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DYNAMIC TWO-CENTER PROBLEM: TRIPLE-COLLISION-LIMIT SOLUTION

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1. Introduction

As we have recently shown[1,2], the regularization of a three-body Born-Oppenheimer-like Hamiltonian[3] at the points of pair collisions leads to a special type of the hyperspherical three-body Hamiltonian. From this transformation it follows that an adiabatic hyperspherical description[4] is a proper variant of the Born-Oppenheimer adiabatic description[3]. There is a lot of numerical justifications of this assertion in the field of atomic[4,5], molecular[6] and mesomolecular physics[7]. The calculations of the $ee e^+$ -system are also very instructive in that sense [8,9].

The key problem in the Born-Oppenheimer adiabatic approach is the quantum-mechanical problem of motion of a light particle (muon or electron) in the field of two fixed centers. The distance between centers is a parameter. When this distance is either negligible or infinite, the solutions of the problem can be given in an analytic form[10]. This is an important point as it is helpful for a numerical analysis of the problem and also provides the classification scheme for quantum states and the so-called correlation diagram which puts into one to one correspondence the solutions from two limiting analytic sets.

After the above-mentioned transformation of the Born-Oppenheimer three-body Hamiltonian one should define the new adiabatic Hamiltonian (dynamic two-center problem). Practically it should be preceded by a partial wave analysis of the total Hamiltonian. This is an important physical feature of the new adiabatic description. Because of that fact the dynamic two-center problem depends on exact quantum numbers of the total angular momentum J and the total parity p (all particles are supposed to be different and spinless). As a result, an analytical analysis of the new adiabatic problem is also Jp-dependent. This is the price one should pay for quality of the new basis. Let us remind that in the original adiabatic method one should use all adiabatic states to restore the exact quantum numbers of the total system.

It was already mentioned that the purpose of the transformation of the Born-Oppenheimer operator/1,2/ was the regularization of its properties in the regions of pair collisions. This transformation produced the change of the adiabatic (slow) variable. The distance between centers (heavy particles) was changed to the hyperradius, which is just known to be the proper variable to treat the triple-collision Fock's singularity. This indicates that the dynamic two-center problem that includes the hyperradius as a parameter is much more natural to account for the proper dynamic description of the total system in that important region.

This paper is aimed at analytic analysis of solutions of the dynamic two-center problem in the limit of the vanishing hyperradius. The solutions should be, of course, closely connected with hyperspherical harmonics/11/. Their particular form, derived in this way, is much more general and easy to use than those known from the literature/12-16/. It includes only well-known polynomials.

In the next section we briefly review the problem of the choice of proper hyperspherical angles. In sect. 3, the Hamiltonian of a three-body system and that for the dynamic two-center problem in the chosen coordinates are given. In sect. 4, analytical solutions of the latter problem in the limit of the vanishing hyperradius are derived. The way to produce the correlation diagram is briefly discussed in the fifth section, which also includes the formula for the degeneracy of the Jp-hyperspherical harmonics. The solutions alternative to those from section 4 are given in the Appendix A.

2. The choice of variables

We start with "molecular" Jacobi coordinates $\vec{X}$ and $\vec{x}$, Fig. 1, for the system of two "heavy" particles a, b and one "light" particle c. The $dt\mu^-$-ion can be taken as a sample system. The hyperradius R for that system is defined by

$$R^2 = X^2 + \frac{m}{M} x^2, \tag{1}$$

where M and m are reduced masses of the systems (a, b) and ($a+b$, c)

$$1/M = 1/m_a + 1/m_b\,; \quad 1/m = 1/m_c + 1/(m_a + m_b) \tag{2}$$

so that $\vec{X}$ is the position vector of b relative to a, and $\vec{x}$ is

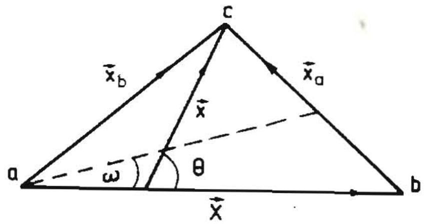

Fig. 1. Type-c Jacobi coordinates. The position of the principal axis of the inertia tensor is given.

