

ОБЪЕДИНЕННЫЙ  
ИНСТИТУТ  
ЯДЕРНЫХ  
ИССЛЕДОВАНИЙ  
ДУБНА

5 70

E4-88-737

**V.G.Soloviev**

**QUASIPARTICLE-PHONON NUCLEAR MODEL  
EQUATIONS  
WITH EFFECTIVE SEPARABLE  
INTERACTIONS  
OF A FINITE RANK**

Submitted to "Ядерная физика"

**1988**

## INTRODUCTION

In most of the calculations within the quasiparticle-phonon nuclear model (QPNM)<sup>1-4/</sup> the use was made of the first rank separable multipole and spin-multipole particle-hole interactions between quasiparticles. The QPNM allows for the tensor forces<sup>4/</sup> and particle-particle interactions<sup>3-5/</sup>. The mathematical formalism of the QPNM is based on separable interactions between quasiparticles.

As is known, nucleon-nucleon potentials can be represented as separable ones. Thus, separable representations are constructed for the Reid potential<sup>6/</sup>, for the Paris<sup>7/</sup> and Bonn<sup>8/</sup> potentials. Separable representations of rank  $n_{\max} \leq 5$  provide a satisfactory approximation for these potentials.

In paper<sup>9/</sup> the standard Paris potential has been compared with its separable representation for nuclear matter and it was concluded that the separable representation of the Paris potential is sufficient for nuclear matter calculations within the Brueckner scheme. It has been asserted in ref.<sup>10/</sup> that the first rank separable interactions written in a proper way are equivalent to the density-dependent stationary Skyrme forces in the random phase approximation (RPA). It should be noted that in calculating the properties of nuclei and nuclear reactions, formulae include matrix elements of effective interactions between single-particle states. The results of calculations of nuclear properties are less sensitive to the radial dependence of forces in comparison with the calculations of two- or few-nucleon systems where the use is made of separable representations of nucleon-nucleon potentials. Therefore, we can conclude that the use of separable interactions of a finite rank in calculating characteristics of complex nuclei is justified. Efficiency of separable interactions in nuclei is to some extent due to the Hartree-Fock-Bogolubov approximation which plays a key role in solving a nuclear many-body problem.

Calculations of the structure of complex nuclei in the QPNM or other microscopic models are usually made with pairing and particle-hole interactions. Sometimes, particle-particle interactions are essential. Thus, the role of particle-

particle interaction is important in describing first  $2_1^+$  and  $3_1^-$  states in spherical nuclei<sup>5,11/</sup>, double  $\beta$  decay<sup>12/</sup> Gamow-Teller  $\beta^+$  decays of spherical<sup>13,14/</sup> and deformed<sup>15/</sup> nuclei and strength functions of the Gamow-Teller (n,p) transitions<sup>14/</sup>. Therefore, general QPNM equations are to be derived for particle-hole and particle-particle interactions.

In the present paper we present a general formulation of the QPNM for the finite rank separable isoscalar and isovector multipole and spin-multipole and isovector tensor particle-hole and particle-particle interactions.

## 1. THE MODEL HAMILTONIAN

The QPNM Hamiltonian includes a mean field of neutron and proton systems as the Saxon-Woods potential, superconducting pairing interactions and effective interactions between quasiparticles. Effective interactions include isoscalar and isovector multipole and spin-multipole and isovector tensor particle-hole and particle-particle interactions. They also include charge-exchange interactions. Earlier, effective interactions were used in a simple separable form ( $n_{\max} = 1$ ). The constants are found from the corresponding experimental data. In this procedure equations neglected in the Hartree-Fock-Bogolubov approximation are partially taken into account. The constants of effective interactions depend on the number of single-particle states included.

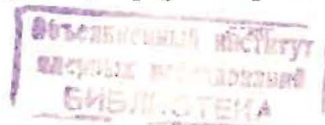
We introduce separable interactions of a finite rank. Consider, for example, the central spin-independent interaction  $V(|\vec{r}_1 - \vec{r}_2|) + (\vec{r}_1^{(1)} \vec{r}_2^{(2)}) V_r(|\vec{r}_1 - \vec{r}_2|)$ .

We expand it over multipoles and write it in the second quantized form

$$\sum_{2'1'} \langle 2'1' | \sum_{\lambda} [R^{\lambda}(r_1 r_2) + (\vec{r}_1^{(1)} \vec{r}_2^{(2)}) R_r^{\lambda}(r_1 r_2)] \frac{4\pi}{2\lambda + 1} \sum_{\mu=-\lambda}^{\lambda} \times \\ \times (-)^{\mu} Y_{\lambda\mu}(1) Y_{\lambda-\mu}(2) | 2'1' \rangle a_1^+ a_2^+ a_2 a_1.$$

It has been suggested in refs.<sup>1-4/</sup> that for the particle-hole interaction

$$R^{\lambda}(r_1 r_2) = \kappa_0^{\lambda} R^{\lambda}(r_1) R^{\lambda}(r_2), \quad R_r^{\lambda}(r_1 r_2) = \kappa_1^{\lambda} R^{\lambda}(r_1) R^{\lambda}(r_2),$$



where  $R^\lambda(r) = r^\lambda$  or  $R^\lambda(r) = \partial/\partial r V(r)$ , where  $V(r)$  is the central part of the Saxon-Woods potential. If one uses a separable interaction of the rank  $n_{\max}$  in the form

$$R^\lambda(r_1 r_2) = \kappa^\lambda \sum_{n=1}^{n_{\max}} R_n^\lambda(r_1) R_n^\lambda(r_2), \quad (1)$$

for the particle-hole interaction and in the form

$$R^\lambda(r_1 r_2) = G^\lambda \sum_{n=1}^{n_{\max}} \tilde{R}_n^\lambda(r_1) \tilde{R}_n^\lambda(r_2) \quad (1')$$

for the particle-particle interaction, expansion over multipoles takes the form

$$\begin{aligned} & \sum_{\lambda\mu} \sum_{n=1}^{n_{\max}} \left\{ \sum_{\tau, \rho=\pm 1} (\kappa_0^\lambda + \rho\kappa_1^\lambda) M_{\lambda\mu n}^+(\tau) \tilde{M}_{\lambda\mu n}(\rho\tau) + \sum_{\tau} (G_0^\lambda + G_1^\lambda) \times \right. \\ & \times \tilde{P}_{\lambda\mu n}^+(\tau) \tilde{P}_{\lambda\mu n}(\tau) + \kappa_1^\lambda (M_{\lambda\mu n}^{\text{CH}})^+ M_{\lambda\mu n}^{\text{CH}} + G_1^\lambda (P_{\lambda\mu n}^{\text{CH}})^+ P_{\lambda\mu n}^{\text{CH}} \\ & \left. + \sum_{\tau} G_1^\lambda \tilde{P}_{\lambda\mu n}^+(\tau) \tilde{P}_{\lambda\mu n}(-\tau) \right\}. \end{aligned}$$

