16 коп.

Редактор Э.В.Ивашкевич. Макет Р.Д.Фоминой.

Подписано в печать 21.10.88.
Формат 60x90/16. Офсетная печать. Уч.-изд.листов 1,07.
Тираж 425. Заказ 41192.

Издательский отдел Объединенного института ядерных исследований.
Дубна Московской области.

объединенный
институт
ядерных
исследований
дубна

E4-88-688

B.Milek

A KICKED QUANTUM SYSTEM INCLUDING THE CONTINUUM

Submitted to "Physics Letters A"

1988

Recently a lot of attempts have been made in order to investigate quantum systems under the influence of external time-dependent fields /1-7/. The question behind that is how dynamical chaos might occur not only in the rather well understood classical case but also in a quantum mechanical system. Dynamical chaos is a random (unpredictable) motion of a completely deterministic mechanical system whose equations of motion do not contain any random parameters or any noise /8/. One problem in this field is the relevance of classical chaos in the corresponding quantum case studied in highly excited systems as a kicked rotator /9/ or, more physically, a hydrogen atom in electro-magnetic fields /1,2/. It was found /10,11/ that the classical diffuse behaviour associated with chaotic time evolution is greatly suppressed in the quantum system. The wave packet remains localized all the time /1, 9/. Another field is the study of simple, pure quantum mechanical models as a spin-1/2 system /5,6/ in order to look for mechanisms which could be capable to weaken the mentioned quantum suppression of dynamical chaos /4,6/. Recently first attention has been paid to the exact incorporation of the continuum /12/ within a schematic quantum system kicked with periodic pulses. The aim of our paper is the presentation of a similar simple quantum model, a separable one-term potential, kicked by an arbitrary modulated external force. Such a potential - well known in few-body physics - describing only one partial wave is of course not very realistic but it simplifies to a large extent practical calculations. It contains the interaction of a particle with one set of quantum numbers in a spherical symmetric field but includes only one bound state and a continuum of scattering states. It has been applied, for example, as a subsystem in more complicated dynamic models in heavy-ion physics /13/ and in fissioning systems /14/. With a set of such separable potentials it is possible to approximate rather complicated local potentials (see for example ref. /15/). The parameters of the separable potential can be chosen in such a way that any given bound state energy, spread of the wave function, or phase shifts for a scattering process can be reproduced. The exact incorporation of the continuum seems to be an advantage of this model and in this sense the model might be considered as a new tool for the investigation of the ionization behaviour of a quantum particle in external time-dependent fields /1,2,7/.

We consider the problem of a quantum particle with mass m in a separable potential including only one term $|\beta\rangle$ with kicked strength (period T) described by the Hamiltonian

$$H=H_0 + g\, F(t) \sum_{\nu=-\infty}^{\infty} \delta(t/T-\nu)\ |\beta\rangle\langle\beta| , \qquad (1)$$

where g is a constant strength parameter and F(t) an arbitrary chosen modulation function. For F(t)=const eq. (1) reduces to a periodically kicked quantum system and for a time-periodic function F with period τ it covers the case of a quasiperiodically driven system, too. The unperturbed Hamiltonian for the particle with momentum **p** reads (h=1 throughout the paper)

$$H_0= \mathbf{p}^2/2m + V_0\ |\beta\rangle\langle\beta| . \qquad (2)$$

The unperturbed system (2) possesses only one bound state $|\Phi_B\rangle$ with energy E_B and a continuum of scattering states $|\Phi_k^+\rangle$ for incomming plane waves $|\mathbf{k}\rangle$ (see for example /13/). These eigenstates can be expressed in terms of the free Green operator $G_0(z)=(z-\mathbf{p}^2/2m)^{-1}$ by

$$|\Phi_B\rangle = n\ G_0(E_B)\ |\beta\rangle \qquad (3)$$

$$|\Phi_k^+\rangle = |\mathbf{k}\rangle + A(\mathbf{k})\ G_0^+(k^2/2m)\ |\beta\rangle \qquad (4)$$

