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UNIFIED SEMIMICROSCOPIC APPROACH SCATTERING OF LOW ENERGY PROTONS AND ALPHA-PARTICLES BY NUCLEI

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1. Introduction

The study of scattering of low energy protons and α-particles by nuclei represents the important source of information concerning the properties of nuclear structure. Recently some new schemes of experimental data analysis including in an obvious way the effective nucleon-nucleon forces and nuclear transition densities were practised on a large scale /1/. The exchange nucleon-nucleon correlations provided by the influence of Pauli principle play the essential role in the interaction of low energy nucleons and α-particles with nuclei. For a long time these correlations have been taken into account explicitly in the calculations of the nucleon-nucleus optical potential (OP) and of the inelastic transition formfactors (ITF), (see f.e./2/). As for alpha-particles interacting with medium and heavy nuclei this explicit account of one-nucleon exchange in the density matrix formalism has been limited only to the case of elastic scattering /3, 4/.

The inelastic scattering of α-particles from nuclei is equally tentative both to the neutron and proton components of nuclear transition densities, while the inelastic low energy proton scattering is more sensitive to the neutron component. Consequently, mutual analysis of inelastic scattering of protons and α-particles by the same target nucleus may provide the information on the isospin structure of inelastic nuclear transitions. Hence, it is actual to develop a new scheme of unified description of low energy nucleon and α-particle interaction with nuclei based on the same effective nucleon-nucleon forces and on the explicit account of exchange nucleon-nucleon correlations in the calculations of nucleon-nucleus and α-particle - nucleus OP's and ITF's.

It has been determined by the studies in the framework of optical model both for nucleons and composite particles that the OP depends on the energy of the ongoing particle /5/. The theoretical study of the energy dependence of nucleon OP has shown that it is provided in general by the exchange and many-particle nucleon-nucleon correlations contribution /6, 7/. As to the composite particles (i.e. α-particles), the role of the exchange correlations in the energy dependence of α-particle OP was an open problem until recently /8/. Thus the investigation of the one-nucleon exchange contribution to the energy dependence of α-particle OP and the theoretical study of the energy dependence of "geometry" of the potential with the comparative analysis of the previously established empirical regularities are actual.

This paper deals with the problems that have been formulated above. We use as a basis the semimicroscopic approach (SMA) that was developed in papers /9 - 11/ for the nucleon-nucleus scattering. Section 2 contains the formalism of the approach which includes the closed expressions for the α-particle OP's and ITF's. Here also the phonon structure of interaction in the system α-particle - target-nucleus is investigated. The experimental data analysis of the scattering of 25.05 MeV protons and 104 Mev α-particle by ^{90}Zr is in the third section. Here the isospin structure of the inelastic transitions is investigated on the base of the standard collective model and of the quasiparticle-phonon model (QPM) of nucleus /12/. The energy dependence of the semimicroscopic α-particle OP is studied in the fourth section , a special attention is paid to the changes of the potential "geometry" with the energy. The section also includes an additional experimental data analysis. And in the last section the main results and conclusions are formulated.

2. The formalism of the SMA

The real part of the interaction potential of α-particle with the target-nucleus can be obtained in the single folding model as /13/:

$$U^{\alpha A}(\vec{R}) = \int U^{NA}(\vec{R}-\vec{r})\rho_{\alpha}(\vec{r})d\vec{r}. \qquad (1)$$

For simplicity we are performing the change of variable in (1):

$$U^{\alpha A}(\vec{R}) = \int U^{NA}(\vec{s})\rho_{\alpha}(\vec{R}-\vec{s})d\vec{s}. \qquad (2)$$

By making the multipole expansion for both right and left parts of expression (2) one can obtain:

$$U_{\lambda}^{\alpha A}(R) = \int_0^{\infty} U_{\lambda}^{NA}(s)\rho_{\alpha\lambda}(R,s)s^2ds. \qquad (3)$$

Keeping in mind the development of a unified description of nucleon and α-particle scattering by nuclei, let us use for the nucleon OP's and ITF's the corresponding expressions obtained in the framework of SMA /10, 11, 14/. We'll get as a result:

$$U_0^{NA}(s) = U_0^D(s) + d\rho_0^2(s) + I_{00}(s)\,\frac{1+\varkappa^2(s)I_{02}(s)I_{00}(s)}{1-\varkappa(s)I_{01}(s)} + \frac{1}{4\pi}\sum_{\lambda}{}'\beta_{\lambda}^2\rho_{\lambda}^2(s) + \frac{1}{4\pi}\sum_{\lambda}{}'\beta_{\lambda}^2 I_{\lambda 1}(s)\varkappa(s)\cdot \left[U_{\lambda 0}^D(s) + 2d\rho_0(s)\rho_{\lambda}(s) + I_{\lambda 0}(s)\right] \qquad (4)$$