The first term allow for the particle-hole (p-h) interaction, the second term allows for the particle-particle (p-p) interaction, the third and fourth allow for the charge-exchange (p-h) and (p-p) interactions and the last for the two-nucleon exchange. Here  $\kappa_0^\lambda$  and  $\kappa_1^\lambda$  are the isoscalar and isovector constants of the p-h interaction of multipolarity  $\lambda$ ;  $G_0^\lambda$  and  $G_1^\lambda$  are the p-p interaction constants,  $\tau=p$  for protons and  $\tau=n$  for neutrons. Introduction of a separable interaction of a finite rank  $n_{\max}$  in comparison with  $n_{\max} = 1$  results in an additional summation over  $n$ . Introduction of a separable interaction of a rank  $n_{\max}$  is justified if  $n_{\max}$  is much less than the rank of the determinant of the RPA secular equation for a nonseparable interaction. Note that a simple separable interaction

$$R^\lambda(r_1 r_2) = \kappa^\lambda R^\lambda(r_1) R^\lambda(r_2),$$

at  $R^\lambda(r) = \sum_n R_n^\lambda(r)$  has large freedom in describing the nuclear structure. The functions  $R_n^\lambda(r)$  are constructed from physical considerations and corresponding experimental data.

Consider excited states of doubly even spherical nuclei. Then, transform the Hamiltonian consisting of the Saxon-Woods potential, pairing and effective interactions between quasiparticles. For this purpose we perform the canonical Bogolubov transformation

$$a_{jm} = u_j a_{jm} + (-)^{j-m} v_j a_{j-m}^+$$

and introduce the operators

$$A^+(jj'; \lambda\mu) = \sum_{mm'} \langle jm j'm' | \lambda\mu \rangle a_{jm}^+ a_{j'm'}^+,$$

$$B(jj'; \lambda\mu) = \sum_{mm'} (-)^{j'-m'} \langle jm j'm' | \lambda\mu \rangle a_{jm}^+ a_{j'-m'},$$

as well as the phonon creation operator

$$Q_{\lambda\mu 1}^+ = \frac{1}{2} \sum_{jj'} \{ \psi_{jj'}^{\lambda 1} A^+(jj'; \lambda\mu) - (-)^{\lambda-\mu} \phi_{jj'}^{\lambda 1} A(jj'; \lambda-\mu) \}. \quad (2)$$

Single-particle states are characterised by quantum numbers  $jm$ ,  $i=1,2,\dots$  are the roots of the RPA secular equation.

Perform the same transformations as in refs.<sup>1,3,4</sup> and represent the QPNM Hamiltonian as

$$H = \sum_{jm} \epsilon_j a_{jm}^+ a_{jm} + H_v + H_{vq}, \quad (3)$$

where  $\epsilon_j$  is the quasiparticle energy on the subshell  $j$ . Here

$$H_v = H_{E_v} + H_{s_v}, \quad (4)$$

$$H_{E_v} = \sum_{\substack{ii' \\ \lambda\mu}} W_{E_{ii'}}^\lambda Q_{\lambda\mu i}^+ Q_{\lambda\mu i'}, \quad (4')$$

$$\begin{aligned} W_{E_{ii'}}^\lambda = & -\frac{1}{4} \frac{1}{2\lambda+1} \sum_{nr} \left\{ \sum_{\rho=\pm 1} [(\kappa_0^\lambda + \rho\kappa_1^\lambda) D_{nr}^{\lambda i} D_{n\rho r}^{\lambda i'} + \right. \\ & \left. + (\kappa_0^{\lambda\lambda} + \rho\kappa_1^{\lambda\lambda}) D_{nr}^{\lambda\lambda i} D_{n\rho r}^{\lambda\lambda i'} \right\} + \end{aligned} \quad (4'')$$

$$+ G_r^\lambda [D_{nr}^{\lambda i-} D_{nr}^{\lambda i'+} + D_{nr}^{\lambda i+} D_{nr}^{\lambda i'-}] + G_r^{\lambda\lambda} [D_{nr}^{\lambda\lambda i-} D_{nr}^{\lambda\lambda i'+} + D_{nr}^{\lambda\lambda i+} D_{nr}^{\lambda\lambda i'-}] \},$$

$$H_{sv} = \sum_{LMi} W_{sil}^L Q_{L Mi}^+ Q_{L Mi} \quad (5)$$

$$W_{sil}^L = -\frac{1}{4} \frac{1}{2L+1} \sum_{nr} \left\{ \sum_{\lambda=L\pm 1} (\kappa_0^{\lambda L} + \rho \kappa_1^{\lambda L}) D_{nr}^{\lambda Li} D_{nr}^{\lambda Li'} - \right.$$

$$- \kappa_T^L \sum_{\rho=\pm 1} (D_{nr}^{L-1Li} D_{nr}^{L+1Li'} + D_{nr}^{L+1Li} D_{nr}^{L-1Li'}) +$$

$$+ \sum_{\lambda=L\pm 1} G_r^{\lambda L} (D_{nr}^{\lambda Li-} D_{nr}^{\lambda Li'+} + D_{nr}^{\lambda Li+} D_{nr}^{\lambda Li'-}) -$$

$$- G_T^L (D_{nr}^{L-1Li-} D_{nr}^{L+1Li'+} + D_{nr}^{L+1Li-} D_{nr}^{L-1Li'+} + D_{nr}^{L-1Li+} D_{nr}^{L+1Li'+} +$$

$$+ D_{nr}^{L+1Li+} D_{nr}^{L-1Li'+}) \}, \quad (5')$$

$$H_{vq} = H_{Evq} + H_{svq}, \quad (6)$$

$$H_{Evq} = \sum_{\lambda\mu i} \sum_{jj'} \{ V_{IEr}^{\lambda\mu i} (jj') [(Q_{\lambda\mu i}^+ + (-)^{\lambda-\mu} Q_{\lambda-\mu i}) B(jj'; \lambda-\mu) + h.c.] + (6')$$

$$+ V_{2Er}^{\lambda\mu i} (jj') [(Q_{\lambda\mu i}^+ - (-)^{\lambda-\mu} Q_{\lambda-\mu i}) B(jj'; \lambda-\mu) + h.c.] \},$$

$$V_{IEr}^{\lambda\mu i} (jj') = -\frac{1}{4} \frac{1}{2\lambda+1} \sum_n \left\{ \sum_{\rho=\pm 1} (\kappa_0^\lambda + \rho \kappa_1^\lambda) f_n^\lambda (jj') v_{jj'}^{(-)} D_{nr}^{\lambda i} - \right.$$

$$- G_r^\lambda f_n^\lambda (jj') (-)^{\lambda-\mu} (u_{jj'}^{(+)} - u_{jj'}^{(-)}) D_{nr}^{\lambda i} -$$

$$- G_r^{\lambda\lambda} f_n^{\lambda\lambda} (jj') (-)^{\lambda-\mu} (u_{jj'}^{(+)} - u_{jj'}^{(-)}) D_{nr}^{\lambda\lambda i-} \}, \quad (6'')$$

