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THREE-DIMENSIONAL RELATIVISTIC EQUATIONS FOR $\pi\pi$, πN AND NN SCATTERING

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INTRODUCTION

At the present time, interest in the $\pi\pi$, πN and NN scattering processes in the energy region up to the second pi-meson creation threshold has been caused by the task of revealing and taking into account the quark degrees of freedom in the low-energy pion-nucleon interaction. For this purpose, in refs.[1-3] the field-theoretical, phenomenological quark models for the meson-nucleon vertex functions have been proposed.

The most general field-theoretical approach, which enables us to obtain the $\pi\pi$, πN and NN scattering t-matrices making use of the known meson-nucleon vertex functions, is the one based on the Bethe-Salpeter equation or its quasipotential reductions[4,5]. However, it is well known that these reductions are not unique and one can obtain different quasipotentials and propagators in the resulting three-dimensional equations due to a different choice of the reduction prescription. These equations differ from each other in the nonrelativistic limit[6] and it is necessary to use different phenomenological parametrization of the vertex functions in order to reproduce experimental results. Furthermore, the quasipotential is often derived with the use of additional simplifying assumptions, which we shall consider, using, as an example, the one-boson exchange model of the NN-interactions[7,8,15], obtained within the framework of the Blankenbecler-Sugar quasipotential approach. It is well known that in this model for nucleon-nucleon scattering the equation for the scattering t-matrix has the form of the Lippmann - Schwinger equation with the relativistic kinematics:

$$\mathcal{T}(\vec{q}\,'\vec{q};E_{\vec{q}}) = V(\vec{q}\,';\vec{q}) + \int d\vec{k}\, V(\vec{q}\,';\vec{k})\, G_o(\vec{k},E_{\vec{q}})\, \mathcal{T}(\vec{k},\vec{q};E_{\vec{q}}), \qquad (1)$$

where $\vec{q}\,'$ and $\vec{q}$ denote relative momenta of nucleons in the c.m. frame, $E_{\vec{q}}$ is the total energy of the system: $E_{\vec{q}} = 2\sqrt{m_N^2 + \vec{q}^{\,2}}$, $G_o(\vec{k};E_{\vec{q}})$ is the three-dimensional propagator for this quasipotential equation, and $V(\vec{q}\,';\vec{q})$ is connected with the kernel of the Bethe-Salpeter equation $K(q'q|P)$ in the following manner:

$$V(\vec{q}\,';\vec{q}) = K(q'q|P)\big|_{q_0'=q_0=0} \equiv K(p_1'p_2';p_1p_2)\big|^{p_1'+p_2'=p_1+p_2=P}_{q_0'=q_0=0} \tag{2}$$

where q_0' and q_0 are the relative energies of nucleons in the c.m. frame, $P=(E_{\vec q};0)$ is the total four-momentum of the system and $p_i=(1/2)P\pm q$ are the four-momenta of individual nucleons.

In the one-boson exchange (OBE) model the following essential simplifying assumption is made[7,8,15]: the off shell behaviour of nucleons is neglected and only the off-shell behaviour of intermediate mesons is taken into account. Thus, in expression (2) for the quasipotential we must use $p'^2 = ((1/2)P\pm q')^2|_{q_0'=0} = m_N^2$ and $p^2=((1/2)P\pm q)^2|_{q_0=0}=m_N^2$ which is equivalent to the assumption $E_{\vec q\,'}-E_{\vec q}=0$. It should be pointed out that though the neglect of the off-shell behaviour of nucleons in expression (2) for the quasipotential provides the hermiticity of the potential $V(\vec q\,';\vec q)$ in equation (1), however, due to this approximation the retardation effects cannot be consistently taken into account in the diagrams which are included in the kernel $K(q',q|P)$ of the Bethe-Salpeter equation. Furthermore, in the OBE model[7], unlike ref.[8], the retardation effects are fully neglected and the condition $E_{\vec q\,'}=E_{\vec q}$ is used in the phenomenological form factors as well as in the propagators in the potentials of the NN interaction.

In this paper the Low equations for the $\pi\pi$, πN and NN scattering obtained in ref.[9] are considered. Unlike the conventional Low equation these equations explicitly contain the s, u, t, $\bar s$, $\bar u$, $\bar t$ channel contributions. These Low-type equations are three-dimensional quadratically nonlinear integral equations, where the crossing symmetry of π-mesons is explicitly taken into account for the t-matrices of the $\pi\pi$ and πN scattering. It is demonstrated that for the case of $\pi\pi$ and NN scattering the potential of the equations suggested is hermitian and these equations can be reduced to the linear Lippmann - Schwinger-type equations (1). The NN-potential obtained will include the effects of the off-shell behaviour of individual nucleons in the meson-nucleon vertex functions and the intermediate mesons will remain on the mass shell.