$$U_L^{NA}(s) = \beta_L\left[1 + I_{01}(s)\varkappa(s)\right]\left[U_L^D(s) + 2d\rho_0(s)\rho_L(s) + I_{L0}(s)\right] + \beta_L I_{L1}(s)\varkappa(s)I_{00}(s) + \frac{1}{\sqrt{4\pi}}d\sum_{\lambda\lambda'}{}'\beta_{\lambda}\beta_{\lambda'}S_{L\lambda\lambda'}\rho_{\lambda}(s)\rho_{\lambda'}(s) + \frac{1}{\sqrt{4\pi}}\sum_{\lambda\lambda'}{}'\beta_{\lambda}\beta_{\lambda'}S_{L\lambda\lambda'}I_{\lambda 1}(s)\varkappa(s)\left[U_{\lambda'}^D(s) + 2d\rho_0(s)\rho_{\lambda'}(s) + I_{\lambda' 0}(s)\right]. \qquad (5)$$

Here $U_0^D(s)$ and $U_{\lambda}^D(s)$ - are nucleon OP and ITF calculated by a simple folding model (without any account of correlations):

$$U_{\lambda}^D(s) = \int_0^{\infty} v_{D\lambda}(s,s')(s')^2\rho_{\lambda}(s')ds' \qquad (6)$$

$$U_0^D(s) = \frac{1}{\sqrt{4\pi}}U_{00}^D(s). \qquad (7)$$

The exchange integrals $I_{\lambda 0}(s)$ are determined for $\lambda \neq 0$ by formula:

$$I_{\lambda 0}(s) = \sqrt{4\pi} \int_0^\infty U_E(t) \rho_{\lambda 00}(s,t) j_0(K_0(s)t) t^2 dt. \quad (8)$$

And in all other cases $I_{\lambda n}(s)$ can be expressed as:

$$I_{\lambda n}(s) = \frac{1}{n!} \int_0^\infty U_E(t) \rho_{\lambda 00}(s,t) j_n(K_0(s)t) t^{n+2} dt. \quad (9)$$

For the density matrix components $\rho_{\lambda 00}(s,t)$ we have:

$$\rho_{\lambda 00}(s,t) = \frac{1}{\sqrt{4\pi}} \int \rho(\vec{s}, \vec{s}+\vec{t}) Y_{\lambda 0}(\omega_{\vec{s}}) d\omega_{\vec{s}} d\omega_{\vec{t}}. \quad (10)$$

As a result of localization performed in accordance with /15/ for nonlocal exchange terms the values of the $I_{\lambda n}(s)$ depend explicitly on nucleon momentum, which is determined by formula:

$$K_0^2(s) = \frac{2\mu}{\hbar^2} [E_N - U_0^D(s) - d\rho_0^2(s) - \frac{1}{2}(1-\tau_{0z}) V_c(s)]. \quad (11)$$

Here $\tau_{0z} = -1$ for protons and $\tau_{0z} = 1$ for neutrons. In Eq. (4) - (9) $U_{D\lambda}(s,s')$ is a component of multipole expansion of the direct part of effective forces, $U_E(t)$ is an exchange part of effective forces and d is a parameter characterising the dependence of these forces on the nuclear matter density distribution. Unmentioned definitions can be found in /10, 11, 14/.

Let us note some features of the ITF $U_L^{NA}(s)$ expansion with the parameter β_λ. This expansion is motivated by the fact that the developed formalism is used for the description of inelastic scattering of protons and α-particles by the target-nucleus with the excitation of phonon states. In this case the use of a standard procedure of transition from the parameter β_λ to the phonon operators and the representative of the interaction operator for the ingoing particle and the

target-nucleus through the phonon operators demand the power expansion of ITF with the parameter β_λ (see also /16/).

The possibility of such expansion in SMA is defined by the choice of density matrix $\rho(\vec{\tau}, \vec{\tau}')$ in the framework of modified Slater approximation /17/. The explicit form of the expressions (4) and (5) was obtained with the account of standard linear in β_λ expression for $\rho_\lambda(\tau)$ /1/ and with the neglection of the dependence of the Fermi momentum K_F on parameter β_λ. The last fact was investigated in /18/ and it was shown that at the first approximation it is possible to skip the dependence of K_F on β_λ for the description of two-phonon states. For simplicity the Eq.(5) contains only the main, linear in $\varkappa(s)$ parts. The explicit view of the parts containing $\varkappa^2(s)$ could be found in /7, 18/. In these two works it is shown that the limitation up to quadratic in $\varkappa(s)$ parts is quite sufficient for the calculations of OP's and ITF's.

If one will set in Eq. (4) and (5) $V_E = 0$ and $d = 0$, i.e. ignore the exchange and many-particle(represented by the density dependence of effective forces) nucleon-nucleon correlations then the $U_L^{NA}(s)$ will be equal to $U_L^D(s)$ and in accordance with Eq. (3), (6), (7) one will obtain for the α-particle OP and ITF the standard double folding model expressions /19/. Thus for the case of α-particle SMA can be considered as the generalization of double folding model for the account of nucleon-nucleon correlations.