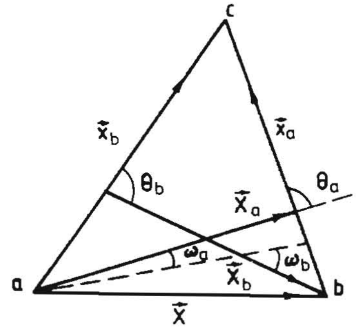

Fig. 2. Type-a and -b Jacobi coordinates. The direction of $\vec{X}_a$ is opposite to the usual one. Angles between the principal axis of the inertia tensor and the related Jacobi vectors $\vec{X}_a$ and $\vec{X}_b$ are given.

that of c with respect to the center of mass of ($a+b$). The Hamiltonian for our system is given by

$$H = -\frac{1}{2M}\frac{1}{R^5}\frac{\partial}{\partial R}R^5\frac{\partial}{\partial R} + \tilde{h}(\hat{O};R) \tag{3}$$

$$\tilde{h} = \hat{t}/2MR^2 + \hat{c}/R \tag{4}$$

$$\hat{c}/R = 1/X - 1/x_a - 1/x_b \tag{5}$$

Here $\tilde{h}$ is the adiabatic hyperspherical Hamiltonian of Macek/4/ which includes R as a parameter, and $\hat{O}$ represents five dimensionless variables. The eigenfunctions $\varphi_n(\hat{O};R)$ and eigenstates $\varepsilon_n(R)$ of the operator $\tilde{h}$ are obtained by solving the Schroedinger equation

$$[\tilde{h} - \varepsilon_n(R)]\varphi_n(\hat{O};R) = 0. \tag{6}$$

The most commonly used set of variables $\hat{O}$ is $\{\alpha, \hat{x}, \hat{X}\}$, where $\hat{x}$ and $\hat{X}$ are polar and azimuthal angles of the vectors $\vec{x}$ and $\vec{X}$, respectively, and

$$\alpha = \arctan(\sqrt{M}X/\sqrt{m}\,x). \tag{7}$$

In this case

$$\hat{t} = -(\sin\alpha\cos\alpha)^{-1}\frac{d}{d\alpha}(\sin\alpha\cos\alpha)\frac{d}{d\alpha} + \frac{\vec{l}^{\,2}}{\cos^2\alpha} + \frac{\vec{L}^{\,2}}{\sin^2\alpha}, \qquad (8)$$

where $\vec{l} = -i\vec{x}\cdot\nabla_{\vec{x}}$, $\vec{L} = -i\vec{X}\cdot\nabla_{\vec{X}}$ and the volume element dv is

$$dv = R^5\, dR\, d\hat{O}$$
$$d\hat{O} = (\sin\alpha\cos\alpha)^2 d\alpha\, d\hat{x}\, d\hat{X}. \qquad (9)$$

The inner product for the solutions of the eigenproblem (6) is defined by

$$(\varphi_i|\varphi_j) = \int d\hat{O}\, \varphi_i^* \varphi_j . \qquad (10)$$

As it explicitly follows from the operator form (8), $\sin\alpha = 0$ and $\cos\alpha = 0$ define singular points of the problem (6). It should be noted that they are pure mathematical singularities that have appeared due to the choice of the independent variables. This artificial complication of the problem can be easily eliminated by introducing the hyperspheroidal coordinates[1]