1. THE LOW EQUATIONS

Let us consider the Low-type equations for the scattering processes $1+2\to 1'+2'$. According to ref.[9], we have following relations:

$$\begin{aligned}
&\delta^{(4)}(p_1'+p_2'-p_1-p_2)[\langle p_1'\alpha_1'p_2'\alpha_2'|\mathcal{T}(\omega=p_1'^{0}+p_2'^{0}-p_1^0-p_2^0)|p_1\alpha_1p_2\alpha_2\rangle - \\
&-\langle p_1'\alpha_1'p_2'\alpha_2'|Y|p_1\alpha_1p_2\alpha_2\rangle] = \delta^{(4)}(p_1'+p_2'-p_1-p_2)\times \\
&\times\{[1+d^{(4)}_{(1'2)}(\widehat{1'2})+d^{(4)}_{(2'2)}(\widehat{2'2})]\sum_n\int\frac{d\omega}{\omega+i\epsilon}(2\pi)^3\,\delta(p_1'^{0}+p_2'^{0}-P_n^0-\omega)\times \\
&\times\delta(\vec p_1\,'+\vec p_2\,'-\vec P_n^0)\langle p_1'\alpha_1'p_2'\alpha_2'|\mathcal{T}(\omega)|n\rangle(-d^{(2)}_{(34)})\langle n|\mathcal{T}^+(\omega)|p_1\alpha_1p_2\alpha_2\rangle + \\
&+[1+d^{(4)}_{(1'\hat{}2)}(\widehat{1'2})+d^{(4)}_{(2'\hat{}2)}(\widehat{2'2})]\sum_n\int\frac{d\omega}{\omega+i\epsilon}(2\pi)^3\,\delta(p_1^0+p_2^0+P^0+\omega)\times \\
&\times\delta(\vec p_1+\vec p_2+\vec P_n)\langle 0|\mathcal{T}(\omega)|p_1\alpha_1p_2\alpha_2 n\rangle\times \\
&\times(-d^{(2)}_{(34)})\langle n\,p_1'\alpha_1'p_2'\alpha_2'|\mathcal{T}^+(\omega)|0\rangle\},
\end{aligned}\tag{3}$$

where α_i denotes the corresponding quantum numbers of the i-th particle, $\langle n|\mathcal{T}(\omega)|m\rangle$ denotes the t-matrix for the m-particle state to the n-particle state transition, e.g., for the t-matrix for the n-particle state to the pion-nucleon state transition we have:

$$\begin{aligned}
&\langle p_N\alpha_N\,p_\pi\alpha_\pi|\mathcal{T}(\omega)|n\rangle = \bar u_{\vec p_N\alpha_N}(0)\,f^*_{\vec p_\pi}(0)(p^2_{\pi\omega}-m_\pi^2)(\not p_N-m_N)\,i\int dx\times \\
&\times e^{iq_{\pi N}x+i\omega\eta_1x_0}\langle 0|\theta(x^0)\Psi(\eta_2x)\Phi(-\eta_1x)|n\rangle + \bar u_{\vec p_N\alpha_N}(0)\,f^*_{\vec p_\pi}(0)(p_\pi^2-m_\pi^2)\times \\
&\times(\not p_{N\omega}-m_N)\,i\int dx\,e^{iq_{\pi N}x-i\omega\eta_2x_0}\langle 0|\theta(-x^0)\Phi(-\eta_1x)\times \\
&\times\Psi(\eta_2x)|n;in\rangle,
\end{aligned}\tag{4}$$

where $\Phi(x)$, $\Psi(x)$ and $f_p(x)$, $u_{p\alpha}(x)$ denote the interacting fields and plane waves[11] of a pi-meson and a nucleon, $q_{\pi N} = \eta_2p_N-\eta_1p_\pi$ is the relative four-momentum of pion-nucleon system: $\eta_1=1-\eta_2=m_N/(m_N+m_\pi)$; $p_{N\omega}=(p_N^0-\omega;\vec p_N)$, $p_{\pi\omega}=(p_{\pi^0}-\omega,\vec p\,)$ and $\theta(x^0)$ is the well-known step function. The expression for the $\mathcal{T}(\omega)$ matrix can easily be related to the Bethe-Salpeter wave function $\chi_n(q_{\pi N})$

$$\chi_n(q_{\pi N}) = \int dx\, e^{iq_{\pi N}x} \langle 0 | T(\Psi(\eta_2 x)\Phi(-\eta_1 x)) | n; \text{in} \rangle \tag{5.1}$$