The second feature of semimicroscopic expressions for OP and ITF is their closed form (contrary to the corresponding expressions for the deformed nuclei /20/), i.e. no any iterations are needed for their calculations. This is extremely essential for the phonon model application to the target-nucleus state description.

Parameters β_λ, included in Eq. (4) and (5) have the meaning of the parameters of static deformation of nucleus.

For the inelastic transitions in nuclei with the dynamic deformation it is necessary to move from parameters β_λ to collective variables $\alpha_{\lambda\mu}$ characterizing the surface vibrations of nucleus and then to introduce operators of creation and annihilation of phonons /21/. By performing these operations we obtain for the interaction of nucleus with phonons the following expression:

$$V(\vec{R}, \hat{b}^{+}_{\lambda\mu}, \hat{b}_{\lambda\mu}) = \sum_{\lambda\mu}{}' f_{\lambda}^{\alpha A}(R) Y_{\lambda\mu}(\theta\varphi)\beta_{\lambda}\hat{\lambda}^{-1}[\hat{b}_{\lambda\mu} + (-1)^{\mu}\hat{b}^{+}_{\lambda-\mu}] +$$

$$+ \sum_{\lambda\mu\lambda_1\lambda_2\mu_1\mu_2} F^{\alpha A}_{\lambda\lambda_1\lambda_2}(R) Y_{\lambda\mu}(\theta\varphi) S(\lambda_1\lambda_2\lambda;\mu_1\mu_2)\beta_{\lambda_1}\hat{\lambda}_1^{-1} \cdot \quad (12)$$

$$\cdot [\hat{b}_{\lambda_1\mu_1} + (-1)^{\mu_1}\hat{b}^{+}_{\lambda_1-\mu_1}]\beta_{\lambda_2}\hat{\lambda}_2^{-1}[\hat{b}_{\lambda_2\mu_2} + (-1)^{\mu_2}\hat{b}^{+}_{\lambda_2-\mu_2}],$$

where $f_{\lambda}^{\alpha A}(R)$ and $F^{\alpha A}_{\lambda\lambda_1\lambda_2}(R)$ are correspondingly the α-particle radial form factors of the 1st and 2nd order:

$$f_{\lambda}^{\alpha A}(R) = \int_0^{\infty}\{[1 + I_{01}(s)\varkappa(s)][U_{\lambda}^{D}(s) + 2d\rho_0(s)\rho_{\lambda}(s) + I_{\lambda 0}(s)] +$$

$$+ I_{\lambda 1}(s)\varkappa(s)I_{00}(s)\}\rho_{\alpha\lambda}(R,s)s^2 ds \quad (12a)$$

$$F^{\alpha A}_{\lambda\lambda_1\lambda_2}(R) = \int_0^{\infty}\{d\rho_{\lambda_1}(s)\rho_{\lambda_2}(s) + I_{\lambda_1 1}(s)\varkappa(s)[U^{D}_{\lambda_2}(s) +$$

$$+ 2d\rho_0(s)\rho_{\lambda_2}(s) + I_{\lambda_2 0}(s)]\}\rho_{\alpha\lambda}(R,s)s^2 ds. \quad (12b)$$

Eqs. (12) have the same phonon structure as the analogous expressions of the standard coupled channel method. The difference is that the radial form factors are not the derivatives of the spherically simmetric part of the potential /16/, but they are calculated according to Eqs. (12a) and (12b) on the base of the effective forces and the transition densities with the account of nucleon-nucleon correlations. Thus the Eqs.(12) may be used for the modification of standard computer codes (f.e. ECIS /22/) for the experimental data analysis in the strong coupling approach.

The Eqs. (12) give the explicit form of interaction of nucleons and α-particles with phonons and include the transition densities $\rho_{\lambda}(s)$ that may be calculated in the framework of different semimicroscopic models /12, 23, 24/. Thus the formalism of the present approach gives the possibility of testing the various nuclear models not only in the description of the electromagnetic transition probabilities but also in the representation of inelastic scattering of low energy nucleons and α-particles from nuclei. This is extremely essential for the investigation of the isospin structure of inelastic transitions and for the extraction of the information concerning the differences in proton and neutron transition densities.

3. Experimental data analysis

We apply the formalism represented above to the analysis of elastic and inelastic scattering of protons with the energy E_p = 25.05 Mev and of 104 MeV α-particle from ^{90}Zr nucleus. In both cases it is possible to consider the direct mechanism as the dominant one. Besides, with such choice of proton and α-particle energy ($E_p \simeq E_{\alpha}/4$) the same nucleon OP's and ITF's can be used for both cases.