$$\xi = \frac{x_a + x_b}{X}, \quad \eta = \frac{x_b - x_a}{X}. \qquad (11)$$

The explicit form of the total Hamiltonian (5) with "hyperspherical angles" ξ and η used as independent variables was given in[1,2]. As ξ, η are internal coordinates, the resulting coordinate frame is a body fixed one. It is well known that the Coriolis coupling term appears as a part of the total Hamiltonian in that case. We have demonstrated earlier[2] that to make this coupling vanish in the regions of pair collisions, one should set $\hat{O} = \{\xi, \eta, \alpha, \beta, \gamma\}$, where α, β, γ define the Euler rotation specifying the body-fixed frame with its unit vectors to coincide with the principal axes of the inertia tensor of a three-body system. The use of this frame is rather familiar[16] in studying three-particle systems. An important feature of that choice in our case[1,2] is that it appeared to be the only possibility after we had demanded that the resulting Hamiltonian should have a proper behaviour in the vicinity of the pair-collision singularities.

Now all six coordinates needed for studying a three-particle system in the center of mass frame are fixed. If our reasoning is valid, we may believe them to be the best available. They are adequate to treat the physical singularities and are convenient to allow

for the symmetry properties of the problem. They are also very satisfactory from the point of view of mathematics as the kinetic energy operator is of a simple form in this case and the singular points of the Schroedinger operator are just singular points of the physical problem.

3. The Hamiltonian

With the hyperradius R defined by (1) the hyperspheroidal coordinates ξ and η given by (11) and Euler angles α, β, γ defining the body-fixed frame with its unit vectors chosen to coincide with the principal axes of the invertia tensor of the system, the total three-body Hamiltonian can be given in the form[2]

$$H = h - \frac{1}{2M}\left(\frac{\partial^2}{\partial R^2} + \frac{5}{R}\frac{\partial}{\partial R}\right) - \frac{3}{2MR^2}. \qquad (12)$$

The operator

$$h = h_0 + T_R + \frac{1}{2MR^2}\cdot\frac{1}{1-\Delta}\left\{\Delta\cdot J_1^2 + iJ_1\left[4\mathcal{L}_1 + 2\rho\Delta\left(\mathcal{L} + \frac{\xi\eta - x}{2s}\right)\right]\right\} \qquad (13)$$

with

$$\mathcal{L} = \frac{s}{\xi^2-\eta^2}\left[(\eta - x\xi)\frac{\partial}{\partial\xi} - (\xi - x\eta)\frac{\partial}{\partial\eta}\right]$$
$$\mathcal{L}_1 = \frac{s}{\xi^2-\eta^2}\left[\eta(\xi^2-1)\frac{\partial}{\partial\xi} + \xi(1-\eta^2)\frac{\partial}{\partial\eta}\right] \qquad (14)$$

should be referred to as a rotational dynamic two-center Hamiltonian. It contains the operator

$$T_R = \frac{1}{2}\left(\frac{J_1^2}{I_1} + \frac{J_2^2}{I_2} + \frac{J_3^2}{I_3}\right) \qquad (15)$$

which is just the Hamiltonian of an asymmetric top with the classical expressions for the principal inertia moments

$$I_1 = I_2 + I_3 = MR^2;\quad I_2 = \frac{1}{2}MR^2\left(1 + \sqrt{1-\Delta}\right) \qquad (16)$$

and J_i are projections of the total angular momentum $\vec{J}$ in the body-fixed frame. When $J = 0$, h coincides with

$$h_0 = -\frac{2}{m}\rho^2\frac{\hat{a}_{\xi\eta}}{R^2} + V. \qquad (17)$$

The differential operator $\hat{a}_{\xi\eta}$ is given by

$$\hat{a}_{\xi\eta} = \frac{1}{\xi^2-\eta^2}\left[\frac{\partial}{\partial\xi}(\xi^2-1)\frac{\partial}{\partial\xi} + \frac{\partial}{\partial\eta}(1-\eta^2)\frac{\partial}{\partial\eta}\right]. \qquad (18)$$