$$\begin{aligned}\langle p_N \alpha_N p_\pi \alpha_\pi | \mathcal{T}(\omega) | n \rangle = \\ = \bar{u}_{\vec{p}_N \alpha_N}(0) f^*_{\vec{p}_\pi}(0)(p^2_{\pi\omega} - m^2_\pi)(\not{p}_N - m_N) \frac{i^2}{2\pi} \int dp^\circ \frac{\chi_n(p^\circ, \vec{q}_{\pi N})}{q^\circ_{\pi N} + \eta_1\omega - p^\circ + i\epsilon} + \\ + \bar{u}_{\vec{p}_N \alpha_N}(0) f^*_{\vec{p}_\pi}(0)(p^2_\pi - m^2_\pi)(\not{p}_{N\omega} - m_N) \frac{i^2}{2\pi} \int dp^\circ \frac{\chi_n(p^\circ; \vec{q}_{\pi N})}{p^\circ - q^\circ_{\pi N} + \eta_2\omega + i\epsilon},\end{aligned} \tag{5.2}$$

$(\hat{\alpha\beta})$ in expression (3) denotes the crossing operator for α and β particles, $d^{(4)}_{(\hat{\alpha\beta})}$ is the corresponding sign + or - which appears in the particle crossing. The matrix Y in relations (3) corresponds to the sum of equal-time commutators of field sources $J_{p\alpha}(x) = f^*_p(0)[\Box_x + m^2_\pi]\Phi(x)$ for bosons and $J_{p\alpha}(x) = \bar{u}_{p\alpha}(0)[i\not{\nabla}_x - m_N]\Psi(x)$ for fermion and the creation operators of interacting fields

$$\begin{aligned}\langle p'_1 \alpha'_1 p'_2 \alpha'_2 | Y | p_1 \alpha_1 p_2 \alpha_2 \rangle = [1 + d^{(2)}_{(\hat{1'1})}(\hat{1'1})][1 + d^{(2)}_{(\hat{2'2})}(\hat{2'2})] \times \\ \times \langle p'_1 \alpha'_1 | [J_{p_2\alpha_2}(0), a^+_{p_1\alpha_1}(0)]_\pm | p_2 \alpha_2 \rangle .\end{aligned} \tag{6}$$

Relations (3) connect the scattering t-matrix on the energy shell $p^\circ_1 + p^\circ_2 = p'^\circ_1 + p'^\circ_2$ with the product of the off-shell t-matrices $P_n \neq p^\circ_1 + p^\circ_2$ which contain all possible n-particle intermediate states with the particles on mass shell. In order to obtain equations from (3) for the scattering t-matrices, we make, the following two assumptions.

1. In the sum over the intermediate states we take into account only the states which contain not more than two particles. This assumption is necessary in order to derive the closed system of equations from the relations obtained within the quantum field theory. From the physical point of view this corresponds to the assumption on the dominating role of one- and two-particle exchanges in the particle interactions in the low and medium energy region[12].

2. Relations (3) are assumed to be valid off-energy shell too, when $P^\circ_{12} = p^\circ_1 + p^\circ_2 \neq P'^\circ_{12} = p'^\circ_1 + p'^\circ_2$. This assumption is necessary in order to obtain equations for the scattering t-matrices from relations (3). It should be pointed out that in the Low equation [10], which is derived on the basis of the t-matrix element $\langle p'_1 \alpha'_1 | J_{p'_2 \alpha'_2}(0) | p_1 \alpha_1 p_2 \alpha_2 \text{in} \rangle$ instead of the

R-matrix element R = S - 1, as in ref. [9], there is no need in such an off-shell continuation. The equations given below can be obtained from the t-matrix elements without the off-shell continuation of relations (3) provided the two-loop corrections in the vertex functions are neglected. However, this way of derivation makes further calculations very complicated; so we shall use relations (3).

Furthermore, under these assumptions for the t-matrix of the NN-scattering from relations (3) we obtain

$$\begin{aligned}\langle \vec{p}'_1 s'_1 \vec{p}'_2 s'_2 | \mathcal{T}(\omega = P^\circ_{1'2'} - P^\circ_{12}) | \vec{p}_1 s_1 \vec{p}_2 s_2 \rangle = \langle \vec{p}'_1 s'_1 \vec{p}'_2 s'_2 | V | \vec{p}_1 s_1 \vec{p}_2 s_2 \rangle + \\ + \sum_{n=d,\,NN} \int \langle \vec{p}'_1 s'_1 \vec{p}'_2 s'_2 | \mathcal{T}(\omega) | n \rangle (2\pi)^3 \frac{d\omega}{\omega + i\epsilon} \delta(p'^\circ_1 + p'^\circ_2 - P_n - \omega) \times \\ \times \delta(\vec{p}'_1 + \vec{p}'_2 - \vec{P}_n) \langle n | \mathcal{T}^+(\omega) | \vec{p}_1 s_1 \vec{p}_2 s_2 \rangle ,\end{aligned} \tag{7.1}$$

