excitation energy of 0.7-2.7 MeV. According to ref.<sup>21)</sup> the energy of the  $\beta^-$  transition is 7.1 MeV and the integrated  $\log \tilde{f}t$  is 4.0. The states  $1^+$  of  $^{80}\text{Ga}$  are described as one-phonon states; and the matrix elements of the GT  $\beta^-$  transitions, by formula (10). The low-lying  $1^+$  states in  $^{80}\text{Ga}$  are not separated from the high-lying ones by the gap, and therefore, the results of calculations depend on the energy interval  $\Delta E$  which contains the levels populated in the  $\beta^-$  decay. The decays observed experimentally<sup>21)</sup> populate the  $1^+$  levels in  $^{80}\text{Ga}$

lying in the interval  $\Delta E = 2$  MeV. The quasiparticle-phonon interactions can shift a part of the GT strength from one energy interval to another.

According to our calculations of the  $\beta^-$  decays of  $^{80}\text{Zn}$ , at  $\alpha_1^{01} = G_1^{01} = 0$  50% of the total strength  $S_{-}$  is concentrated on the low-lying states in the interval  $\Delta E = 3$  MeV with  $\log \tilde{f}t = 2.4$ . For  $\alpha_1^{01} = -23/\text{A MeV}$ ,  $G_1^{01} = 0$  the total GT strength  $S_{-}$  decreases by 0.34%; in the low-lying region 0.5% of the total GT strength lies at  $\Delta E = 2$  MeV with  $\log \tilde{f}t = 4.3$  and 2.2% of the total strength lies at  $\Delta E = 3$  MeV with  $\log \tilde{f}t = 3.5$ . For  $\alpha_1^{01} = -23/\text{A MeV}$  and  $G_2^{01} = 7.5/\text{A MeV}$  the total GT strength decreases by 1% and  $\log \tilde{f}t$  for the  $\beta^-$  transitions to the low-lying regions changes slightly. This calculation indicates that the p-p interactions slightly affect the  $\beta^-$  transitions.

We have calculated the matrix elements of the two-neutrino double  $\beta$  decay of  $^{128,130}\text{Te}$  with the constants  $\alpha_1^{01}$  and  $G_1^{01}$  which were used to calculate  $\log \tilde{f}t$  for the  $\beta^+$  transitions shown in Table 1. The calculated matrix elements and the lifetimes are in agreement with the results of ref.<sup>3)</sup> and with the experiment. The necessity of taking into account the p-p interaction to suppress the strength of the two-neutrino double  $\beta$  decay has been confirmed in ref.<sup>22)</sup>. The separable interaction in our calculations,  $\delta$ -function in ref.<sup>3)</sup> and effective forces calculated from the Bonn potential in ref.<sup>22)</sup> did not influence the results of calculations. The difference in the constants of the p-h and p-p interactions in our calculation is due to numerical factors. It is to be mentioned that in ref.<sup>23)</sup> the suppression of the double  $\beta$  decay is caused by ground state correlations due to quadrupole vibrations.

#### 4. On Renormalisation of the Weak Axial-Vector

##### Constant $g_A$ in Nuclei

A large part of the total GT strength of  $S_+$  manifests itself in the  $\beta^+$  decay of neutron-deficit nuclei. Therefore, it is assumed that from the  $\log ft$  values of the GT  $\beta^+$  decays one can find the ratio of the constants  $|g_A/g_v|$  of the axial-vector and vector weak interactions, thus determining the renormalisation in nuclei of the constant  $g_A$ . Experiments (4-9, 20) on  $\beta^+$  decays of neutron-deficit nuclei revealed fast GT transitions with  $\log ft < 4$ . The comparison of the RPA calculations with p-h interactions with the experimental data indicated a large renormalisation of the constant  $g_A$ . It was obtained in refs. (4-6, 9) that  $|g_A/g_v| = 0.6 - 0.8$  which changed while passing from one nucleus to another. The investigation of the  $\beta^+$  decay of  $^{147}\text{mDy}$  in ref. (20) gave  $|g_A/g_v| = 0.7 \pm 0.1$ . The comparison of the experimental data with more accurate calculations including the renormalisation of the vertex and the configuration mixing in the particle-particle channel allowed the authors of ref. (24) to get  $|g_A/g_v| = 0.7 - 1.0$ . A comprehensive comparison in ref. (25) of the experimentally observed GT  $\beta^+$  decays with the shell model calculations showed that in the middle of the  $sd$ -shell  $|g_A/g_v| = 0.95$ .