All other quantities appearing in the definition of H are functions of coordinates and Jacobi masses:

$$\varkappa = (m_\beta - m_\alpha)/(m_\beta + m_\alpha); \quad \tilde{\alpha} = m/4M$$

$$\rho = 1 + \tilde{\alpha}(\xi^2 + \eta^2 - 2\varkappa\xi\eta + \varkappa^2 - 1)$$

$$s = [(\xi^2-1)(1-\eta^2)]^{1/2} \qquad (19)$$

$$\Delta = 4\tilde{\alpha}\frac{s^2}{\rho^2}$$

The volume element is now

$$dv = R^5 dR \, \frac{\xi^2-\eta^2}{\rho^2} \, d\xi \, d\eta \, \sin\beta \, d\alpha \, d\beta \, d\gamma. \qquad (20)$$

We look for a physical solution of the Schroedinger equation

$$H\Psi = E\Psi \qquad (21)$$

with the well-defined total angular momentum J and total parity p. This is supplied with the partial-wave representation of the wave function Ψ in the form[2]

$$\Psi^{JpM_J}(\vec{X},\vec{x}) = \sum_{m=0}^{J} B_m^{JpM_J}(\alpha,\beta,\gamma)\, \Phi_m^{Jp}(R,\xi,\eta). \qquad (22)$$

The angular part of the wave function has the form

$$B_m^{JpM_J}(\alpha,\beta,\gamma) = \frac{(-i)^m}{4\pi}\left(\frac{2J+1}{1+\delta_{0m}}\right)^{1/2}\left[\mathcal{D}^J_{-m-M_J}(\gamma,\beta,\alpha) + p(-1)^J\mathcal{D}^J_{m-M_J}(\gamma,\beta,\alpha)\right]. \qquad (23)$$

It contains the Wigner $\mathcal{D}$-functions as defined in [17] with $|M_J| \le J$ and $m \le J$ being the projections of J onto the space and body-fixed z-axis, respectively. The projection of the Schroedinger equation (21) onto the states (23) leads to the system of $J+1$ (for normal parity states, i.e. when $p=(-1)^J$, or J when $p=-(-1)^J$). The resulting Schroedinger equation with the matrix Hamiltonian

$$\mathbb{H}^{Jp} = \mathbb{h}^{Jp} - \frac{1}{2M}\left(\frac{\partial^2}{\partial R^2} + \frac{5}{R}\frac{\partial}{\partial R}\right) - \frac{3}{2MR^2} \qquad (24)$$

includes the matrix operator of the dynamic two-center problem for rotational states $\mathbb{h}^{Jp}$; this is just the operator (13) averaged over the angular states (23). The operator of an asymmetric rotator T_R couples the states (23) for $m' = m \pm 2$ and the Coriolis-type operator from (13) includes also $m' = m \pm 1$ coupling.

In the adiabatic picture the components of the vector-column solutions of the projected Schroedinger equation

$$(\mathbb{H}^{Jp} - E^{Jp})\,\Phi^{Jp}(R,\xi,\eta) = 0 \qquad (25)$$

are searched in the form

$$\Phi_m^{Jp}(R,\xi,\eta) = \varphi_m^{Jp}(\xi,\eta;R)\,\chi_m^{Jp}(R), \qquad (26)$$

where the components φ_m^{Jp} are solutions of the Jp-projected dynamic two-center problem with $\mathbb{h}^{Jp}$ from (24)

$$[\mathbb{h}^{Jp} - \varepsilon^{Jp}(R)]\,\varphi^{Jp} = 0. \qquad (27)$$

In this equation the hyperradius R is included as a parameter. In the next section we shall derive the solution of the Schroedinger equation (27) in the limit of $R \to 0$.