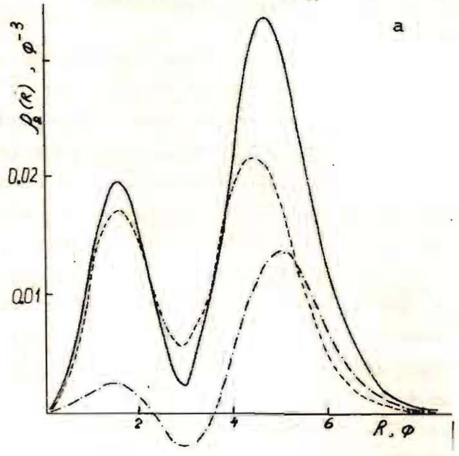
Let us consider briefly the calculational scheme. As a part of the effective nucleon-nucleon forces independent of the matter density distribution of the nucleus the Schmid-Wildermuth forces were used /25/ which successfully described the free low energy NN and $\alpha\alpha$ scattering and also the cluster properties of light nuclei. In /10/ this interaction was modified by the introducing of the density dependence in the following form:

$$V_{\rho}(\vec{r}, \vec{r}') = d\,\delta(\vec{r} - \vec{r}')\rho\left(\frac{\vec{r} + \vec{r}'}{2}\right). \quad (13)$$

The parameter "d" here is to be defined from the description of the elastic scattering cross-sections by minimization of the χ^2. The two parameters of Fermi density charge distribution were extracted from electron scattering data for ^{90}Zr /5/ (in present calculations we did not take into account the differences between proton and neutron distributions for target nucleus ground state). As to the matter density distribution for α-particle the Gaussian representation was used with the oscillator parameter b = 1.39 fm /26/. It is possible to use the transition densities either from the collective model ($\beta_{\lambda}\rho_{\lambda}(r) = \beta_{\lambda} r\, d\rho_0(r)/dr$), or to calculate them in the framework of QPM /12/.

The neutron and proton components of the transition densities for $|2^+\rangle$ and $|4^+\rangle$ states of ^{90}Zr are shown on Fig. 1. The method of calculation is described in detail in /27/. It should be noted that the calculations of proton components were done with the isoscalar and isovector constants of multipole interaction chosen from the description of experimental values of reduced probabilities B(Eλ). And the calculations of neutron components were done with the already fixed values of isoscalar and isovector constants. Thus, as has been mentioned above, the

Fig.1. Transition densities, for ^{90}Zr nucleus calculated in quasiparticle-phonon model (QPM): Hatched line - the proton component, Hatched-dotted line - the neutron component, full line - the sum of protons and neutron components. a) for $|2^+\rangle$ state, b) for $|4^+\rangle$ state.



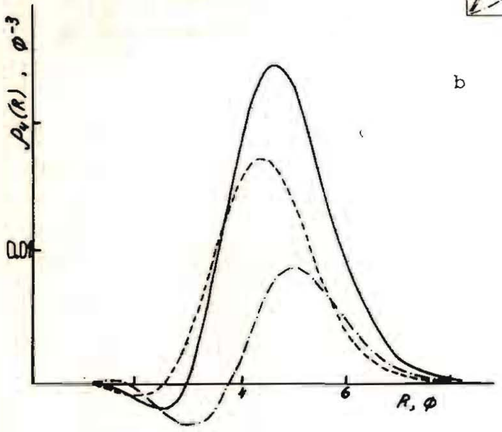


Table 1

E_α, Mev	40	60	80	100	120	150	200	250	300
U_0, Mev	241.7	238.6	235.6	232.7 128.9*)	230.3	226.6	221.6	217.5	214.1
U (6 fm) MeV	48.65	47.57	46.54	45.58 48.14*)	44.79	43.58	42.02	40.2	39.84

*) The values of the potential for the Woods-Saxon parametrization at the E_α = 104 MeV were taken from the paper /29/.

analysis of scattering of particles from nuclei gives the possibility of checking the model predictions for the neutron transition densities components. One can see from Fig. 1 that $\rho_2(r)$ calculated on the base of QPM for both neutrons and protons has (in contrast to macroscopic transition densities) the oscillating structure for the inner region of nucleus. But the cross-sections of scattering are not sensitive to these structures due to the presence of absorbtion. For both the states the maximum of $\rho_\lambda^{(n)}(r)$ is shifted to the surface region in comparison with the maximum of $\rho_\lambda^{(p)}(r)$ and the values in maximum are significantly greater for the proton components $\rho_\lambda(r)$.

The real part of proton and α-particle OP's and ITF's were calculated on the base of mentioned choice of the effective forces, $\rho_0(r)$, $\rho_2(r)$ and $\rho_4(r)$ and the formalism of previous section. For the other parts of OP (coulomb, imaginary and spin-orbit terms) the standard optical model predictions were used. The corresponding parameters for protons were taken from /28/ and for α-particles from /29/.

The results of the calculations of differential cross-sections of 25.05 MeV protons and of 104 MeV α-particles elastic and inelastic scattering from ^{90}Zr nucleus are shown together with experimental data in Fig. 2 and Fig. 3. Full lines correspond to the QPM transition densities. There are also the results of calculation with collective model transition densities for $|4^+\rangle$ state excited in α-particle scattering . **This is the case when the difference between above results is the largest.** In other cases the difference is unsignificant. The corresponding results for the macroscopic transition densities are not shown but the optimal values of the parameters β_λ extracted from the analysis of experimental cross-sections are represented in Table 1.