with

$$n = \langle\beta|\ G_0^2(E_B)\ |\beta\rangle^{-1/2} \qquad (5)$$

and

$$A(\mathbf{k}) = V_0\ \langle\beta|\mathbf{k}\rangle\ /(1 - V_0\ \langle\beta|\ G_0^+(k^2/2m)\ |\beta\rangle , \qquad (6)$$

where the usual notation $G_0^+(z)=G_0(z+i\varepsilon)$ has been used. The set of eqs.(3) and (4) is orthogonal and complete so that we are able to expand the wave function after the ν-th kick in the unperturbed basis:

$$|\psi(\nu)\rangle = a_B^{\nu}\ |\Phi_B\rangle + \int d\mathbf{k}\ a^{\nu}(\mathbf{k})\ |\Phi_k^+\rangle . \qquad (7)$$

On the other hand the wave function just before the ν-th kick is given by /6/

$$|\psi(\nu)\rangle = e^{-iH_0T}\ e^{-igT\ F((\nu-1)T)\ |\beta\rangle\langle\beta|}\ |\psi(\nu-1)\rangle. \qquad (8)$$

Employing the expansion (7) and using the operator idendity

$$e^{i\alpha|\beta\rangle\langle\beta|} = 1 + |\beta\rangle\langle\beta|\ (e^{i\alpha} - 1) , \qquad (9)$$

one finally gets the quantum map between the states before the ν-th and $(\nu-1)$-th kick

$$e^{iE_BT}\ a_B^{\nu} = a_B^{\nu-1}\ (1 + P^{\nu-1}\ |\langle\beta|\Phi_B\rangle|^2) + P^{\nu-1}\ \langle\Phi_B|\beta\rangle\ I^{\nu-1} \qquad (10)$$

$$e^{ik^2/2m\ T}\ a^{\nu}(\mathbf{k}) = a^{\nu-1}(\mathbf{k}) + a_B^{\nu-1}\ P^{\nu-1}\ \langle\beta|\Phi_B\rangle\ \langle\Phi_k^+|\beta\rangle + P^{\nu-1}\ \langle\Phi_k^+|\beta\rangle\ I^{\nu-1} \qquad (11)$$

with

$$P^{\nu-1} = e^{-igT\ F((\nu-1)T)} - 1 \qquad (12)$$

and

$$I^{\nu-1} = \int d\mathbf{k}\ a^{\nu-1}(\mathbf{k})\ \langle\beta|\Phi_k^+\rangle. \qquad (13)$$

The practical calculation of the unknown coefficient a_B^{ν} and the unknown function $a^{\nu}(\mathbf{k})$ starts with the computation of the remaining matrix elements $\langle\beta|\Phi_B\rangle$ and $\langle\beta|\Phi_k^+\rangle$. Since the operator G_0, appearing in both of them, involves only the momentum operator, it is convenient to work in momentum representation. If one supposes the state $|\beta\rangle$ to be a harmonical oscillator state $|nlm\rangle$ with the radial node number n, angular quantum numbers l,m and oscillator parameter b, all arising matrix elements in eqs. (3) - (13) can be evaluated analytically (see for example Appendix A of ref. /15/).

One should note that due to the action of G_0 in eqs. (3) and (4) the asymptotic behaviour in space representation of the eigenfunctions of H_0 corresponds to the correct behaviour for finite depth potentials /15/.

What is left to do in practice at a computer is mainly the calculation of the, in fact one-dimensional, integrals $I^{\nu-1}$ in eq.(13). As an advantage, their integral kernels are proportional to complex conjugated oscillator functions $\langle\beta|\mathbf{k}\rangle$ (see eqs. (4) and (6)) which are for large values of the magnitude of $\mathbf{k}$ rapidly decreasing functions ($\propto e^{-(k^2/2b^2)}$). Consequently, the numerical integration after the analytical evaluation of the matrix elements of G_0^+ makes no serious trouble. So we conclude that the problem of the kicked quantum system (1) and (2) with full inclusion of the continuum has been solved for simple form factors to a large extent analytically for an arbitrary modulation function F (for example periodical, quasiperiodical or stochastic modulations).