$$V_{2Er}^{\lambda\mu i} (jj') = -\frac{1}{4} \frac{1}{2\lambda+1} \sum_n \left\{ \sum_{\rho=\pm 1} (\kappa_0^{\lambda\lambda} + \rho \kappa_1^{\lambda\lambda}) f_n^{\lambda\lambda} (jj') v_{jj'}^{(+)} D_{nr}^{\lambda\lambda i} - \right.$$

$$- G_r^\lambda f_n^\lambda (jj') (-)^{\lambda-\mu} (u_{jj'}^{(+)} - u_{jj'}^{(-)}) D_{nr}^{\lambda i+} -$$

$$- G_r^{\lambda\lambda} f_n^{\lambda\lambda} (jj') (-)^{\lambda-\mu} (u_{jj'}^{(+)} - u_{jj'}^{(-)}) D_{nr}^{\lambda\lambda i+} \}, \quad (6''')$$

$$H_{svq} = \sum_{LMi} \sum_{jj'} \{ V_{1sr}^{LMi} (jj') [(Q_{LMi}^+ - (-)^{L-M} Q_{L-Mi}) B(jj'; L-M) + h.c.] +$$

$$+ V_{2sr}^{LMi} (jj') [(Q_{LMi}^+ + (-)^{L-M} Q_{L-Mi}) B(jj'; L-M) + h.c.] \}, \quad (7)$$

$$V_{1sr}^{LMi} (jj') = -\frac{1}{4} \frac{1}{2L+1} \sum_n \left\{ \sum_{\lambda=L\pm 1} (\kappa_0^{\lambda L} + \rho \kappa_1^{\lambda L}) f_n^{\lambda L} (jj') v_{jj'}^{(+)} D_{nr}^{\lambda Li} - \right.$$

$$- \kappa_T^L \sum_{\rho=\pm 1} \rho v_{jj'}^{(+)} [D_{nr}^{L-1Li} f_n^{L+1L} (jj') + D_{nr}^{L+1Li} f_n^{L-1L} (jj')] -$$

$$- \sum_{\lambda=L\pm 1} G_r^{\lambda L} f_n^{\lambda L} (jj') (-)^{L-M} (u_{jj'}^{(+)} - u_{jj'}^{(-)}) D_{nr}^{\lambda Li+} -$$

$$- G_T^L (-)^{L-M} (u_{jj'}^{(+)} - u_{jj'}^{(-)}) [f_n^{L+1L} (jj') D_{nr}^{L-1Li+} + f_n^{L-1L} (jj') D_{nr}^{L+1Li+}] \}, \quad (7')$$

$$V_{2sr}^{LMi} (jj') = \frac{1}{4} \frac{1}{2L+1} \sum_n \left\{ \sum_{\lambda=L\pm 1} G_r^{\lambda L} f_n^{\lambda L} (jj') (-)^{L-M} (u_{jj'}^{(+)} - u_{jj'}^{(-)}) D_{nr}^{\lambda Li-} + \right.$$

$$+ G_T^L (-)^{L-M} (u_{jj'}^{(+)} - u_{jj'}^{(-)}) [f_n^{L+1L} (jj') D_{nr}^{L-1Li-} + f_n^{L-1L} (jj') D_{nr}^{L+1Li-}] \}. \quad (7'')$$

Here  $\kappa_0^{\lambda L}$  and  $\kappa_1^{\lambda L}$  are the isoscalar and isovector constants of the spin-multipole (p-h) interaction,  $G_r^\lambda$  and  $G_r^{\lambda L}$  are the constants of the multipole and spin-multipole (p-p) interaction;  $\kappa_T^L$ ,  $G_T^L$  is the constant of the tensor (p-h) and (p-p) interaction. Then,

$$D_{nr}^{\lambda i} = \sum_{jj'} f_n^\lambda (jj') u_{jj'}^{(+)} g_{jj'}^{\lambda i}, \quad D_{nr}^{\lambda Li} = \sum_{jj'} f_n^{\lambda L} (jj') u_{jj'}^{(-)} w_{jj'}^{Li},$$

$$D_{nr}^{\lambda i-} = \sum_{jj'} f_n^\lambda (jj') v_{jj'}^{(-)} g_{jj'}^{\lambda i}, \quad D_{nr}^{\lambda i+} = \sum_{jj'} f_n^\lambda (jj') v_{jj'}^{(+)} w_{jj'}^{\lambda i},$$

$$D_{nr}^{\lambda Li-} = \sum_{jj'} f_n^{\lambda L} (jj') v_{jj'}^{(-)} g_{jj'}^{Li}, \quad D_{nr}^{\lambda Li+} = \sum_{jj'} f_n^{\lambda L} (jj') v_{jj'}^{(+)} w_{jj'}^{Li}, \quad (8)$$

$$f_n^\lambda (jj') = \langle j || i^\lambda R_n^\lambda(r) Y_{\lambda\mu} || j' \rangle,$$

$$f_n^{\lambda L} (jj') = \langle j || i^\lambda \tilde{R}_n^\lambda(r) \{ \sigma Y_{\lambda\mu} \}_{LM} || j' \rangle,$$

$$u_{jj'}^{(+)} = u_j v_{j'} \pm u_{j'} v_j, \quad v_{jj'}^{(\pm)} = u_j u_{j'} \pm v_j v_{j'}, \quad g_{jj'}^{\lambda i} = \psi_{jj'}^{\lambda i} + \phi_{jj'}^{\lambda i}, \quad w_{jj'}^{\lambda i} = \psi_{jj'}^{\lambda i} - \phi_{jj'}^{\lambda i},$$

$\sum_{jj'}$  means summation over single-particle levels of the neutron at  $r = n$  or proton at  $r = p$  systems.

Note that for the RPA solutions the condition is fulfilled under averaging over the phonon vacuum

$$\langle \{ \sum_{jm} \epsilon_j a_{jm}^+ a_{jm} + H_v \} Q_{\lambda\mu i}^+ Q_{\lambda\mu i}^+ \rangle = 0.$$

Therefore, Hamiltonian (3) has no terms  $\sim Q_{\lambda\mu i}^+ Q_{\lambda\mu i}^+$  and  $Q_{\lambda\mu i} Q_{\lambda\mu i}$ .

The calculations within the QPNM were made in three steps. The first step is the calculations within the independent quasiparticle model: single-particle energies and the wave functions of the Saxon-Woods potential are found and the superconducting pairing correlations are taken into account (see ref. 18'). As a basis, the QPNM uses not single-particle but one-phonon states including collective, weakly collective and two-quasiparticle states. Therefore, the second step is the calculations in the RPA of one-phonon states forming the basis. At this stage all the QPNM constants are fixed. The third step is the inclusion of the quasiparticle-phonon interaction responsible for fragmentation of quasiparticle and collective motions and thus for complication of the nuclear state with increasing excitation energy.