Let us discuss the obtained results. It could be seen, that with the same optimum value of the parameter "d" equal to 442 MeV·fm^6, the good description of angular distributions of elastically scattered protons and α-particles has been obtained. It should be noted that the value of parameter "d" is well matched with the value d = 488 MeV·fm^6, which was obtained in /30/ for the isotopes $^{94,96,100}Mo$. For the α-particle scattering description no variation of the absorbtion

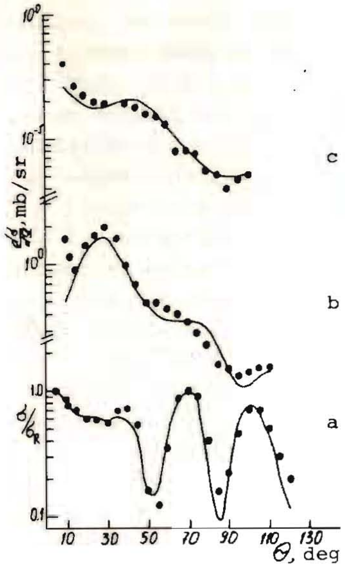
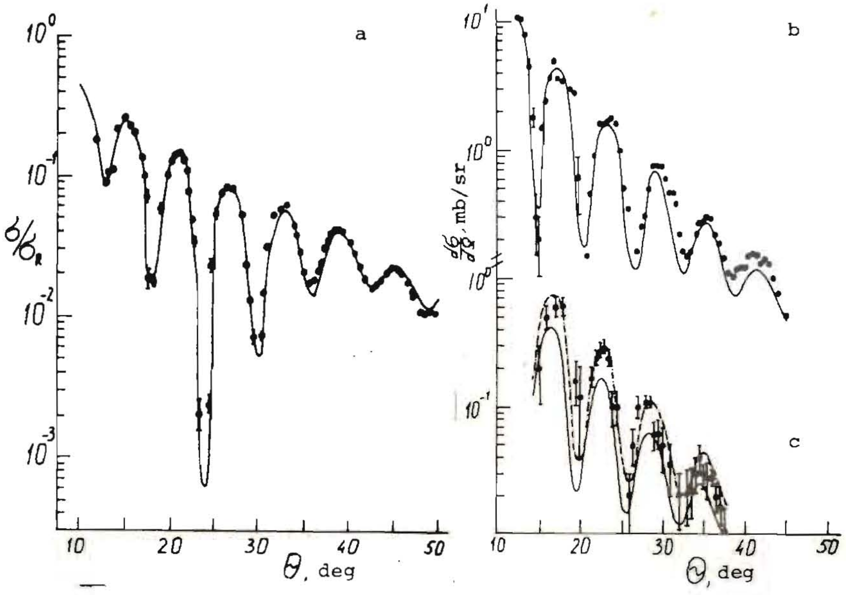


Fig.2. Angular distributions of 25.05 MeV proton scattering from ^{90}Zr calculated in SMA with the use of QPM transition densities: a) elastic scattering, b) inelastic scattering ($|2^+\rangle$, E_x = 2186 keV), c) inelastic scattering ($|4^+\rangle$, E_x = 3077 keV). Dots - the experimental data /28/.

Fig.3. The same as Fig.2 for 104 MeV α-particles. Hatched-dotted line - calculations with the transition densities of collective model. Dots - the experimental data /29/.



potential parameters has been made and for the protons the improvement of experimental data description was achieved by increasing W_D from 1.5 MeV /28/ to 2.5 MeV.

The values of deformation parameters β_2 and β_4 were extracted from the fits to the experimental angular distributions. As in the formalism of SMA the information on the deformation of matter distribution and not of a potential is introduced, the values of β_2 extracted from the description of (p, p') and (α, α') scattering in the present work appeared to be greater than the corresponding values for the deformation parameters of proton and α-particle potential obtained in /29, 31/. As has been mentioned in the Introduction the inelastic scattering of α-particles is sensitive equally well to the distribution both of protons and neutrons, while the scattering of low energy protons is more sensitive to the neutron distribution. Due to this fact the value of β_2 extracted from the analysis of (α, α') in SMA, and which is greater than the optimum value of β_2 from the description of (p, p') scattering, gives that for the ^{90}Zr state $|2^+\rangle$ $\beta_p / \beta_n > 1$. This result also is consistent with the fact that $\beta_{nn'} > \beta_{pp'}$ and $\beta_{em} > \beta_{pp'}$ (see Table 1). The values of β_4 are very close to corresponding values obtained in /28/.