First numerical investigations have been performed for the particularly simple case n=0 and l=0. As the initial state we choose a stationary bound state of H_0, i. e. $a_B{}^0=1$ and $a^0(\mathbf{k})=0$. The parameters have been adjusted to typical values of the nuclear physics describing a weakly bound valence nucleon: $E_B=-8.3$ MeV and b=3 fm. The modulation function F at time $t=\nu T$ has been chosen as

$$F(\nu T) = \cos(2\pi\nu x) , \qquad (14)$$

where $x=T/\tau$ is the ratio of the two periods of the quasiperiodically driven system. The special case x=1 refers to the pure periodic excitation of the system. In order to characterize the period T of the kicks , a parameter δ has been introduced by $\delta=2\pi\, T^{-1}/|E_B|$ which is just the ratio of the external kicking frequency and the binding energy. The physical quantities after the ν-th kick like the total ionization probability W_I, the probability for the occupation of the bound state W_B, the differential emission spectrum $dW_I/d\varepsilon$ for particle emission with energy $\varepsilon=k^2/2m$, and the total energy E are defined, following the usual procedures, as

$$W_I = \int_0^\infty dk\, k^2\, |a^\nu(k)|^2 \qquad (15)$$

$$W_B = |a_B{}^\nu|^2 \qquad (16)$$

$$dW_I/d\varepsilon = (2m)^{3/2}\, \varepsilon^{1/2}\, |a^\nu((2m\varepsilon)^{1/2})|^2 \qquad (17)$$

$$E = W_B\, E_B + \int_0^\infty d\varepsilon\; \varepsilon\; dW_I/d\varepsilon\ . \qquad (18)$$

For irrational x values (incommensurate case) compared to rational x values (commensurate case) it has been found in a quasiperiodically driven two-state model /5,6/ that the rapid decay of the autocorrelation function defined in ref. /16/ could be a strong signal for the onset of quantum chaos. In the spirit of these investigations it is convenient to introduce a similar correlator as in ref. /6/ including the continuum

$$C(\nu) = \Big|\lim_{N\to\infty} 1/N \sum_{\mu=0}^{N} [(a_B{}^\mu)^*\, a_B{}^{\nu+\mu} + \int_0^\infty dk\, k^2\, (a^\mu(k))^*\, a^{\nu+\mu}(k)]\Big| . \qquad (19)$$

With the above definitions of the physical quantities we have examined the system for all parameter combinations with $x=1, 1/3, 1/\pi$ and $\delta=0.01$ (very subthreshold ionization), $3, \pi, 3.5$ and for a small - compared to the binding energy - intensity parameter of the external field g=1 MeV and for an extremely large field parameter g=400 MeV. The calculations were done with a mesh in k-space of 0.01 fm^{-1} and a maximal k-value of 2.0 fm^{-1}. These values ensure the conservation of the norm $W_I + W_B = 1$ within an error smaller than 10^{-6} for all considered time. The maximal kick number in the calculations was N=10000. All calculations have been performed at the CDC-6500 computer at Dubna.

As a general feature concerning the total ionization probability we found in accordance with ref. /6/ that the product

ρ=gT is the main control parameter for two quite different regimes in the dynamical behaviour of the system. For small values of ρ (in the order of 1) it does not matter whether x is a rational or irrational number. The total emission probability exhibits a rather regular, oscillating shape in time as it can be seen, for example, in the upper part of Fig. 1. for the special case δ=3. The main difference between the results for x=1/3 and x=1/π are only quantitative differences in magnitude and shape of the smooth functions in time. The magnitude of the oscillations of w_I depends on the parameter combination and reaches values between 0.05 and 0.98 for the parameter values under consideration. This situation