$$\begin{aligned}\langle \vec{p}'_1 s'_1 \vec{p}'_2 s'_2 | V | \vec{p}_1 s_1 \vec{p}_2 s_2 \rangle = \langle \vec{p}'_1 s'_1 \vec{p}'_2 s'_2 | Y | \vec{p}_1 s_1 \vec{p}_2 s_2 \rangle + \\ + [1 - (\hat{12})] \sum_{m=\pi,\sigma,\rho,\omega,2\pi,\dots} \sum \sum \int \langle \vec{p}'_1 s'_1 | \mathcal{T}(\omega) | \vec{p}_2 s_2 m \rangle (2\pi)^3 \delta(\vec{p}'_1 - \vec{p}_2 - \vec{P}_m) \times \\ \times \delta(p'^\circ_1 - p^\circ_2 - P^\circ_m - \omega) \frac{d\omega}{\omega} \langle m \vec{p}'_2 s'_2 | \mathcal{T}^+(\omega) | \vec{p}_1 s_1 \rangle + \\ + [1 - (\hat{12})] \sum_{m=\pi,\sigma,\rho,\omega,2\pi,\dots} \sum \sum \int \langle \vec{p}'_2 s'_2 | \mathcal{T}(\omega) | \vec{p}_1 s_1 m \rangle (2\pi)^3 \delta(\vec{p}'_1 - \vec{p}_2 + \vec{P}_m) \times \\ \times \delta(p'^\circ_1 - p^\circ_2 + P^\circ_m + \omega) \frac{d\omega}{\omega} \langle m \vec{p}'_1 s'_1 | \mathcal{T}^+(\omega) | \vec{p}_2 s_2 \rangle ,\end{aligned} \tag{7.2}$$

where we have omitted $\bar{s}$-channel contributions containing the d and NN intermediate states which are supposed to make small contributions to the NN interaction up to 1 GeV. From expression (7.2) we conclude that the two-pion exchange terms of the NN potential are determined by the $\langle \vec{p}' s' | \mathcal{T}(\omega) | \vec{p} s \vec{p}_\pi \alpha \vec{p}_{\pi'} \alpha' \rangle$ $N' \to N + \pi + \pi'$ transition vertex functions with the on-shell pions. If the two-pion exchange quasipotential [7] is constructed, then the two-pion term in this potential will be determined by the πN-matrix too. Consequently, within the given approach to the constructing of the NN potential the two-pion part of the potential should be much less than the corresponding term that is obtained in the quasipotential approach.

In equation (7.1) for the NN-scattering t-matrices, nucleons in the in- and out-states remain on the mass shell $p'^2 = p^2 = m_N^2$, $p'^{\circ} > 0$ and $p^{\circ} > 0$. Mesons in the intermediate states of the NN potential are on the mass shell too. However, according to expression (4), the off-energy shell extrapolation parameter ω determines the off-mass-shell extrapolation of one of the nucleons in the in- and out-states in the expression for these matrix elements. In particular, for the meson-nucleon vertex function from expression (7.2) we obtain

$$<\vec{p}\,'s'|\mathcal{T}(\omega)|\vec{p}\,s\,m> =$$
$$= i\bar{u}_{\vec{p}\,'\vec{s}\,'}(0)\,(\not{p}'_{\omega} - m_N)\int d^4x\, e^{\frac{i}{2}(p'+p)x + \frac{i}{2}x^{\circ}\omega} <0|\theta(x^{\circ})\Psi(\frac{1}{2}x)\bar{\Psi}(-\frac{1}{2}x)|m;in> \times$$
$$\times(\not{p} - m_N)\,u_{\vec{p}\,s}(0) + i\bar{u}_{\vec{p}\,'\vec{s}\,'}(0)\times \tag{8}$$
$$\times(\not{p}' - m_N)\int d^4x\, e^{\frac{i}{2}(p'+p)x - \frac{i}{2}\omega x^{\circ}} <0|\theta(-x^{\circ})\Psi(-\frac{1}{2}x)\bar{\Psi}(\frac{1}{2}x)|m;in>\times$$
$$\times(\not{p}_{\omega} - m_N)\,u_{\vec{p}\,s}(0),$$

where $m = \sigma, \rho, \omega, \pi, 2\pi, \ldots$ and $\omega = p'^{\circ} - p^{\circ} - P^{\circ}_m$ or $p^{\circ} - p'^{\circ} - P^{\circ}_m$; $p'_{\omega} = (p'^{\circ} - \omega, \vec{p}\,')$ and $p_{\omega} = (p^{\circ} - \omega, \vec{p})$.