The results that are represented in Figs. 2 - 3 demonstrate the good description of angular distributions both for protons and α-particles for the excitation of $|2^+\rangle$ state. Thus it is not necessary to renormalize the neutron component of transition density calculated in QPM. By calculating the ratio of moments of transition densities (Fig. 1) we obtain M_n/M_z = 0.96 for $|2^+\rangle$ state. From this fact the ratio β_n / β_p = 0.77 follows. This conclusion is in good agreement with the previous analysis which has used the macroscopic transition densities and also with the conclusions of effective charge model /33/.

For the $|4^+\rangle$ excited state the calculations also give the good description of angular distributions and the magnitude of cross-sections for protons while for α-particles the theoretical predictions are underestimated. This discrepancy can be due to the fact that in a present calculations the sum of neutron and proton components of transition densities was used. The introduction in an explicit way of the neutron and proton differences of transition densities into the calculational scheme may be essential for the description of $|4^+\rangle$ state.

In /34/ it was noted that for the inelastic scattering of α-particles from nuclei it is very essential to take into account not the static density dependence of the effective forces (SDD) but the so-called dynamic density dependence (DDD). The transition from SDD to DDD as was shown in /34/, leads to the increase of values of dynamic deformation parameters extracted from the inelastic scattering analysis by the factor of 1.5. Let us consider the mentioned effect in SMA. The use of SDD means that Eq. (13) includes only spherically symmetrical part of $\rho(\vec{r})$, i.e.:

$$V_\rho(\vec{r},\vec{r}\,') = \alpha\, \delta(\vec{r}-\vec{r}\,')\,\rho_0\left(\left|\frac{\vec{r}+\vec{r}\,'}{2}\right|\right). \tag{13a}$$

The well-known folding procedure with the interaction (13a) gives to the interaction potential of particle and nucleus the contribution which is equal to $\alpha\rho_0(r)\rho(\vec{r})$. The corresponding contribution to the ITF is equal to $\alpha\rho_0(r)\rho_\lambda(r)$. The account of DDD means the use of $V_\rho(\vec{r},\vec{r}\,')$ in the form of Eq. (13). In this case the contribution of the density dependent effective forces to the ITF will be equal to $2\alpha\rho_0(r)\rho_\lambda(r)$ that is taken into account in Eq.'s (5, 12a, 12b). Thus the DDD is presented in the approach of this paper. Because the signs of $2\alpha\rho_0(s)\rho_\lambda(s)$ and $U^D_\lambda(s) + I_{\lambda 0}(s)$ are different (see Eq. (12a, b)), the transition from SDD to DDD in SMA also leads to the increase of values of the parameters. But this increase is not so dramatic as in the paper /34/, which is due to the fact that in SMA the exchange nucleon-nucleon correlations are taken into account while in /34/ the latter are not considered.

4. Energy dependence of α-particle nucleus potential

In order to investigate the role of exchange nucleon-nucleon correlations in the energy dependence of the real part of α-particle - nucleus potential we have made the calculations for the system (α + ^{90}Zr) in the energy region from zero up to 300 MeV. The part of the calculations is represented in

Fig.4. Angular distributions of α-particle elastic scattering from ^{90}Zr calculated in SMA (with the account of the exchange nucleon-nucleon correlations and the density dependence of the effective forces). Hatched line - the calculation without the mentioned above effects. Dots - the experimental data /37/.

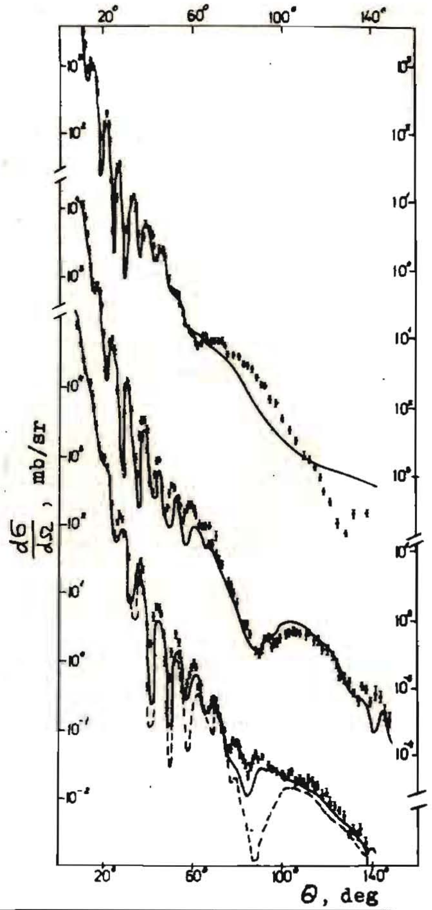


Table 2

λ	β_λ	Process	Energy, MeV	Reference
2	0.094	B(E2)		/32/
2	0.085	(n, n')	11	/31)
2	0.070	(p. p')	12.7 - 61.2	/31/
2	0.086	(p, p')	25.05	present work
2	0.086	(α, α')	104	/29/
2	0.12	(α, α')	104	present work
4	0.053	(p, p')	25.05	/28/
4	0.045	(p, p')	25.05	present work
4	0.045	(α, α')	104	present work

Table 2. The essential feature of the calculated potential is the deviation from the Woods-Saxon form for the radial dependence. One can see from the data of Table 2 that inside the nucleus at E = 100 MeV the values of phenomenological and microscopic potentials differ almost by the factor of two, while at the surface region the difference is about 5%.