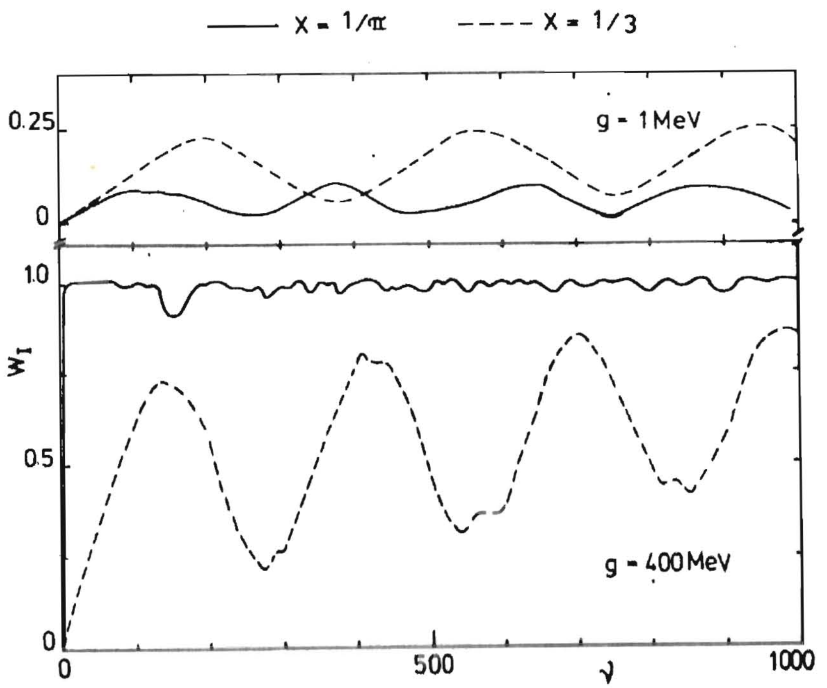

Fig. 1. Dependence of the ionization probability on the kick number for two frequency ratios x (δ=3.). Upper part: "regular" regime (gT ≈ 0.3), lower part: "irregular" regime (gT ≈ 100.).

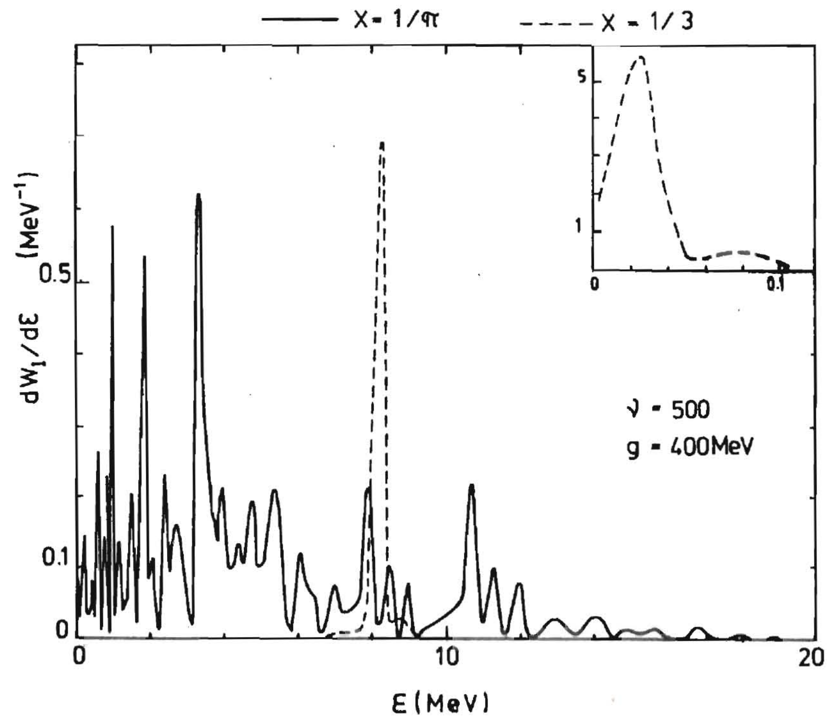

Fig. 2. Differential ionization probability versus emission energy after 500 kicks in the "irregular" regime (lower part of Fig. 1.).

changes drastically if one adjusts ρ to be of an order of 100 either by increasing the external field strength or diminishing the frequency of the external field (enlarging of T, not shown in the figures). Now the emission probability jumps from 0 to 1 after a few kicks for the incommensurate value of x. For larger times (up to 10000 kicks) W_I remains in the vicinity of 1 (full curve in the lower part of Fig. 1). On the other hand the response of the system for commensurate ratios of the characteristic frequencies and large ρ values does not differ qualitatively from the case ρ≈1 (dashed curve in the lower part of Fig. 1).