4. Triple-collision-limit solution for the hyperspherical adiabatic states

When $R \to 0$, the kinetic energy operator of a three-body Hamiltonian behaves like $1/R^2$ in comparison with $1/R$ behaviour of the potential energy operator for the Coulomb interaction. So, as it is well known, the hyperspherical adiabatic eigenfunctions can be given in that limit in terms of hyperspherical harmonics[11]. In a general case they depend on five variables and there exist several popular choices of that set. The simplest analytical form of hyperspherical harmonics is obtained when laboratory-frame coordinates connected with any possible pair of Jacobi vectors are used. We have already introduced a set of that sort, $\hat{O} = \{\alpha, \hat{x}, \hat{X}\}$, in **Sect.2** of this paper. In that case the harmonics are the solutions of the eigenvalue equation

$$[\hat{t} - K(K+4)]\, \mathcal{Y}^{JpM_J}_{K\ell L(c)}(\alpha, \hat{x}, \hat{X}) = 0. \tag{28}$$

where $\hat{t}$ is given by the expression (8). The quantum numbers K and ℓ, L are those of grand and pair angular momenta and the letter C in **parentheses** means that the C -set (Fig. 1) of Jacobi coordinates is used. For $\mathcal{Y}^{JpM_J}_{K\ell L(c)}(\alpha, \hat{x}, \hat{X})$ we have up to the normalization factor /11/

$$\mathcal{Y}^{JpM_J}_{K\ell L(c)}(\alpha, \hat{x}, \hat{X}) = f_{K\ell L}(\alpha)\, \mathcal{Y}^{JpM_J}_{\ell L}(\hat{x}, \hat{X}), \tag{29}$$

where $f_{K\ell L}(\alpha)$ is the Jacobi polynomial

$$f_{K\ell L}(\alpha) = (\cos\alpha)^{\ell} (\sin\alpha)^{L} F(-n, (K+\ell+L+4)/2, L+3/2; \sin^2\alpha), \tag{30}$$

$n = (K-L-\ell)/2$ should be non-negative and

$$\mathcal{Y}^{JpM_J}_{\ell L} = \sum_{mm'} Y_{\ell m}(\hat{x}) Y_{Lm'}(\hat{X}) (\ell L m m' | J M_J)[1 + p(-1)^{\ell+L}]. \tag{31}$$

The Clebsh-Gordan coefficients appear in this expression and the last factor accounts for the proper parity of the solution. As we have chosen the other set of hyperspherical angles, namely $\hat{O} = \{\xi, \eta, \alpha, \beta, \gamma\}$, we should transform the expression (29) properly. This transformation includes the rotation of the coordinate system with Euler angles α, β, γ and the substitution into the final expression of the ξ, η - combinations for all the functions of the internal coordinates. The procedure is straightforward, the only important step will be explained after presenting the final expression which reads

$$\mathcal{Y}^{JpM_J}_{K\ell L(c)}(\xi, \eta, \alpha, \beta, \gamma) = \sum_{m'=0}^{J} \Psi^{Jp}_{\ell L m'(c)}(\xi, \eta)\, B^{JpM_J}_{m'}(\alpha, \beta, \gamma), \tag{32}$$

where

$$\Psi^{Jp}_{\ell L m'(c)}(\xi, \eta) = (-1)^{\ell} f_{K\ell L}(\alpha) \sum_{m} W^{Jp\ell}_{mL} \cdot P_{\ell m}(\theta) \cdot d^{Jp}_{mm'}(\omega) \tag{33}$$

with the coefficients

$$W^{Jp\ell}_{mL} = p(-1)^{J+\ell+m} \sqrt{2-\delta_{0m}}\, (\ell J -m\, m | L 0). \tag{34}$$