In the expression for the NN potential (7.2) it is easy to observe that unlike the NN potential, obtained on the basis of the conventional Low equation [13] with the use of cluster decomposition, the potential V (7.2) is hermitian provided the equal-time commutators Y are hermitian. The seagull term is usually hermitian when it is calculated with the use of the simplest phenomenological Lagrangians [13].

From relations (3) for the t-matrix of the $\pi\pi$ scattering we obtain

$$<\vec{p}\,'_1\alpha'_1\vec{p}\,'_2\alpha'_2|\mathcal{T}(\omega = P^{\circ}_{1'2'} - P^{\circ}_{12})|\vec{p}_1\alpha_1\vec{p}_2\alpha_2> = <\vec{p}\,'_1\alpha'_1\vec{p}\,'_2\alpha'_2|V|\vec{p}_1\alpha_1\vec{p}_2\alpha_2> -$$
$$-\sum_{n=\sigma,\rho,\pi\pi}\sum\int<\vec{p}\,'_1\alpha'_1\vec{p}\,'_2\alpha'_2|\mathcal{T}(\omega)|n>(2\pi)^3\delta(p'^{\circ}_1 + p'^{\circ}_2 - P^{\circ}_n - \omega)\frac{d\omega}{\omega + i\epsilon}\times$$
$$\times\delta(\vec{p}\,'_1 + \vec{p}\,'_2 - \vec{P}_n)<n|\mathcal{T}^{+}(\omega)|\vec{p}_1\alpha_1\vec{p}_2\alpha_2> - [1 + (\hat{12})]\sum_{n=\pi\pi}\times$$
$$\times\{\int<\vec{p}\,'_1\alpha'_1|\mathcal{T}(\omega)|\vec{p}_1\alpha_1\,n>(2\pi)^3\delta(p'^{\circ}_1 - p^{\circ}_1 - P^{\circ}_n - \omega)\frac{d\omega}{\omega}\times$$

$$\delta(\vec{p}\,'_1 - \vec{p}_1 - \vec{P}_n)<n\,\vec{p}\,'_2\alpha'_2|\mathcal{T}^{+}(\omega)|\vec{p}_2\alpha_2> + h.c.\} -$$
$$-\sum_{m=\sigma,\rho,\pi\pi}\sum\int<0|\mathcal{T}(\omega)|m\,\vec{p}_1\alpha_1\vec{p}_2\alpha_2>\frac{d\omega}{\omega}(2\pi)^3\,\delta(\vec{p}\,'^{\circ}_1 + p'^{\circ}_2 + P^{\circ}_m + \omega)\times \tag{9.1}$$
$$\times\delta(\vec{p}\,'_1 + \vec{p}\,'_2 + \vec{P}_m)<m\,\vec{p}\,'_1\alpha'_1\vec{p}\,'_2\alpha'_2|\mathcal{T}^{+}(\omega)|0>,$$

$$<\vec{p}\,'_1\alpha'_1\vec{p}\,'_2\alpha'_2|V|\vec{p}_1\alpha_1\vec{p}_2\alpha_2> = [1 + (\hat{12})][1 + (\hat{1'2'})]\times$$
$$\times<\vec{p}\,'_1\alpha'_1|[J_{\vec{p}\,'_2\alpha'_2}(0), a^{+}_{\vec{p}_1\alpha_1}(0)]|\vec{p}_2\alpha_2> -$$

$$-[1 + (\hat{12})]\{\sum_{m=\sigma,\rho}\int<\vec{p}\,'_1\alpha'_1|\mathcal{T}(\omega)|\vec{p}_1\alpha_1 m>\frac{d\omega}{\omega}(2\pi)^3\,\delta(\vec{p}\,'_1 - \vec{p}_1 - \vec{P}_m)\times$$
$$\times\delta(p'^{\circ}_1 - p^{\circ}_1 - P^{\circ}_m - \omega)<m\,\vec{p}\,'_2\alpha'_2|\mathcal{T}^{+}(\omega)|\vec{p}_2\alpha_2> + h.c.\}, \tag{9.2}$$