## 2. RANDOM PHASE APPROXIMATION FOR STATES OF AN ELECTRIC TYPE

Now we derive equations for calculating in the RPA the energies and wave functions of one-phonon states

$$Q_{\lambda\mu i}^+ \Psi_0, \quad (9)$$

where  $\Psi_0$  is the ground state wave function of a doubly even nucleus which is determined as a phonon vacuum. Normalisation (9) has the form

$$\frac{1}{2} \sum_{jj'} [ (\psi_{jj'}^{\lambda 1})^2 - (\phi_{jj'}^{\lambda 1})^2 ] = 1. \quad (9')$$

To describe one-phonon states of an electric type, i.e. states with  $\lambda^\pi = 1^-, 2^+, 3^-, 4^+, \dots$ , we use the following part of the Hamiltonian (3):

$$\sum_{jm} \epsilon_j a_{jm}^+ a_{jm} + H_{Ev}. \quad (10)$$

Now we find an average value of (10) over the state (9) and using the variational principle

$$\delta \{ \langle Q_{\lambda\mu i} \{ \sum_{jm} \epsilon_j a_{jm}^+ a_{jm} + H_{Ev} \} Q_{\lambda\mu i}^+ \rangle - \frac{\omega_{\lambda i}}{2} [ \sum_{jj'} g_{jj'}^{\lambda i} w_{jj'}^{\lambda i} - 2 ] \} = 0 \quad (11)$$

get the following system of equations:

$$D_{nr}^{\lambda i} = \sum_{n'=1}^{n_{\max}} \{ \sum_{\rho=\pm 1} [ (\kappa_0^\lambda + \rho \kappa_1^\lambda) X_{nn'}^{\lambda i}(\tau) D_{n'\rho\tau}^{\lambda i} + (\kappa_0^{\lambda\lambda} + \rho \kappa_1^{\lambda\lambda}) X_{nn'}^{\lambda i\lambda}(\tau) D_{n'\rho\tau}^{\lambda i\lambda} ] + G_\tau^\lambda [ X_{nn'}^{\lambda i-}(\tau) D_{n'\tau}^{\lambda i-} + X_{nn'}^{\lambda i\omega-}(\tau) D_{n'\tau}^{\lambda i+} ] + G_\tau^{\lambda\lambda} [ X_{nn'}^{\lambda i\lambda+}(\tau) D_{n'\tau}^{\lambda i-} + X_{nn'}^{\lambda i\lambda\omega+}(\tau) D_{n'\tau}^{\lambda i+} ] \}, \quad (12)$$

$$D_{nr}^{\lambda\lambda i} = \sum_{n'=1}^{n_{\max}} \{ \sum_{\rho=\pm 1} [ (\kappa_0^{\lambda\lambda} + \rho \kappa_1^{\lambda\lambda}) X_{nn'}^{\lambda i-}(\tau) D_{n'\rho\tau}^{\lambda\lambda i} + (\kappa_0^{\lambda\lambda} + \rho \kappa_1^{\lambda\lambda}) X_{nn'}^{\lambda i\lambda}(\tau) D_{n'\rho\tau}^{\lambda i} ] + G_\tau^\lambda [ X_{nn'}^{\lambda i\lambda-}(\tau) D_{n'\tau}^{\lambda i+} + X_{nn'}^{\lambda i\lambda\omega-}(\tau) D_{n'\tau}^{\lambda i-} ] + G_\tau^{\lambda\lambda} [ X_{nn'}^{\lambda\lambda i\omega}(\tau) D_{n'\tau}^{\lambda\lambda i-} + X_{nn'}^{\lambda\lambda i+}(\tau) D_{n'\tau}^{\lambda\lambda i+} ] \}, \quad (12')$$

$$D_{nr}^{\lambda i+} = \sum_{n'=1}^{n_{\max}} \{ \sum_{\rho=\pm 1} [ (\kappa_0^\lambda + \rho \kappa_1^\lambda) D_{n'\rho\tau}^{\lambda i} X_{nn'}^{\lambda i\omega+}(\tau) + (\kappa_0^{\lambda\lambda} + \rho \kappa_1^{\lambda\lambda}) X_{nn'}^{\lambda i\lambda-}(\tau) D_{n'\rho\tau}^{\lambda\lambda i} ] + G_\tau^\lambda [ X_{nn'}^{\lambda i\gamma+}(\tau) D_{n'\tau}^{\lambda i+} + X_{nn'}^{\lambda i\gamma\omega}(\tau) D_{n'\tau}^{\lambda i-} ] + G_\tau^{\lambda\lambda} [ X_{nn'}^{\lambda i\lambda\gamma+}(\tau) D_{n'\tau}^{\lambda\lambda i+} + X_{nn'}^{\lambda i\lambda\gamma\omega}(\tau) D_{n'\tau}^{\lambda\lambda i-} ] \}, \quad (13)$$

$$D_{nr}^{\lambda i-} = \sum_{n'=1}^{n_{\max}} \{ \sum_{\rho=\pm 1} [ (\kappa_0^\lambda + \rho \kappa_1^\lambda) X_{nn'}^{\lambda i-}(\tau) D_{n'\rho\tau}^{\lambda i} + (\kappa_0^{\lambda\lambda} + \rho \kappa_1^{\lambda\lambda}) X_{nn'}^{\lambda i\lambda\omega-}(\tau) D_{n'\rho\tau}^{\lambda\lambda i} ] + G_\tau^\lambda [ X_{nn'}^{\lambda i\gamma-}(\tau) D_{n'\tau}^{\lambda i-} + X_{nn'}^{\lambda i\gamma\omega}(\tau) D_{n'\tau}^{\lambda i+} ] + G_\tau^{\lambda\lambda} [ X_{nn'}^{\lambda i\lambda\gamma-}(\tau) D_{n'\tau}^{\lambda\lambda i-} + X_{nn'}^{\lambda i\lambda\gamma\omega}(\tau) D_{n'\tau}^{\lambda\lambda i+} ] \}. \quad (13')$$

Equations for  $D_{nr}^{\lambda i \pm}$  are derived from equations (13) and (13') for  $D_{nr}^{\lambda i \pm}$  by substituting in each term of matrix element  $f_n^{\lambda, (jj')}$  by  $f_{nn}^{\lambda, (jj')}$ . Here and in the next section we use the notation

$$X_{nn'}^{\lambda i}(\tau) = \frac{1}{2\lambda + 1} \sum_{jj'}^{\tau} \frac{f_n^{\lambda, (jj')} f_{n'}^{\lambda, (jj')} (u_{jj'}^{(+)})^2 \epsilon_{jj'}}{\epsilon_{jj'}^2 - \omega_{\lambda 1}^2},$$

$$X_{nn'}^{\lambda i-}(\tau) = \frac{1}{2\lambda + 1} \sum_{jj'}^{\tau} \frac{f_n^{\lambda, (jj')} f_{n'}^{\lambda, (jj')} u_{jj'}^{(+)} v_{jj'}^{(-)} \epsilon_{jj'}}{\epsilon_{jj'}^2 - \omega_{\lambda 1}^2},$$

$$X_{nn'}^{\lambda i \omega \pm}(\tau) = \frac{1}{2\lambda + 1} \sum_{jj'}^{\tau} \frac{f_n^{\lambda, (jj')} f_{n'}^{\lambda, (jj')} u_{jj'}^{(+)} v_{jj'}^{(\pm)} \omega_{\lambda 1}}{\epsilon_{jj'}^2 - \omega_{\lambda 1}^2}, \quad (14)$$