Apart from the Jacobi polynomials $f_{K\ell L}(\alpha)$ given by (3) the form (32) includes the associated Legendre polynomials of $\cos\theta = (\hat{x} \cdot \hat{X})$

$$P_{\ell m}(\theta) = \sqrt{\frac{2\ell+1}{2} \frac{(\ell-m)!}{(\ell+m)!}}\, \frac{1}{2^{\ell} \ell!} (-\sin\theta)^{m} \frac{d^{\ell+m}}{d(\cos\theta)^{\ell+m}} (\cos^2\theta - 1)^{\ell} \tag{35}$$

and the combination of the Rose d -functions $d^{J}_{mm'}$ from $d^{Jp}_{mm'}(\omega)$ in the form

$$d^{Jp}_{mm'}(\omega) = \frac{(-1)^{m}}{\sqrt{(1+\delta_{0m})(1+\delta_{0m'})}} \{ d^{J}_{mm'}(-\omega) + p(-1)^{J+m'} d^{J}_{m-m'}(-\omega) \}. \tag{36}$$

Three scalar combinations of hyperspheroidal coordinates ξ and η appear in (29). They are given by

$$\sin\alpha = 1/\sqrt{\rho}$$
$$\sin\theta = \sqrt{\tilde{\alpha}}\, \xi / (\rho - 1) \tag{37}$$
$$\sin 2\omega = \frac{2\tilde{\alpha}\, \rho (\xi\eta - x)\, \xi}{\sqrt{1-\Delta}}.$$

Two of them (α and θ) are frequently used in the construction of the hyperspherical harmonics. The success of the present derivation is due to the parametrization of the (α, β, γ) rotation in terms of ω (Fig. 1) that is the angle between vector $\vec{X}$, connecting "centers" a and b , and the nearest principal axis of the inertia tensor of a three-body system.

The direct comparison of the rotated harmonics (29) and the adiabatic anzats for the total three-body eigenfunction (26) shows that $\Psi^{Jp}_{\ell L m'(c)}(\xi, \eta)$ are the solutions of the dynamic two-center problem in the vicinity of the point of triple collision.

5. Correlation diagrams

The eigenvalues $\varepsilon^{Jp}_{n}(R)$ of the dynamic two-center problem (27), i.e. hyperspherical adiabatic potential curves, reproduce in the limit of $R \to \infty$ the spectra of $a+c$ and $b+c$ subsystems. In the limit of $R \to 0$ the spectrum of the problem (27) is defined by the quantum number K as it follows from (28) and (4). Different $\mathcal{E}_n^{Jp}(R)$ are not allowed to cross for finite R because of non-degeneracy of the problem. So, the lowest value of the spectrum of (27) for $R\to\infty$ corresponds to the lowest possible value of K which depends on the Jp -pair. If the degeneracy of the spectra of clusters $a+c$ and $b+c$ is known and if the degeneracy of the spectrum of the same problem in the limit of $R\to 0$ can be found, we can determine the value of K corresponding to any given energy in the limit of $R\to\infty$. The corresponding scheme is usually referred to as a correlation diagram. Examples of diagrams like that for $\{Jp\}=\{01\}$ and $\{Jp\}=\{11\}$ hyperspherical adiabatic states of the $dt\mu^-$-ion are presented in our paper[18]. They can be easily constructed for any state, if a general expression for the degeneracy $N(K,J,p)$ of $\mathcal{E}^{Jp}(R)$ in the limit of $R\to 0$ is derived. We complete the section with this result:

$$N(K,J,p)=\frac{1}{2}\left(K-J+1+\frac{1+p(-1)^J}{2}\right)\left(J+\frac{1+p(-1)^J}{2}\right). \tag{38}$$

6. Summary

Now the hyperspherical adiabatic method is widely used both for the approximate semianalytic analysis[18,19] and calculation of a rather different three-body systems[20-23]. In this paper we have used the earlier proposed set of hyperspheroidal coordinates and derived the eigenfunctions of the relevant adiabatic Schroedinger equation in the vicinity of the point of triple collision. The solution is, as it should be, a special form of the rotated hyperspherical harmonics. Its final expression, when compared with similar forms known from the literature[12-16], appears to be the most general and easy to construct. Though the original purpose of this paper was to prepare the trial functions for further semianalytic analysis of the adiabatic problem (27) along the lines reported earlier[18], we hope that the derived form for rotated hyperspherical harmonics can also be useful for other investigations[20-23].