As a function of energy the constructed potential inside the nucleus can be approximated by two types of expressions - linear and quadratic:

$$U_{(E)} = U_o - \gamma E \quad U_o = 247.8\ \text{MeV} \quad \gamma = 0.15 \tag{14a}$$

$$U_{(E)} = U_o - \gamma E + \beta E^2 \quad U_o = 247.8\ \text{MeV} \quad \gamma = 0.15 \quad \beta = 0.00019\ \text{MeV}^{-1}. \tag{14b}$$

Eq. (14a) gives the good description of the energy dependence of the potential below 100 MeV. As for the region $\Delta E = 100 - 300$ MeV the approximation works with the 35% accuracy. Quadratic approximation (14b) is much better and gives 6% accuracy.

The comparison of data of the 2nd and the 3rd lines of Table 2 shows, that for the inner part of nucleus the potential depends on energy lesser than on the surface. This fact allows one to take the following form of the potential

$$U(E,R) = U_o[1 - \alpha(R)E]f(R). \tag{15}$$

Here $\alpha(R_{1/2}) > \alpha(0)$. One can find from the described analysis that for the radius $R_{1/2}$ $\alpha(R_{1/2}) = 1.7\alpha(0)$, i.e. the contribution of the exchange correlations to the energy dependence of potential is increased by a factor of 1.7 that speaks about the localization of the exchange correlations at the surface region. Eq. (15) can also be interpreted as the energy dependence of radius $R_{1/2}$. Note that previously through the empirical analysis in several cases the energy dependent potentials with $R_{1/2}(E)$ were used (see examples in /35/). For nucleons such dependence was justified by semimicroscopic investigations /6, 7, 9/, while for α-particles this problem on the SMA basis was not studied.

Present calculations allow one to reveal the energy dependence of the radius $R_{1/2}$ for alpha-nucleus potential. It is possible to use the two types of approximations again:

$$R_{1/2}(E) = R_{0\,1/2} - \gamma_R E \quad R_{0\,1/2} = 4.69\ fm \quad \gamma_R = 0.001\ fm\cdot MeV^{-1} \tag{16a}$$

$$R_{1/2}(E) = R_{0\,1/2} - \gamma_R E + \beta_R E^2 \quad \gamma_R = 0.0011\ fm\cdot MeV^{-1} \quad \beta_R = 0.0000012\ fm\cdot MeV^{-2}. \tag{16b}$$

Eq. (16a) analogous to (14a) gives good agreement with our SMA results up to 100 MeV energy of α-particles. As for the 100-300 MeV energy region - the accuracy is poor (up to 35%). The quadratic approximation works with the accuracy 1% in the whole energy region (0 - 300 MeV).

Thus the SMA supports the conclusions of the empirical analysis on the energy dependence of radius $R_{1/2}$ of α-particle - nucleus potential, the decrease of $R_{1/2}$ with the increase of energy /36, 37/. It can be shown that the functional dependence of Eq. (16a) type introduced into a standard optical model is equivalent to the functional dependence of the potential of Eq. (15). In fact by making the Taylor expansion of the potentials $U(R_{1/2}(E), R)$ in the vicinity of $R_{1/2} = R_{o\,1/2}$ one can get:

$$U(R_{1/2}(E), R) = U(R_{0\,1/2}, R) - \gamma(R)E, \tag{17}$$

where

$$\gamma(R) = \gamma_R\, \partial U(R_{0\,1/2}, R)/\partial R_{0\,1/2}. \tag{17a}$$

Supposing the Woods-Saxon radial dependence for $U(R)$ we obtain from Eq. (17), (17a):

$$U(R_{1/2}(E), R) = U(R_{0\,1/2}, R)[1 - \alpha(R)E] \tag{18}$$

$$\alpha(R) = \frac{\gamma_R}{a}\cdot\frac{\exp((R - R_{0\,1/2})/a)}{1 + \exp((R - R_{0\,1/2})/a)}. \tag{18a}$$

One can see that $\alpha(R_{1/2}) > \alpha(0)$ and that the maximum of the function $\alpha(R)$ is situated in the surface region of the nucleus. This conclusion corresponds to SMA results (see discussion of Eq. (15) and Table 2).