Our calculations of  $\log ft$ , tabulated in Table 1, have been performed for two cases  $|g_A/g_v| = 1$  and 1.25. To get the same  $\log ft$  in the calculations with  $|g_A/g_v| = 1.25$ , the constant is increased from  $G_1^{01} = 7.5/A$  MeV to 7.9/A MeV, apart from the decay of  $^{146}\text{Dy}$  for which  $G_1^{01} = 7.7/A$  MeV. It follows from the calculations with  $|g_A/g_v| = 1$  that from 30% up to 80% of the total GT strength  $S_+$  is concentrated on the low-lying states.

In the calculations with  $|g_A/g_v| = 1.25$ , the low-lying states contain from 20% to 77% of  $S_+$ . With increasing  $G_1^{01}$  from 7.5/A MeV by approximately 5%  $S_+$  decreases by 3% and a part of the GT strength is shifted to the high-lying states.

According to our calculations with the separable p-h and p-p interactions, a satisfactory description of the experimental values of  $\log ft$  with  $|g_A/g_v| = 1$  and 1.25 has been obtained. With the new parameter  $G_1^{01}$  we get a correct description of the experimental data with the renormalisation of the constant  $g_A$  and without it. Our calculations show that from the available experimental data on the  $\beta^+$  decays of neutron-deficit nuclei with  $A = 96$  and 146-152 one cannot conclude that the constant  $g_A$  is renormalised considerably. We think that the effective constant  $g_A$  should not change when passing from one nucleus to another. Note that in some calculations, for instance, in ref. (26) medium dependence of  $g_A$  turns out to be roughly 5%.

#### 5. Strength Functions of (n,p) Transitions

The Gamow-Teller resonance excited in (p,n) reactions is well studied experimentally. The model independent sum rule (1) contains the total strength function  $S_+$  related to the  $0^0$  (n,p) cross section and  $\beta^+$  decay.

Let us study now the influence of the p-h and p-p interactions on  $S_-$  and  $S_+$ . For the GT transitions we calculate the sums  $\sum_i B^{(+)}(i, i)$  depending on the upper limit of the energy  $\omega_i$ . The sums over all the states  $i$  are equal to  $S_+$ . The results of calculations in the RPA of  $\sum_i B^{(+)}(i, i)$  and  $\sum_i B^{(-)}(i, i)$  for  $^{90}\text{Zr}$  are shown in figs. 1 and 2. In the calculations we

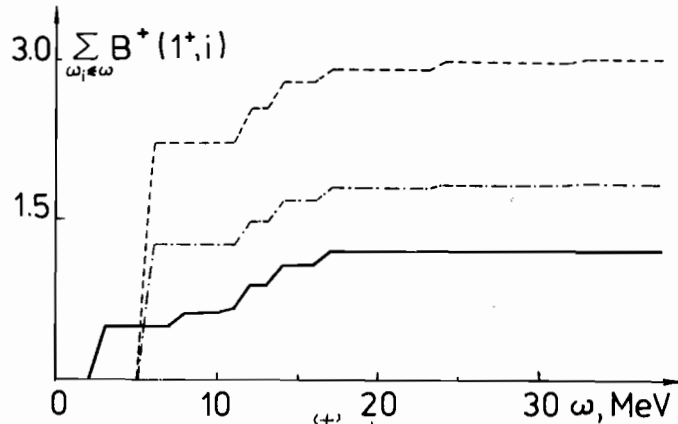


Fig. 1. Calculated sums  $\sum_i B^{(+)}(1^+, i)$  as a function of the upper limit of  $\omega_i$  for  $(n, \rho)$  transitions to  $^{90}\text{Zr}$ .

Notation: dashed curve at  $G_1^{01} = 0$  ;  
dash-dotted curve at  $G_1^{01} = -23/A, G_2^{01} = 0$  ;  
solid curve at  $G_1^{01} = -23/A, G_2^{01} = 7.5/A$ .

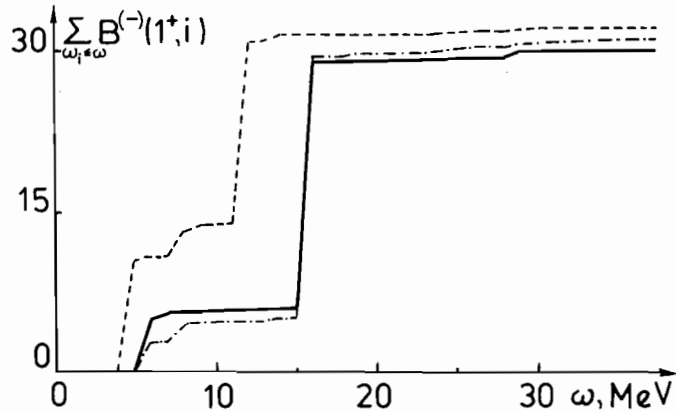


Fig. 2. Calculated sums  $\sum_i B^{(-)}(1^+, i)$  as a function of the upper limit of  $\omega_i$  for  $(\rho, n)$  transitions to  $^{90}\text{Zr}$ .