These two different regimes can be observed even more clearly in the differential emission probabilities. For low ρ values in all performed calculations and for high ρ values with commensurate frequency ratios the differential emission spectrum is governed by one or a few extremely narrow peaks with fixed positions in time. But in the "irregular" regime with high ρ values and incommensurate frequency ratios the spectrum exhibits a broad band behaviour with a lot of peaks and with complicated structure. As an example, in Fig. 2. two spectra after 500 periods of the kicking potential are demonstrated for the same parameter values used for the lower part of Fig. 1. While for $x=1/3$ (dashed curve) only two peaks exist at 0.02 MeV and 8.5 MeV (please note the inset in Fig. 2. for the very low energy region), the spectrum for $x=1/\pi$ (full line) is dislocalized and covers the energy region from 0 to 20 MeV. An adequate feature could be stated for the autocorrelation functions $C(\nu)$. In Fig. 3. one can see that in difference to the bound spin 1/2 system /5,6/ all correlations are damped with time. But again there are big qualitative and quantitative differences in the time-behaviour. While for low ρ values (two upper parts of Fig. 3.) the more or less quasiperiodic autocorrelation functions are damped only weakly, in the "irregular" regime the correlator decays smoothly and very rapidly. This interesting property should be examined more carefully in further investigations.

Is the stated transition in the dynamics of the considered model system controlled by the dimensionless parameter ρ from a "regular" regime characterized by an oscillating emission probability, a spectrum with a few sharp resonances and a weakly decaying autocorrelation function to an "irregular" regime characterized by a fully ionized system, a broad band emission spectrum and strongly decaying autocorrelation function a signal of dynamical quantum chaos? At first sight one might think : yes, it could be. But a very important physical quantity, the total energy of the particle, has not shown in our calculations an increase in time as it has been assumed to be essentially for a chaotic regime in refs. /1,3,6/. The energy in the "irregular" case up to very large time is limited and does not exceed a certain value. Such a

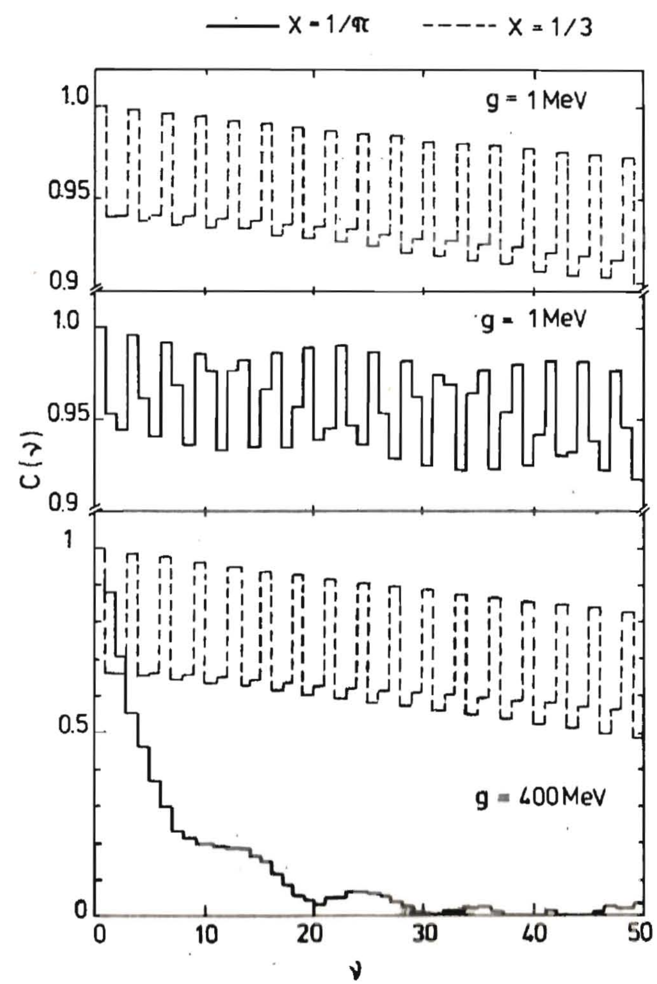

Fig. 3. Auto-correlation functions in the "regular" (two upper parts) and in the "irregular" regime (lower part). The parameters are the same as in Fig. 1.

property has been observed in all calculations and is connected with the fact that the differential emission spectrum,showing a broad band behaviour in a certain energy interval in the continuum, also does not differ significantly from zero above a maximum emission energy. But in comparison with ref. /7/ an asymptotic value of the energy could not be established. In all calculations the energy remains time-dependent up to 10000 kicks.