$$X_{nn'}^{\lambda i v \pm}(\tau) = \frac{1}{2\lambda + 1} \sum_{jj'}^{\tau} \frac{f_n^{\lambda, (jj')} f_{n'}^{\lambda, (jj')} (v_{jj'}^{(\pm)})^2 \epsilon_{jj'}}{\epsilon_{jj'}^2 - \omega_{\lambda 1}^2},$$

$$X_{nn'}^{\lambda i v \omega}(\tau) = \frac{1}{2\lambda + 1} \sum_{jj'}^{\tau} \frac{f_n^{\lambda, (jj')} f_{n'}^{\lambda, (jj')} v_{jj'}^{(+)} v_{jj'}^{(-)} \omega_{\lambda 1}}{\epsilon_{jj'}^2 - \omega_{\lambda 1}^2},$$

$$X_{nn'}^{\lambda i \lambda}(\tau) = \frac{1}{2\lambda + 1} \sum_{jj'}^{\tau} \frac{f_n^{\lambda, (jj')} f_{n'}^{\lambda \lambda, (jj')} u_{jj'}^{(+)} u_{jj'}^{(-)} \omega_{\lambda 1}}{\epsilon_{jj'}^2 - \omega_{\lambda 1}^2},$$

$$X_{nn'}^{\lambda i \lambda \pm}(\tau) = \frac{1}{2\lambda + 1} \sum_{jj'}^{\tau} \frac{f_n^{\lambda, (jj')} f_{n'}^{\lambda \lambda, (jj')} u_{jj'}^{(\pm)} v_{jj'}^{(\mp)} \epsilon_{jj'}}{\epsilon_{jj'}^2 - \omega_{\lambda 1}^2},$$

$$X_{nn'}^{\lambda i \lambda \omega \pm}(\tau) = \frac{1}{2\lambda + 1} \sum_{jj'}^{\tau} \frac{f_n^{\lambda, (jj')} f_{n'}^{\lambda \lambda, (jj')} u_{jj'}^{(+)} v_{jj'}^{(\pm)} \omega_{\lambda 1}}{\epsilon_{jj'}^2 - \omega_{\lambda 1}^2}, \quad (14')$$

$$X_{nn'}^{\lambda i \lambda v \pm}(\tau) = \frac{1}{2\lambda + 1} \sum_{jj'}^{\tau} \frac{f_n^{\lambda, (jj')} f_{n'}^{\lambda \lambda, (jj')} (v_{jj'}^{(\pm)})^2 \epsilon_{jj'}}{\epsilon_{jj'}^2 - \omega_{\lambda 1}^2},$$

$$X_{nn'}^{\lambda i \lambda v \omega}(\tau) = \frac{1}{2\lambda + 1} \sum_{jj'}^{\tau} \frac{f_n^{\lambda, (jj')} f_{n'}^{\lambda \lambda, (jj')} v_{jj'}^{(+)} u_{jj'}^{(-)} \omega_{\lambda 1}}{\epsilon_{jj'}^2 - \omega_{\lambda 1}^2},$$

$$X_{nn'}^{\lambda \lambda' i}(\tau) = \frac{1}{2L + 1} \sum_{jj'}^{\tau} \frac{f_n^{\lambda L, (jj')} f_{n'}^{\lambda' L, (jj')} (u_{jj'}^{(-)})^2 \epsilon_{jj'}}{\epsilon_{jj'}^2 - \omega_{\lambda 1}^2},$$

$$X_{nn'}^{\lambda \lambda' i+}(\tau) = \frac{1}{2L + 1} \sum_{jj'}^{\tau} \frac{f_n^{\lambda L, (jj')} f_{n'}^{\lambda' L, (jj')} u_{jj'}^{(-)} v_{jj'}^{(+)} \epsilon_{jj'}}{\epsilon_{jj'}^2 - \omega_{L1}^2},$$

$$X_{nn'}^{\lambda \lambda' i \omega}(\tau) = \frac{1}{2L + 1} \sum_{jj'}^{\tau} \frac{f_n^{\lambda L, (jj')} f_{n'}^{\lambda' L, (jj')} u_{jj'}^{(-)} v_{jj'}^{(-)} \omega_{L1}}{\epsilon_{jj'}^2 - \omega_{L1}^2}, \quad (14'')$$

$$X_{nn'}^{\lambda \lambda' i v \pm}(\tau) = \frac{1}{2L + 1} \sum_{jj'}^{\tau} \frac{f_n^{\lambda L, (jj')} f_{n'}^{\lambda' L, (jj')} (v_{jj'}^{(\pm)})^2 \epsilon_{jj'}}{\epsilon_{jj'}^2 - \omega_{L1}^2},$$

$$X_{nn'}^{\lambda \lambda' i v \omega}(\tau) = \frac{1}{2L + 1} \sum_{jj'}^{\tau} \frac{f_n^{\lambda L, (jj')} f_{n'}^{\lambda' L, (jj')} v_{jj'}^{(+)} v_{jj'}^{(-)} \omega_{L1}}{\epsilon_{jj'}^2 - \omega_{L1}^2},$$

Equations (12)-(13') include the functions (14'') with  $L = \lambda$  and  $\lambda' = \lambda$  and  $\epsilon_{jj'} = \epsilon_j + \epsilon_{j'}$ .

From equations (12), (12'), (13), (13') and the equation for  $D_{nr}^{\lambda \lambda i \pm}$  we derive the secular equation for the one-phonon energies  $\omega_{\lambda 1}$  as an equality to zero of the determinant of the rank  $12 n_{\max}$ . With the use of separable interactions of the rank  $n_{\max}$ , the rank of the determinant increases  $n_{\max}$  times in comparison with simple separable interactions. If one does not take into account spin-multipole interactions with  $L = \lambda$ , the rank of the determinant equals  $6 n_{\max}$ . If particle-particle interactions are disregarded, the rank of the determinant is  $4 n_{\max}$ . With the inclusion of multipole p-h interactions the rank of the determinant is  $2 n_{\max}$ . The phonon amplitudes  $\psi_{jj'}^{\lambda 1}$  and  $\phi_{jj'}^{\lambda 1}$  are calculated by using the condition (9').