Notation: As in fig. 1.

used a truncated space of single-particle states; therefore, at  $x_1^{01} = G_1^{01} = 0$  we have  $S_- - S_+ = 29$  instead of 30. The inclusion of the p-h interaction with  $x_1^{01} = -23/A$  MeV diminishes  $S_+$  1.6 times. Inclusion of the p-p interaction with  $G_1^{01} = 7.5/A$  MeV shifts a part of the strength to the energy less than 5 MeV and decreases  $S_+$  1.5 times, i.e. the p-h and p-p interactions decrease  $S_+$  approximately 2.5 times. For  $\sum_i B^{(-)}(1^+, i)$  the p-h interactions shift maximum of the GT resonance from 11 to 15 MeV and increase the strength at maximum,  $S_-$  decreasing by 4%. The inclusion of the p-p interaction decreases  $S_-$  by 3% more. The p-p interaction strongly influences  $B^{(+)}(1^+, i)$  and  $S_+$  slightly influences  $B^{(-)}(1^+, i)$  and  $S_-$ .

Cross sections for  $^{54}\text{Fe}(n, p)^{54}\text{Mn}$  have been measured in ref.<sup>11)</sup> at small angles with the use of the charge-exchange facility at TRIUMF. This determination of the GT strength  $S_+$  allows a full test of the sum rule (1). Their measurements give  $S_+ = 3.8$ . In ref.<sup>27)</sup>  $S_- = 7.8$  has been obtained from the  $^{54}\text{Fe}(p, n)^{54}\text{Co}$  reaction. As a result,  $S_- - S_+ = 4.0 \pm 2.1$ . This is 67% of the sum rule value of 6, i.e. the quenching of the GT strength takes place. According to the calculations<sup>28)</sup>,  $S_+ = 9.1$  for a two-particle, four-hole (2p-4h) model. The calculations<sup>29)</sup> taking account of the 1p-1h configurations provide  $S_+ = 9.4$  and more refined calculations give  $S_+ = 7.4$ . However, this still overestimates twice the experimental value.

We have calculated the  $S_+$  for  $^{54}\text{Fe}$  at  $x_1^{01} = G_1^{01} = 0$  and  $x_1^{01} = -23/A$  MeV,  $G_1^{01} = 0$  and  $x_1^{01} = -23/A$  MeV and  $G_1^{01} = 7.5/A$  MeV. The results of calculations are shown in Table 4. The p-h RPA calculations of  $S_+$  overestimate the experimental value almost twice. With the p-p interactions included, the  $S_+$  becomes close to the experimental value  $S_+ = 3.8$  obtained in ref.<sup>11)</sup>. Note that

for nuclei with small difference  $N-Z$  the influence of the p-p interaction on  $S_+$  is greater than in nuclei with large difference.

Interest in the strength functions of (n,p) transitions (see ref.<sup>10</sup>) stimulated us to calculate  $S_+$  for  $^{140}\text{Ce}$ ,  $^{120}\text{Sn}$  and  $^{90}\text{Zr}$ , the fragmentation of the GT resonance (branch  $\sigma^+(\epsilon)$ ) for which has been calculated in ref.<sup>15</sup>). It is seen from Table 4 that the calculations with the p-p interaction lead to a 1.5 - 2.0 decrease of  $S_+$ . Since a truncated space of single-particle states has been used, the sum rule (1) is not exhausted completely. The inclusion of the p-p interactions, as is seen from Table 4, does not lead to stronger deviations

Table 4. Effect of the p-h and p-p interactions on  $S_-$  and  $S_+$

Target	$x_1^{01} A$	$G_1^{01} A$	$S_-$	$S_+$
$^{140}\text{Ce}$	0	0	73.4	2.7
	-23	0	72.2	1.1
	-23	7.5	71.0	0.5
$^{120}\text{Sn}$	0	0	60.3	2.7
	-23	0	59.2	1.5
	-23	7.5	58.0	0.8
$^{90}\text{Zr}$	0	0	32.1	3.0
	-23	0	30.8	1.9
	-23	7.5	29.9	1.2
$^{54}\text{Fe}$	0	0	16.4	10.3
	-23	0	12.3	6.6
	-23	7.5	10.3	4.2

from the sum rule (1) in comparison with  $G_1^{01}=0$ . It is seen from Table 4 that  $S_-$  is slightly influenced by the p-p interaction.

For nuclei far from stability, shown in Table 1, the p-p interactions taken into account diminish  $S_+$  2-4 times; the effect of the p-p interactions turned out to be stronger than in nuclei shown in Table 4, which confirms the conclusion on the relation between the neutron excess and influence of the p-p interactions.