The constructed semimicroscopic potentials were tested for the several values of energies in the description of experimental cross-sections of elastic scattering of α-particles in the wide angular region (see Fig. 4). The good overall agreement was obtained to the experiment in the description of the main features of angular distributions: oscillations at small angles, the shift of the critical angle for those oscillations with the change of the energy, the dramatic diminishing of cross-sections at E_α = 79 MeV and 99.5 MeV for the angles greater than the critical angle, peculiar form of angular distributions in the angular region from $\theta = 80^\circ$ to $\theta = 140^\circ$ for E_α = 59 MeV and 79 MeV. In the same time there are some discrepancies between the calculated and the experimental cross-sections. But one must remember that the calculations were done without fitting of any parameter concerning the real part of the OP, the latter was calculated by a scheme of the previous section, the density dependence parameter "d" was fixed at the value 442 MeV·fm^6 (see sect. 3). Fitting was allowed only for the imaginary part of the OP.

In Fig. 4 results are presented also for the calculations without exchange and many-particle nucleon-nucleon correlations (batched line). So one can see the effect of nucleon-nucleon correlations on angular distributions both for oscillations and for the characteristic behaviour in the backward hemisphere. It should be noted that such behaviour was connected with the so-called "rainbow" scattering /38/. In paper /8/ this phenomenon was shown to be dominated by the exchange correlations. Thus the present analysis confirms this conclusion.

Let us discuss briefly the possible ways of the improvement of the calculational scheme. Firstly the more detailed account of density dependent factor including the dependence on matter density of ingoing α-particle is needed /39, 40/. At the other hand it is necessary to take into account for the interaction of α-particles with nuclei in a more completed way the effects of one-nucleon exchange /4, 8/. Until now these problems were treated separately:in papers /39, 40/ the effects of one nucleon exchange were not taken into account explicitly, and in papers /4, 8/ the density independent forces were considered.

It was mentioned in /39/ that the usual iteration procedure for the calculation of the exchange term for the potential of α-particle - nucleus interaction is rather ambiguous, that makes difficult the wide use of such scheme to the experimental data analysis and to the extraction of model-independent information on nuclear density. The calculational scheme for the exchange term which is presented here does not need any iteration procedures (see also /8/) and may appear the basis for the modified method of the analysis of elastic and inelastic scattering of low energy α-particles from nuclei.

5. Conclusion

Let us formulate the main results of the present work:

1) The unified formalism for the description of elastic and inelastic scattering of low energy nucleons and α-particles by nuclei has been developed in the coupled-channel approximation and on the base of semimicroscopic approach. The optical potentials and the inelastic transition form factors are constructed in a closed form with the account of the exchange nucleon-nucleon correlations and the density dependence of the effective forces.

2) The numerical realisation of the developed approach has been made for the description of scattering of protons with the energy 25.05 MeV and of α-particles with the energy 104 MeV from the target-nucleus ^{90}Zr on the base of the same effective nucleon-nucleon forces.

3) It was established from the analysis of the dynamic deformation parameters for proton and neutron density distributions that $\beta_p/\beta_n > 1$. The calculations made with the transition densities of the quasi-particle phonon model have confirmed this result. The value of the hexadecapole deformation parameter is in a good agreement with the published data.

4) The energy dependence has been investigated in a wide energy range for the semimicroscopic α-particle-nucleus potential. Simple approximations have been obtained for the energy dependence $U_0(E)$ and $R_{1/2}(E)$ and it was established that the strongest dependence corresponds to the surface region of the nucleus and is provided by the contribution of the nucleon-nucleon correlations.

5) The analysis of angular distributions of elastically scattered α-particles from ^{90}Zr nucleus has been made for several values of the bombarding energy in the wide angular region. The satisfactory agreement has been obtained for the description of experimental cross-sections of elastic scattering and the essential role of the nucleon-nucleon correlations in the nature of nuclear "rainbow" scattering has been shown.

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Дао Тиен Кхоа и др. E4-88-673

Единое полумикроскопическое описание рассеяния протонов и α-частиц низких энергий на ядрах

Развит формализм единого полумикроскопического описания взаимодействия нуклонов и α-частиц низких энергий с ядрами. Проводится анализ упругого и неупругого рассеяния протонов с энергией 25,05 МэВ и α-частиц с энергией 104 МэВ на ядре ^{90}Zr. Извлекается информация о различиях в деформации протонного и нейтронного распределений. Исследуется энергетическая зависимость полумикроскопического α-частичного потенциала, в том числе зависимость от энергии "геометрии" потенциала. Анализируется изменение угловых распределений упругорассеянных α-частиц с ростом энергии и вклад в сечения рассеяния нуклон-нуклонных корреляций.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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Unified Semimicroscopic Approach to Scattering of Low Energy Protons and Alpha-Particles by Nuclei

The unified approach has been developed to the description of the interaction of low energy nucleons and α-particles with nuclei. The analysis of elastic and inelastic scattering of 25.05 MeV protons and 104 MeV α-particles from ^{90}Zr is made. The differences in deformations of neutron and proton density distributions for ^{90}Zr nucleus are extracted. The energy dependence of the obtained α-particle - nucleus semimicroscopic potential is investigated, including the energy dependence for the "geometry" of the potential. The features of angular distributions of elastic α-particle scattering and the role of nucleon-nucleon correlations are analysed as a function of α-particle energy.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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