Appendix A

The analytic form (29) for space-fixed hyperspherical harmonics and its body-fixed partner (32) were defined for the c-type Jacobi coordinates, Fig. 1. For two other possible choices of the Jacobi vectors, Fig. 2, the relevant space-fixed harmonics are given by the same analytic expression but with different arguments and indices:

$$\mathcal{Y}^{JpM_J}_{KL_al_a}(\alpha_a,\hat{x}_a,\hat{X}_a)=f_{KL_al_a}(\alpha_a)\,\mathcal{Y}^{JpM_J}_{L_al_a}(\hat{x}_a,\hat{X}_a) \tag{29a}$$

$$\mathcal{Y}^{JpM_J}_{KL_bl_b}(\alpha_\beta,\hat{x}_b\hat{X}_b)=f_{KL_bl_b}(\alpha_\beta)\,\mathcal{Y}^{JpM_J}_{L_bl_b}(\hat{x}_b,\hat{X}_b) \tag{29b}$$

with

$$tg\,\alpha_a=\sqrt{M_a}\,X_a/\sqrt{m_a}\,x_a \tag{7a}$$

$$tg\,\alpha_b=\sqrt{M_b}\,X_b/\sqrt{m_b}\,x_b \tag{7b}$$

and[1,2]

$$M_a=Mp_a,\quad m_a=m/p_a;\ p_a=1+\tilde{\alpha}(1-\varkappa)^2 \tag{2a}$$

$$M_b=Mp_b,\quad m_b=m/p_b;\ p_b=1+\tilde{\alpha}(1+\varkappa)^2. \tag{2b}$$

The corresponding rotated partners will be $\mathcal{Y}^{JpM_J}_{KL_al_a}(\xi,\eta,\alpha,\beta,\gamma)$ and $\mathcal{Y}^{JpM_J}_{KL_bl_b}(\xi,\eta,\alpha,\beta,\gamma)$ by analogy with the expression (32) after the above-mentioned change of the arguments of the polynomials and indices in (32)-(36). The physical (or mathematical) meaning of all of them is clear from comparison with the c-case and from Fig. 2. We just give their exact ξ,η -form for the completeness

$$\begin{aligned} \sin\alpha_a &= \sqrt{\frac{\tilde{\alpha}}{\rho p_a}}(\xi-\eta)\\ \sin\theta_a &= \sqrt{\frac{p_a}{p}}\ \frac{S}{\xi-\eta}\ \frac{1}{\cos\alpha_a}\\ tg(2\omega_a) &= \frac{\sin 2\theta_a}{\cos 2\theta_a+ctg^2\alpha_a} \end{aligned} \tag{37a}$$

In order to change from a- to b-case, the sign of η and $\varkappa$ should be changed. Three different sets of the rotated hyperspherical harmonics (29), (29a) and (29b) are interconnected by the Raynal-Revai transformations[11]. One more known orthogonal transformation used in this paper is that of Chang and Fano[24], given by (33). With that remark one can consider the total matrix transformation from (34) as a generalisation of the above-mentioned Chang-Fano transformation.

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Динамическая задача двух центров:
решение в точке тройного удара

Получено решение динамической задачи двух центров /адиабатического гиперсферического гамильтониана/ в окрестности точки тройного удара. Оно является специальной формой гиперсферических гармоник для задачи трех тел с точными квантовыми числами полного момента, полной четности и парных якобиевских угловых моментов. Эта форма является наиболее общей и простой из имеющихся в литературе.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Dynamic Two-Center Problem:
Triple-Collision-Limit Solution

We have derived analytic solutions of the dynamic two-center problem (adiabatic hyperspherical hamiltonian) in the vicinity of the triple-collision point. They are rotated hyperspherical harmonics with quantum numbers of the total angular momentum, parity and pair Jacobi angular momenta. Their particular form is both general and simplest available.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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