### 3. RANDOM PHASE APPROXIMATION FOR STATES OF A MAGNETIC TYPE

To describe one-phonon states of a magnetic type, i.e. states with  $L^{\pi} = 1^+, 2^-, 3^+, 4^-, \dots$  we use the following part of the Hamiltonian (3)

$$\sum_{jm} \epsilon_j a_j^+ a_{jm} + H_{sv}. \quad (15)$$

We find an average value of (15) over (9) and using the variational principle in the form (11) we get for  $\lambda = L \pm 1$  the following system of equations:

$$\begin{aligned} D_{nr}^{\lambda Li} &= \sum_{n'=1}^{n_{\max}} \left\{ \sum_{\lambda'=L \pm 1} (\kappa_0^{\lambda'L} + \rho \kappa_1^{\lambda'L}) X_{nn'}^{\lambda \lambda' i}(\tau) D_{n'\rho\tau}^{\lambda' Li} - \right. \\ &\quad \left. - \kappa_T^L \sum_{\rho=\pm 1} \rho [X_{nn'}^{\lambda L-1i}(\tau) D_{n'\rho\tau}^{L+1Li} + X_{nn'}^{\lambda L+1i}(\tau) D_{n'\rho\tau}^{L-1Li}] + \right. \\ &\quad \left. + \sum_{\lambda'=L \pm 1} G_T^{\lambda' i} [X_{nn'}^{\lambda \lambda' i}(\tau) D_{n'\tau}^{\lambda' Li+} + X_{nn'}^{\lambda \lambda' i\omega}(\tau) D_{n'\tau}^{\lambda' Li-}] - \right. \\ &\quad \left. - G_T^L [X_{nn'}^{\lambda L-1i+}(\tau) D_{n'\tau}^{L+1Li+} + X_{nn'}^{\lambda L+1i+}(\tau) D_{n'\tau}^{L-1Li+} + \right. \\ &\quad \left. + X_{nn'}^{\lambda L-1i\omega}(\tau) D_{n'\tau}^{L+1Li-} + X_{nn'}^{\lambda L+1i\omega}(\tau) D_{n'\tau}^{L-1Li-}] \right\}, \\ D_{nr}^{\lambda Li+} &= \sum_{n'=1}^{n_{\max}} \left\{ \sum_{\lambda'=L \pm 1} (\kappa_0^{\lambda'L} + \rho \kappa_1^{\lambda'L}) X_{nn'}^{\lambda \lambda' i+}(\tau) D_{n'\rho\tau}^{\lambda' Li} - \right. \\ &\quad \left. - \kappa_T^L \sum_{\rho=\pm 1} \rho [X_{nn'}^{\lambda L-1i+}(\tau) D_{n'\rho\tau}^{L+1Li} + X_{nn'}^{\lambda L+1i+}(\tau) D_{n'\rho\tau}^{L-1Li}] + \right. \\ &\quad \left. + \sum_{\lambda'=L \pm 1} G_T^{\lambda' L} [X_{nn'}^{\lambda \lambda' v+}(\tau) D_{n'\tau}^{\lambda' Li+} + X_{nn'}^{\lambda \lambda' i v\omega}(\tau) D_{n'\tau}^{\lambda' Li}] - \right. \\ &\quad \left. - G_T^L [X_{nn'}^{\lambda L-1iv+}(\tau) D_{n'\tau}^{L+1Li+} + X_{nn'}^{\lambda L+1iv+}(\tau) D_{n'\tau}^{L-1Li+} + \right. \\ &\quad \left. + X_{nn'}^{\lambda L-1iv\omega}(\tau) D_{n'\tau}^{L+1Li-} + X_{nn'}^{\lambda L+1iv\omega}(\tau) D_{n'\tau}^{L-1Li-}] \right\}, \\ D_{nr}^{\lambda Li-} &= \sum_{n'=1}^{n_{\max}} \left\{ \sum_{\lambda'=L \pm 1} (\kappa_0^{\lambda'L} + \rho \kappa_1^{\lambda'L}) X_{nn'}^{\lambda \lambda' i\omega}(\tau) D_{n'\rho\tau}^{\lambda' Li} - \right. \end{aligned} \quad (16)$$

$$\begin{aligned} &\quad - \kappa_T^L \sum_{\rho=\pm 1} \rho [X_{nn'}^{\lambda L-1i\omega}(\tau) D_{n'\rho\tau}^{L+1Li} + X_{nn'}^{\lambda L+1i\omega}(\tau) D_{n'\rho\tau}^{L-1Li}] + \\ &\quad + \sum_{\lambda'=L \pm 1} G_T^{\lambda' L} [X_{nn'}^{\lambda \lambda' i v\omega}(\tau) D_{n'\tau}^{\lambda' Li+} + X_{nn'}^{\lambda \lambda' i v}(\tau) D_{n'\tau}^{\lambda' Li-}] - \\ &\quad - G_T^L [X_{nn'}^{\lambda L-1iv\omega}(\tau) D_{n'\tau}^{L+1Li+} + X_{nn'}^{\lambda L+1iv\omega}(\tau) D_{n'\tau}^{L-1Li+} + \\ &\quad + X_{nn'}^{\lambda L-1iv}(\tau) D_{n'\tau}^{L+1Li-} + X_{nn'}^{\lambda L+1iv}(\tau) D_{n'\tau}^{L-1Li-}] \}. \end{aligned} \quad (16'')$$

The functions  $X_{nn'}^{\lambda}(\tau)$  are determined by formulae (14'').

From eqs. (16), (16'), (16'') with  $\lambda = L \pm 1$ ,  $\tau = p$  and  $\tau = n$  we get the secular equation for  $\omega_{Li}$  as an equality to zero of the determinant of the rank  $12 n_{\max}$  Tensor interactions do not increase the rank of the determinant. It is determined by spin-multipole interactions. If the p-p interactions are neglected, the rank equals  $4 n_{\max}$ . In many papers the calculation has been performed with  $\lambda = L-1$ ; then, the rank of the determinant is  $2 n_{\max}$ .

If the RPA secular equations for states of electric and magnetic types are solved and the energies  $\omega_{\lambda i}$  and phonon amplitudes  $\psi_{jj}^{\lambda i}$  and  $\phi_{jj}^{\lambda i}$  are found, the Hamiltonian (3) appears to be uniquely determined. It contains no any free parameters and no unfixed constants.

Equations for charge-exchange one-phonon states can be derived in a similar way. For simple separable p-h interactions they are given in ref.<sup>14/</sup>; for the (p-h) + (p-p) Gamow-Teller interactions they have been derived in ref.<sup>14/</sup>.

#### 4. BASIC EQUATIONS OF THE MODEL

In the QPNM the excited state wave functions are given as a series in the number of phonon operators; in odd nuclei each term is multiplied by the quasiparticle operator. The approximation implies break off this series. In the calculations performed earlier, except for ref.<sup>17/</sup>, the wave functions consist of one- and two-phonon terms.