## 6. Conclusion

The investigation within the QPNM has shown that the inclusion of the p-p interaction decreases the total GT strength

$S_+$  1.5 - 4.0 times. The p-p interactions influence considerably the integrated  $\log \tilde{f}t$  for the  $\beta^+$  decays of neutron-deficit nuclei. As a result, a good description of the experimental values of  $\log \tilde{f}t$  is obtained under the assumption that  $|g_A/g_V| = 1.0$  and 1.25.

The calculations, including the p-h and p-p interactions, of the  $S_+$  strength and  $\log \tilde{f}t$  for the  $\beta^+$  transitions at  $|g_A/g_V|=1$  have been made with fixed constants  $x_1^{01}=-23A/\text{MeV}$  and  $G_1^{01}=7.5/A$  MeV. The effect of the p-p interaction is global in nature; it is independent of the detailed properties of the low-lying states populated in the  $\beta^+$  decay.

The investigations have shown that the p-p interactions slightly influence the strength functions  $S_-$  manifesting themselves in the (p,n) reactions and  $\beta^-$  decay probabilities. The quenching of the GT resonance strength is not influenced by the p-p interactions.

Due to a strong effect of the p-p interaction on the  $\beta^+$  decay probabilities, one cannot conclude unambiguously that the constant  $g_A$  of the axial-vector weak interaction is renormalised in nuclei. It would be important to find the constant  $G_I^{01}$  of the GT p-p interaction from other, apart from the  $\beta^+$  decays, experimental data. Further experimental studies of the allowed and first forbidden  $\beta^+$  and  $\beta^-$  transitions are needed for clarifying the renormalisation of the constant  $g_A$  in atomic nuclei. More exact calculations of the energies and wave functions of the states between which the  $\beta^+$  transitions proceed are required as well.

The p-p as well as p-h interactions influence some nuclear characteristics. It is essential that the effect of the p-p interactions on many properties of spherical and deformed nuclei should further be studied.

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Описание гамов-теллеровских  $\beta^+$ -распадов и силовых функций (n, p) переходов в сферических ядрах

Исследовано влияние частично-частичного взаимодействия на вероятности гамов-теллеровских  $\beta^+$ -распадов и на силовые функции (n, p) переходов. Рассчитаны в RPA с сепарабельными частично-дырочными (p-h) и частично-частичными (p-p) взаимодействиями суммарные  $\log ft$ -величины для GT распадов  $^{152}\text{Yb}$ ,  $^{152,150}\text{Er}$ ,  $^{148,146}\text{Dy}$  и  $^{90}\text{Pb}$ . Получено хорошее согласие с экспериментальными данными с константами  $\kappa_1^{01} = 23/A$  МэВ и  $G_1^{01} = 7,5/A$  МэВ при  $|g_A/g_V| = 1$  и  $G_1^{01} = 7,9/A$  МэВ при  $|g_A/g_V| = 1,25$ . Показано, что на имеющихся экспериментальных данных по  $\beta^+$  распадам ядер, удаленных от полосы стабильности, нельзя сделать утверждение о сильной перенормировке в ядрах константы  $g_A$  аксиально-векторного слабого взаимодействия. Показано, что p-p взаимодействия оказывают сильное влияние на полную силу  $S_+(n, p)$  переходов путем уменьшения ее в  $1,5 \div 3,0$  раза. Для реакции  $^{54}\text{Fe}(n, p)^{54}\text{Mn}$  рассчитанное значение  $S_+ = 4,2$  согласуется с экспериментальным.

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Gamow-Teller  $\beta^+$  Decays and Strength Functions of (n, p) Transitions in Spherical Nuclei

The effect of the particle-particle interaction on the Gamow-Teller  $\beta^+$  decay and strength functions of (n, p) transitions is studied. The integrated  $\log ft$  values for the GT decays of  $^{152}\text{Yb}$ ,  $^{152,150}\text{Er}$ ,  $^{148,146}\text{Dy}$  and  $^{90}\text{Pb}$  are calculated in the RPA with separable particle-hole and particle-particle interactions. A good description of  $\log ft$  is obtained with the constants  $\kappa_1^{01} = 23/A$  MeV and  $G_1^{01} = 7,5/A$  MeV at  $|g_A/g_V| = 1$  and  $G_1^{01} = 7,9/A$  MeV at  $|g_A/g_V| = 1,25$ . From the available experimental data on  $\beta^+$  decay of nuclei far from stability one cannot conclude that the weak axial-vector constant  $g_A$  is strongly renormalised in nuclei. The particle-particle interaction is shown to influence strongly the total strength  $S_+$  of (n, p) transitions by diminishing it (1.5-3.0) times. For the reaction  $^{54}\text{Fe}(n, p)^{54}\text{Mn}$  at forward angles the calculated value  $S_+ = 4.2$  is in agreement with the experimental one.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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