Finally, we conclude that the kicked separable potential with a three-dimensional form factor including one bound state and a continuum of scattering states has been treated almost analytically. This model allows a large variety of investigations concerning quantum chaos, dislocalization of a quantum particle in periodic, quasiperiodic or otherwise time-dependent external fields, and the time-behaviour of the energy gain in such fields. First results indicate in agreement with ref. /5,6/ that for large values of a phase (external field strength parameter multiplied by the kicking period) it could exist an "irregular" regime for the dynamics (incommensurate ratio of two external frequencies). Such a regime is characterized by rapidly decaying autocorrelation functions, the onset of nearly complete ionization already after a few periods and a broad band irregular spectrum in the basis of the unperturbed scattering functions.

Several extensions of the presented investigations can be made. The incorporation of realistic cosine-functions rather than delta-pulses for the periodically driven system has been already performed /17/. On the other hand, the extension of the potential to a finite rank potential with two or more bound states and the continuum is straightforward and in progress. Further, the quasienergy spectrum of the model can be studied, at least for periodic and quasiperiodic driving potentials with commensurate frequency ratios.

The author gratefully acknowledges very stimulating and helpful discussions with Prof. R. Reif and Prof. W. John from the Technische Universität, Dresden.

References

/1 / G. Casati, B. V. Chirikov, D. L. Sheplyansky, and I. Guarneri, Phys. Rep. 154 (1987) 77 and references therein.
/2 / K. A. H. van Leeuen, G. V. Oppen, S. Renwick, J. B. Bowlin, P. M. Koch, R. V. Jensen, O. Rath, D. Richards, and J. G. Leopold, Phys. Rev. Lett. 55 (1985) 2231,
R. V. Jensen, Phys. Rev. Lett. 54 (1985) 2057.
/3 / M. V. Berry, Proc. R. Soc. Lond. A 413 (1987) 183.
/4 / F. M. Izrailev, Phys. Lett. A 125 (1987) 251.
/5 / Y. Pomeau, B. Dorizzi, and B. Grammaticos, Phys. Rev. Lett. 56 (1986) 681.
/6 / P. W. Milloni, J. R. Ackerhalt, and M. E. Goggin, Phys. Rev. A 35 (1987) 1714.
/7 / J.-M. Lopez-Castillo, C. Tannous, and J.-P. Jay-Gerin, Phys. Lett. A 123 (1987) 316.
/8 / B. V. Chirikov, Proc. R. Soc. Lond. A 413 (1987) 1845.
/9 / G. Casati, B. V. Chirikov, J. Ford, and F. M. Izrailev, Lecture Notes in Physics (Springer) 93 (1079) 334.
/10/ D. R. Grempel, R. E. Prange, and S. Fishman, Phys. Rev. A 29 (1984) 1639.
/11/ H. Frahm and M. J. Mikeska, Phys. Rev. Lett. 60 (1988) 3.
/12/ K. Rzazewski and J. Mostowski, Phys Rev. A35 (1987) 4414.
/13/ J. Revai, Nucl, Phys. A 438 (1985) 512,
B. Milek, R. Reif, and J. Revai, Phys. Lett. 150B (1985) 65.
/14/ B. Milek, R. Reif, and J. Revai, Phys. Rev. C 37 (1988) 1027.
/15/ B. Milek and R. Reif, Nucl. Phys. A 458 (1986) 354.
/16/ G. Casati and I. Guarneri, Phys. Rev. Lett. 50 (1983) 640.
/17/ J. Revai, private communication.

Received by Publishing Department
on September 16, 1988.