The excited state wave function of a doubly spherical nucleus is

$$\Psi_\nu(JM) = \left\{ \sum_i R_i Q_{JM1}^+ + \sum_{\substack{\lambda_1 i_1 \\ \lambda_2 i_2}} P_{\lambda_2 i_2}^{\lambda_1 i_1} \sum_{\mu_1 \mu_2} \langle \lambda_1 \mu_1 \lambda_2 \mu_2 | JM \rangle \times \right. \quad (17)$$

$$\left. \times Q_{\lambda_1 \mu_1 i_1}^+ Q_{\lambda_2 \mu_2 i_2}^+ \right\} \Psi_0,$$

where  $\nu$  is the state number with given  $JM$ , and  $\langle \lambda_1 \mu_1 \lambda_2 \mu_2 | JM \rangle$  is the Clebsch-Gordan coefficient. Allowance is made for the fact that phonons consist of quasiparticle operators, and therefore, satisfy complicated permutation relations<sup>3,4'</sup>. Indeed, averaging over the phonon vacuum we have

$$\sum_{\substack{\mu \mu_2 \\ \mu' \mu'_2}} \langle \lambda' \mu' \lambda'_2 \mu'_2 | JM \rangle \langle \lambda \mu \lambda_2 \mu_2 | JM \rangle \langle Q_{\lambda'_2 \mu'_2 i'_2} Q_{\lambda' \mu' i'} Q_{\lambda \mu i}^+ Q_{\lambda_2 \mu_2 i_2}^+ \rangle =$$

$$= \delta_{\lambda \lambda'} \delta_{i i'} \delta_{\lambda_2 \lambda'_2} \delta_{i_2 i'_2} + \delta_{\lambda \lambda'_2} \delta_{i i'_2} \delta_{\lambda_2 \lambda' i'_2} + K^J(\lambda'_2 i'_2, \lambda' i' | \lambda i, \lambda_2 i_2).$$

Following refs.<sup>3,4'</sup>, the diagonal terms  $K^J$  denoted by  $K^J(\lambda i, \lambda_2 i_2)$  are considerably larger than nondiagonal ones. In the diagonal in  $K^J$  approximation the normalization condition (17) has the form

$$\sum_i R_i^2 + 2 \sum_{\lambda_1 i_1 \lambda_2 i_2} (P_{\lambda_2 i_2}^{\lambda_1 i_1})^2 [1 + K^J(\lambda_1 i_1, \lambda_2 i_2)] = 1. \quad (18)$$

An average value of the Hamiltonian (3) over the state (17) in the diagonal in  $K^J$  approximation is

$$(\Psi_\nu^*(JM) H \Psi_\nu(JM)) = \sum_i \omega_{Ji} R_i^2 + 2 \sum_{\lambda_1 i_1 \lambda_2 i_2} (P_{\lambda_2 i_2}^{\lambda_1 i_1})^2 [\omega_{\lambda_1 i_1} +$$

$$+ \omega_{\lambda_2 i_2} + \Delta\omega(\lambda_1 i_1, \lambda_2 i_2)] [1 + K^J(\lambda_1 i_1, \lambda_2 i_2)] + \quad (19)$$

$$+ 2 \sum_{\lambda_1 i_1 \lambda_2 i_2} R_i P_{\lambda_2 i_2}^{\lambda_1 i_1} U_{\lambda_2 i_2}^{\lambda_1 i_1}(Ji) [1 + K^J(\lambda_1 i_1, \lambda_2 i_2)],$$

where

$$\Delta\omega(\lambda_1 i_1, \lambda_2 i_2) = \sum_{i_3} [W_{\lambda_1 i_3}^{\lambda_1} K^J(\lambda_2 i_2, \lambda_1 i_3 | \lambda_1 i_1, \lambda_2 i_2) + \quad (19')$$

$$+ W_{\lambda_2 i_3}^{\lambda_2} K^J(\lambda_2 i_3, \lambda_1 i_1 | \lambda_1 i_1, \lambda_2 i_2)],$$

$$(\Psi_\nu^*(JM) H_{vq} \Psi_\nu(JM)) = 2 \sum_{\lambda_1 i_1 \lambda_2 i_2} R_i P_{\lambda_2 i_2}^{\lambda_1 i_1} U_{\lambda_2 i_2}^{\lambda_1 i_1}(Ji) [1 + \quad (19'')$$

$$+ K^J(\lambda_1 i_1, \lambda_2 i_2)].$$

$W_{\lambda_1 i_2}^{\lambda_1}$  has the form (4'') for states of an electric type and (5') for states of a magnetic type;  $U_{\lambda_2 i_2}^{\lambda_1 i_1}$  includes the functions  $V_{1E\tau}^{\lambda \mu i}(Jj')$ ,  $V_{2E\tau}^{\lambda \mu i}(Jj')$ ,  $V_{1S\tau}^{LMi}(Jj')$ ,  $V_{2S\tau}^{LMi}(Jj')$ .

Using the variational principle

$$\delta\{(\Psi_\nu^*(JM) H \Psi_\nu(JM)) - \eta_\nu [(\Psi_\nu^*(JM) \Psi_\nu(JM)) - 1]\} = 0,$$

we get the equations

$$(\omega_{Ji} - \eta_\nu) R_i - \sum_{\lambda_1 i_1 \lambda_2 i_2} P_{\lambda_2 i_2}^{\lambda_1 i_1}(Ji) U_{\lambda_2 i_2}^{\lambda_1 i_1}(Ji) [1 + K^J(\lambda_1 i_1, \lambda_2 i_2)] = 0,$$

$$2(\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} + \Delta\omega(\lambda_1 i_1, \lambda_2 i_2) - \eta_\nu) P_{\lambda_2 i_2}^{\lambda_1 i_1} + \sum_i R_i U_{\lambda_2 i_2}^{\lambda_1 i_1}(Ji) = 0. \quad (20)$$

Then, the secular equation for the energies  $\eta_\nu$  of the states described by the wave function (17) becomes

$$\det \| (\omega_{Ji} - \eta_\nu) \delta_{ii'} - \frac{1}{2} \sum_{\lambda_1 i_1 \lambda_2 i_2} \frac{U_{\lambda_2 i_2}^{\lambda_1 i_1}(Ji) U_{\lambda_2 i_2}^{\lambda_1 i_1}(Ji') [1 + K^J(\lambda_1 i_1, \lambda_2 i_2)]}{\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} + \Delta\omega(\lambda_1 i_1, \lambda_2 i_2) - \eta_\nu} \| = 0. \quad (21)$$

From eqs. (20) and condition (18) for each value of  $\eta_\nu$  we find  $R_i$  and  $P_{\lambda_2 i_2}^{\lambda_1 i_1}$ . The form of eqs. (20) and (21) coincides with that of eqs. for simple ( $n_{\max} = 1$ ) separable interactions<sup>4,5'</sup>. Effect of the Pauli principle in two-phonon terms (17) leads, as before, to the factor  $1 + K^J(\lambda_1 i_1, \lambda_2 i_2)$  in



(21) and shifts of the two-phonon poles  $\Delta\omega(\lambda_1 i_1, \lambda_2 i_2)$ . A specific description of  $0^+$  states is presented in ref. /4/. Formulae for charge-exchange states in doubly even spherical nuclei have a form similar to (20) and (21).

It should be emphasized that with the use of finite rank separable interactions the rank of the determinant (21) does not increase in comparison with simple separable interactions. The inclusion of finite rank separable interactions, (p-h) tensor and (p-p) multipole, spin-multipole and tensor interactions makes the expressions for  $\Delta\omega(\lambda_1 i_1, \lambda_2 i_2)$  and  $U_{\lambda_2 i_2}^{\lambda_1 i_1}(J_i)$  more complicated in comparison with the case when (p-h) multipole and spin-multipole simple separable interactions are taken into account. This complication of the functions turns out to be unessential in computer calculations.