In equation (9.1) we have picked up all the terms that have singularities on the real axis of the compex energy plane and in the pion-pion potential (9.2) the seagull terms and one-meson exchange $u, t, \bar{u}, \bar{t}$ terms are included. From the explicit expressions (9.1) and (9.2) one can conclude that the crossing of any pi-meson from the out-state with the pi-meson from the in-state leaves the $\pi\pi$ scattering t-matrix unchanged. Furthermore, the potential V (9.2) is hermitian. The hermiticity of the potential V in the given formulation of equation for the scattering t-matrices stems from the presence of identical particles in the in- and out-states, leading to the hermiticity of the $\bar{u} + u$ and $\bar{t} + t$ channel terms in the NN and $\pi\pi$ interaction potentials (7.2) and (9.2). It can easily be seen that in the case of πN scattering the πN interaction potential is not hermitian

$$<\vec{p}\,'_N s'_N\,\vec{p}\,'_{\pi}\alpha'_{\pi}|\mathcal{T}(\omega = P'^{\circ}_{\pi N} - P^{\circ}_{\pi N})|\vec{p}_N s_N\vec{p}_{\pi}\alpha_{\pi}> = <\vec{p}\,'_{\pi}\alpha'_{\pi}\,\vec{p}\,'_N s'_N|V|\vec{p}_N s_N\vec{p}_{\pi}\alpha_{\pi}> -$$
$$-\sum_{n=N,\pi N}\int\frac{d\omega}{\omega + i\epsilon}(2\pi)^3\delta(p'^{\circ}_N + p'^{\circ}_{\pi} - P^{\circ}_n - \omega)\,\delta(\vec{p}\,'^{\circ}_N + \vec{p}\,'_{\pi} - \vec{P}_n)\times$$
$$\times<\vec{p}\,'_N s'_N\vec{p}\,'_{\pi}\alpha'_{\pi}|\mathcal{T}(\omega)|n><n|\mathcal{T}^{+}(\omega)|\vec{p}_N s_N\vec{p}_{\pi}\alpha_{\pi}> +$$

$$
+\sum_{m=\bar{N}}\int\frac{d\omega}{\omega+i\epsilon}(2\pi)^3\delta(p_N'^{o}+p_\pi'^{o}+P_m^{o}+\omega)\,\delta(\vec{p}_N+\vec{p}_\pi+\vec{P}_m)\times
$$

$$
\times<0|\mathcal{T}(\omega)|\vec{p}_N s_N\vec{p}_\pi a_\pi m><m\vec{p}_N's_N'\vec{p}_\pi' a_\pi|\mathcal{T}^+(\omega)|0>-
$$

$$
-\sum_{m=\pi N}\int\frac{d\omega}{\omega+i\epsilon}(2\pi)^3\delta(\vec{p}_N'-\vec{p}_\pi-\vec{P}_m)(2\pi)^3\delta(p_N'^{o}-p_\pi^{o}-P_n^{o}-\omega)\times \tag{10.1}
$$

$$
\times<\vec{p}_N's_N'|\mathcal{T}(\omega)|\vec{p}_\pi a_\pi m><m\vec{p}_\pi' a_\pi'|\mathcal{T}^+(\omega)|\vec{p}_N s_N>,
$$

$$
<\vec{p}_N's_N'\vec{p}_\pi' a_\pi|V|\vec{p}_N s_N\vec{p}_\pi a_\pi>=<\vec{p}_N's_N'\vec{p}_\pi' a_\pi'|Y|\vec{p}_N s_N\vec{p}_\pi a_\pi>+
$$

$$
+\sum_{n=N}\int\frac{d\omega}{\omega}(2\pi)^3\delta(\vec{p}_N'-\vec{p}_\pi-\vec{P}_n)<\vec{p}_N's_N'|\mathcal{T}(\omega)|\vec{p}_\pi a_\pi n>\times
$$

$$
\times\delta(p_N'^{o}-p_\pi^{o}-P_n^{o}-\omega)<n\vec{p}_\pi' a_\pi'|\mathcal{T}^+(\omega)|\vec{p}_N a_N>-
$$

$$
-\sum_{n=\bar{N}}\int\frac{d\omega}{\omega}(2\pi)^3\delta(\vec{p}_\pi'-\vec{p}_N-\vec{P}_n)<\vec{p}_\pi' a_\pi'|\mathcal{T}(\epsilon)|\vec{p}_N s_N n>\times \tag{10.2}
$$

$$
\times\delta(p_\pi'^{o}-p_N^{o}-P_n^{o}-\omega)<n\vec{p}_N's_N'|\mathcal{T}^+(\omega)|\vec{p}_\pi a_\pi>+
$$

$$
+\sum_{m=\sigma,\rho,\pi\pi}\sum\Big\{\int\frac{d\omega}{\omega}(2\pi)^3\delta(\vec{p}_N'-\vec{p}_N-\vec{P}_m)\delta(p_N'^{o}-p_N^{o}-P_m^{o}-\omega)\times
$$

$$
\times<\vec{p}_N's_N'|\mathcal{T}(\omega)|\vec{p}_N s_N m><m\vec{p}_\pi' a_\pi'|\mathcal{T}^+(\epsilon)|\vec{p}_\pi a_\pi>+h.c.\Big\}.
$$

It is interesting to compare (10.1,2) equations for the scattering t-matrix with the corresponding equation from ref.[4]. It is easy to observe that taking into account the off-shell behaviour of nucleons in the matrix elements (4) leads to additional seagull terms and to the πN scattering t- and t-channel σ, ρ, $\pi\pi$ meson exchange terms in the πN potential. The πN potential is not hermitian due to the u- and $\bar{u}$-channel terms. The simplest way to obtain the needed hermiticity of the potential of the πN interaction is to replace in these terms of expression (3) $\delta(p_N'^{o}-p_\pi^{o}-P_n^{o}-\omega)$ and $\delta(p_N'^{o}-p_\pi^{o}+P_n^{o}+\omega)$ by $(1/2)[\delta(p_N'^{o}-p_\pi^{o}-P_n^{o}-\omega)+$

$+\delta(p_\pi'^{o}-p_N^{o}+P_n^{o}+\omega)]$ and $(1/2)[\delta(P_n^{o}+\omega+p_N'^{o}-p_\pi^{o})+$ $+\delta(p_\pi'^{o}-p_N^{o}-P_n^{o}-\omega)]$. This replacement does not change relation (3) on the energy shell, however we have to make an additional assumption, namely, we have to assume that equation (10.1,2) with the hermitian potential holds.

2. CONNECTION WITH THE LIPPMANN - SCHWINGER EQUATIONS

From the quantum scattering theory[15] we know that the linear integral equation (1) is equivalent to the following nonlinear integral equation:

$$
\mathcal{T}^{(\pm)}(\vec{q}',\vec{q},E_{\vec{p}})=V(\vec{q}',\vec{q})+\sum_n\int d^3\vec{k}\,V(\vec{q}';\vec{k})<\vec{k}|\phi_n>\frac{1}{E_{\vec{p}}-E_n}<\phi_n|\vec{k}'>d^3\vec{k}'V(\vec{k}',\vec{q})+
$$

$$
+\int d^3\vec{p}\,d^3\vec{k}\,V(\vec{q}',\vec{k})<\vec{k}|\Psi_{\vec{p}}^{(\pm)}>G_o(\vec{p},E_{\vec{q}})<\Psi_{\vec{p}}^{(\pm)}|\vec{k}'>d^3\vec{k}'V(\vec{k}',\vec{q}), \tag{11}
$$

where $V(\vec{q}',\vec{q})$ is the hermitian potential, ϕ_n and Ψ form the complete set of eigenfunctions for the discrete and continuous spectra of the corresponding hamiltonian and $G_o(\vec{p},E_{\vec{p}})$ is the Green function for the noninteracting fields:

$$
G_o^{(\pm)}(\vec{p},E_{\vec{q}})=\frac{1}{E_{\vec{q}}-E_{\vec{p}}\pm i\epsilon}; \tag{12}
$$

$\mathcal{T}^{(\pm)}(\vec{q}',\vec{q};E_{\vec{q}})$ is connected with the wave functions from the continuous spectrum in the following manner:

$$
\mathcal{T}^{(\pm)}(\vec{q}',\vec{q};E_{\vec{q}})=\int d^3\vec{k}\,V(\vec{q}',\vec{k})<\vec{k}|\Psi_{\vec{q}}^{(\pm)}>=(E_{\vec{q}}-E_{\vec{q}'})<\vec{q}'|\Psi_{\vec{q}}^{(\pm)}>. \tag{13}
$$

Comparing equation (7.1) for the NN-scattering t-matrix with equation (11), one can conclude that equation (7.1) is equivalent to the Lippmann - Schwinger equation[1] with the relativistic kinematics provided the hermitian NN potential (7.2) has the single eigenstate in the discrete spectrum which corresponds to the deuteron state.

The equation for the $\pi\pi$ scattering t-matrix (9.1) explicitly contains the $\pi\to\pi_1\pi_2\pi_3$ transition matrix that is related to the $\pi\pi$ scattering t-matrix. The terms with these scattering matrices are included in the potential (9.2) and are denoted by $W(\mathcal{T}^+,\mathcal{T})$. Further, from equations (9.1,2) the following connected system of linear integral equations can be obtained:

$$\mathcal{T}^{(\pm)}_{i+1} = W(\mathcal{T}^{+}_{i},\mathcal{T}_{i}) + V + [W(\mathcal{T}^{+}_{i},\mathcal{T}_{i}) + V]G^{(\pm)}_{0}\,\mathcal{T}_{i}, \qquad (14)$$

where V is the pion-pion potential defined according to expression (9.2). At the first step of the solution of equations (14)

$$W(\mathcal{T}^{+}_{0},\mathcal{T}_{0}) = 0, \qquad (15)$$

is assumed.