#### CONCLUSION

Many characteristics of low- and high-lying states of spherical and deformed nuclei have been calculated in the QPNM. High-lying states have been calculated by using the strength function method /1-4/. The obtained description of low-lying states of spherical and deformed nuclei is in agreement with experimental data. The predictions made for the structure of some states of deformed nuclei have been later confirmed by experimental data. In describing nonrotational states of deformed nuclei the QPNM is more advantageous (see ref. /18/) over the interacting boson model /19/.

The QPNM was used to calculate the fragmentation of one-quasiparticle states, to describe neutron strength functions and widths of giant resonances in spherical and deformed nuclei. The first calculations of  $\gamma$  decay of a deep hole state were also made in the QPNM /20/. It is to be noted that the description of fragmentation of quasiparticle and collective motions is one of the central problems of nuclear theory. Fragmentation is being calculated in the QPNM, finite Fermi-system theory, nuclear field theory and by a direct diagonalisation in the space of 2p-2h states. The results of these investigations are presented in a series of reviews, for instance, refs. /2-4, 21-24/. In the present paper we have obtained the basic QPNM equations for (p-h) and (p-p) isoscalar and isovector multipole and spin-multipole and isovector tensor finite rank separable interactions. The finite rank of separable interactions makes the RPA equations more complicated,

which is unimportant in computer calculations. Most important is the fact that allowance for the finite rank separable equations does not result in any essential complication of equations for calculating the fragmentation of quasiparticle and collective motions. This implies that the QPNM may serve as a basis for calculating many properties of atomic nuclei and spectroscopic factors of nuclear reactions.

In conclusion I should like to note that in solving complex problems as the nuclear many-body problem is, one should try to extract the most important degrees of freedom and to find the decisive part of effective nuclear forces rather than to formulate the problem in the most general form.

#### REFERENCES

1. Soloviev V.G. - Sov.J.Part.Nucl., 1978, v.9, p.580; Soloviev V.G. - Nucleonica, 1978, v.13, p.1149.
2. Malov L.A., Soloviev V.G. - Sov.J.Part.Nucl., 1980, v.11, p.301.
3. Vdovin A.I., Soloviev V.G. - Sov.J.Part.Nucl., 1983, v.14, p.237; Voronov V.V., Soloviev V.G. - Sov.J.Part.Nucl., 1983, v.14, p.1380; Vdovin A.I., Voronov V.V., Soloviev V.G., Stoyanov Ch. - Sov.J.Part.Nucl., 1985, v.16, p.245.
4. Soloviev V.G. - Progress in Particle and Nucl.Physics, 1987, v.19, p.107; Soloviev V.G. Theory of Atomic Nuclei. Quasiparticles and Phonons. M. Energoizdat, 1988.
5. Vdovin A.I., Dambasuren D., Soloviev V.G., Stoyanov Ch. - Izv.AN SSSR, ser.fiz., 1976, v.40, p.2183; Nguyen Dinh Thao, Stoyanov Ch. - Izv.AN SSSR, ser.fiz., 1982, v.46, p.2157.
6. Baldo M., Ferriera L.S., Verste T. - Phys.Rev., 1986, v.C33, p.1587; Baldo M., Ferriera L.S. - Phys.Lett., 1987, v.B198, p.307.
7. Haidenbauer J., Plessas W. - Phys.Rev., 1984, v.C30, p.1822; - Phys.Rev., 1985, v.C32, p.1424.
8. Haidenbauer J., Koike Y., Pleassas W. - Phys.Rev., 1986, v.C33, p.439.
9. Lampl H., Weigel M.K. - Phys.Rev., 1986, v.C33, p.1834.
10. Suzuki T., Sagawa H. - Prog.Theor.Phys., 1981, v.65, p.565.
11. Birbrair B.L., Erokhina K.T., Lemberg I.Kh. - Nucl.Phys., 1972, v.A145, p.129.

12. Vogel P., Zirnbauer M.R. - Phys.Rev.Lett., 1986, v.57, p.3148.
13. Kuzmin V.A., Soloviev V.G. - Pisma Zh.Eksp.Teor.Fiz., 1988, v.47, p.68; JINR Preprint P4-83-406, Dubna, 1988; Suhonen J., Faessler A., Taigel T., Tomoda T. - Phys.Lett., 1988, v.B202, p.147.
14. Kuzmin V.A., Soloviev V.G. - Nucl.Phys., 1988, v.A486, p.118.
15. Soloviev V.G., Sushkov A.V. JINR Preprint P4-88-509, Dubna, 1988.
16. Soloviev V.G. Theory of Atomic Nuclei, Nuclear Models. M.: Energoizdat, 1981.
17. Soloviev V.G. - Yad.Fiz., 1984, v.40, p.1163.
18. Soloviev V.G. - Z.Phys.A - Atomic Nuclei, 1986, v.324, p.393;  
Soloviev V.G. - Yad.Fiz., 1988, v.47, p.332.
19. Arima A. In: Capture Gamma-Ray Spectroscopy, 1987, Ist.of Phys. Conference Series Number 88, ed. K.A.Abraghams and P.Van Assche, 1988, p.1.
20. Ponomarev V.Yu., Soloviev V.G., Vdovin A.I., Stoyanov Ch. - Phys.Lett., 1987, v.B183, p.237.
21. Speth J., van der Woude A. - Rep.Prog.Phys., 1981, v.44, p.719.
22. Bertsch G.F., Bortignon P.F., Broglia R.A. - Rev.Mod.Phys., 1983, v.55, p.287.
23. Wambach J. - Rep.Prog.Phys., 1988, v.51, p.989.
24. Gales S., Stoyanov Ch., Vdovin A.I. - Physics Reports, 1988, v.166., p.125.

Received by Publishing Department  
on October 10, 1988.

Соловьев В.Г.

E4-88-737

Уравнения квазичастично-фононной модели ядра с эффективными конечного ранга сепарабельными взаимодействиями

Получены основные уравнения квазичастично-фононной модели ядра для конечного ранга сепарабельных изоскалярных и изовекторных мультипольных и спин-мультипольных и изовекторных тензорных частично-дырочных и частично-частичных взаимодействий между квазичастицами. Для четно-четных сферических ядер показано, что значительное усложнение, обусловленное конечным  $n_{max} > 1$  рангом сепарабельных взаимодействий, имеет место при вычислении однофононных состояний в приближении хаотичных фаз. Учет сепарабельных взаимодействий с  $n_{max} > 1$  не приводит к существенному усложнению при вычислении фрагментации квазичастичных и коллективных состояний. Утверждается, что модель может служить основой для вычисления многих характеристик сложных ядер.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1988

Soloviev V.G.

E4-88-737

Quasiparticle-Phonon Nuclear Model Equations with Effective Separable Interactions of a Finite Rank

The quasiparticle-phonon nuclear model equations are obtained for the finite rank separable isoscalar and isovector multipole and spin-multipole and isovector tensor particle-hole and particle-particle interactions between quasiparticles. For doubly even spherical nuclei allowance for separable interactions of the rank  $n_{max}$  leads to a considerable complication of equations in the random phase approximation. Separable interactions with  $n_{max} > 1$  do not cause any significant complication in calculating the fragmentation of quasiparticle and collective states. It is asserted that the model can serve as a basis for calculating many characteristics of complex nuclei.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1988