If the converging procedure for solving equation (14) is found, then the obtained solution represents the correct physical solution provided equation (9.1) holds, i.e., the resulting Green function $G=(G_0^{-1} - V - W)^{-1}$ has simple poles at the masses $\pm m_\sigma, \pm m_\rho$ and the left-hand and right-hand cuts on the real axis of energy in the interval $(-\infty,\ 2m_\pi)$ and $(2m_\pi,\ \infty)$.

3. CONCLUSION

The integral equations (1) and (14) are the linear integral equations with the hermitian potentials (7.2) and (9.2) for the NN and $\pi\pi$ scattering t-matrices. These equations are derived from the corresponding Low-type equations (7.1) and (9.1) for the NN and $\pi\pi$ scattering t-matrices. In order to obtain a similar equation for the πN scattering t-matrix, one has to change propagators in the u and $\bar{u}$ channel term of the pion-nucleon potential (10.2) off-energy shell $p'^{o}_{\pi} + p'^{o}_{N} \neq \neq p^{o}_{\pi} + p^{o}_{N}$. Similar replacements are usually assumed in the quasipotential approach to the reactions $1 + 2 \to 1' + 2'$ in order to achieve hermiticity. Unlike the quasipotential formulations, the potentials obtained will be hermitian in the case when the vertex functions are complex and there is no need in taking into account of certain terms from the full Bethe-Salpeter Green function.

The expressions of the $\pi\pi$, πN and NN interaction potentials are determined by the Bethe-Salpeter wave functions $\Gamma_{NNM} = \langle 0|\,T(\Psi_N(x)\,\Psi_N(y))\,|M;in\rangle$ and $\Gamma_{\pi\pi M} = \langle 0|T(\Phi_a(x)\,\Phi_a(y))|M;in\rangle$. The construction of these vertex functions in the phenomenological quark field-theoretical model is equivalent to the determination of the following vertex functions [16]:

$$\Gamma_{NNM} = \int[d^4\rho][d^4\omega]\bar{U}_N(x,\rho)\langle 0|\,T(\Psi_N(x,\rho)\,\Psi_N(y,\omega))|\,M;in\rangle U_N(y,\omega)$$
$$\Gamma_{\pi\pi M} = \int[d^4\rho][d^4\omega]\bar{U}_\pi(x,\rho)\langle 0|T(\Phi_a(x,\rho)\,\Phi_a(y,\omega)]|\,M;in\rangle U_\pi(y,\omega), \qquad (16)$$

where $x_i = x + \rho_i$, $y_i = y + \omega_i$ and $\sum_i \rho_i = \sum_i \omega_i = 0$ denote the coordinates of the individual quark fields, from which the nucleon and meson fields $\Psi_N(x,\rho) = T(q_1(x_1)\,q_2(x_2)\,q_3(x_3))$ and $\Phi(x,\rho) = T(q_1(x)\,q_2(y))$ are built up, and $U(x,\rho)$ are the Bethe-Salpeter wave functions for the nucleon-quark and meson-quark systems. It is easy to observe that expression (16) contains the quark-exchange effects between individual nucleon and pi-meson fields. Let us point out that if these quark-exchange effects between all four hadron fields are taken into account in the full Green function, which serves as a basis in deriving relations (3) [9], then we obtain the NN, $\pi\pi$ and πN potentials (7.2), (9.2) and (10.2) as well as the corresponding terms with the meson and quark exchanges. Consequently, in this formulation the meson and quark exchange interactions are contained in the potential additively and independently from each other.

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Мачавариани А.И. E4-88-610
Трехмерные релятивистские уравнения
для задач $\pi\pi$-, πN- и NN-рассеяния

Рассмотрены различные уравнения типа Лоу для t-матриц $\pi\pi$-, πN- и NN-рассеяния. Проанализирована их связь с уравнениями типа Липпмана-Швингера с релятивистской кинематикой и проведено сравнение с соответствующими квазипотенциальными уравнениями.

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Machavariani A.I. E4-88-610
Three-Dimensional Relativistic Equations
for $\pi\pi$, πN and NN Scattering

Different Low-type equations for the $\pi\pi$, πN and NN scattering t-matrices are considered. Their connection with the Lippmann - Schwinger equation with the relativistic kinematics is analysed and the comparison is made with the corresponding quasipotential equations.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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