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I.N.Mikhailov, E.G.Nadjakov,
M.Aiche¹, Ch.Briançon², N.Schulz¹, V.Vanin¹

**DIPOLE OCTUPOLE CORRELATIONS
IN A BOSON MODEL AND EVIDENCE
FOR THEIR EXISTENCE IN NUCLEI**

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¹CRN, Strasbourg, France

²CSNSM, Orsay, France

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1. Boson model

Introduction of new degrees of freedom in the interacting boson model (IBM), other than the monopole s and quadrupole d ones, has been tried already in the first papers on IBM. However in the last years it was realised that these degrees of freedom can be chosen so that the $U(3)$ subgroup and thus the rotational limit remain there. This has been achieved e.g. in the spdf boson model ^{/1,2/}, and in a unified IBM ^{/3/}, both developed algebraically only in the rotational limit.

In 1986 two of us suggested an extension of the IBM called interacting multi boson model (IMBM) ^{/4,5/}, showing that a definite simple choice of the boson space preserves not only the vibrational and rotational limits, but other intermediate limits as well. Thus it has been demonstrated that IMBM conserves the basic feature of IBM to give a simple and clear description of transitional nuclei as well.

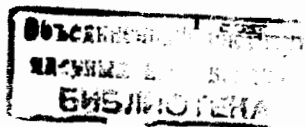
It has been shown that such an advantageous boson choice can be achieved by including bosons b^j of space parity $(-)^j$, one of each multipolarity $j = 0(1), 2(3), \dots, n-2$ and/or $j = 1(0), 3(2), \dots, n-1$. In the first case we obtain an initial $U^{0(1),2(3),\dots,n-2} (n(n-1)/2)$ group; in the second, a $U^{1(0),3(2),\dots,n-1} ((n+1)n/2)$ group; and in both the cases, a

$$U^{0,1,\dots,n-2,n-1} (nn) \supset$$

$$U^{0(1),2(3),\dots,n-2} (n(n-1)/2) \times U^{1(0),3(2),\dots,n-1} ((n+1)n/2) \quad (1)$$

group. In particular, at $n = 4$ we obtain the spdf boson model initial group $U^{0,1,2,3}(16) \supset U^{0,2}(6) \times U^{1,3}(10)$, including the usual sd $U^{0,2}(6)$ and the pf $U^{1,3}(10)$ model groups. The group structure of $U^{0,2}(6)$ is well known, and of $U^{1,3}(10)$ it has been derived in reference ^{/4/}; the boson hamiltonian and transition operators have been also introduced there.

It has been shown further that a simple pf model hamiltonian, with two parameters left in each case, can roughly reproduce the



positive and negative parity yrast bands of actinide isotones in the vibrational, intermediate (transitional) and rotational cases ^{15/}.

In what follows we are going to apply the following simplified hamiltonians in the vibrational limit, either:

$$h = \mathcal{E} (\hat{n}_3^1 + 2\hat{n}_5^2 + 3\hat{n}_7^3)/3 + \beta_7 \hat{\omega}_7^3 \quad (2)$$

or:

$$h = \mathcal{E} (\hat{n}_3^1 + 2\hat{n}_5^2 + 3\hat{n}_7^3)/3 + \beta_3 \hat{\omega}_3^{1,2,3}. \quad (3)$$

Both are corresponding to the group chains presented by the first lines of formulae (3) and (4) in reference ^{14/}. The first term has been chosen to provide a smooth yrast line and to ensure a maximal degeneracy, and also in this way a maximal mixing of the pdf degrees of freedom, in order to look if it is allowed by experiment. Hamiltonian (2) gives the same yrast level energy for the same $n^1 + 2n^2$. A small additional term in (2), e.g. $\beta_5 \hat{\omega}_5^2$, can make lower n^2 values advantageous. In the following, with (2) only, we shall substitute such a term by the condition: $2n^2 \leq N - n^1 - n^3$, which is stronger than: $n^2 \leq N - n^1 - n^3$ following from the total boson number rule (see the end of this section). Hamiltonian (3) gives moreover the same yrast level energy for the same linear combination $n^1 + 2n^2 + 3n^3$ of the p boson number n^1 , d boson number n^2 and f boson number n^3 . For corrections in the intermediate limit we are going to use:

$$h = \beta_5 \hat{\omega}_5^{1,2,3}. \quad (4)$$

This is corresponding to the group chains presented by any of the first two lines of formulae (3) and any of the three lines before the last one of formulae (4) in reference ^{14/}.

In our formulae $\hat{n}_r^t = \hat{n}^t$ and $\hat{\nu}_r^t$ are the first and second order Casimir operators of $U^t(r)$, $\hat{\omega}_r^t$ is the second order Casimir operator of $O^t(r)$. The embedding of the subgroups in each chain of formulae (4) in reference ^{14/} to find the level quantum numbers, and the Casimir operator eigenvalues $n_r^t = n^t$ (numbers of t bosons), ν_r^t , ω_r^t in terms of these quantum numbers to find the level energies, have been discussed before ^{14,5/}.

One additional important point for the application of IBM is related to the well known ad hoc rule accepted in IBM: the total boson

number N is equal to the valence nucleon (or hole) pair number \bar{N} . It has been noticed ^{15/} that if such total boson number is accepted for the spdf model in actinides, one would miss the highest collective level spins observed. Deviations from that rule have been used ^{12,5/}. But this can be understood if one remembers the shell model reasons in favour of that rule. In fact, to build nucleon pairs related to negative parity e.g. pf bosons, to the valence subshell of a single parity π_+ with M_+ state pairs and N_+ nucleon (or hole) pairs, one has to add a near subshell of opposite parity $\pi_- = -\pi_+$ with M_- state pairs and N_- nucleon (or hole) pairs. N_+ and N_- are accepted to be of the same nucleon or hole type, the type being chosen so that $N_+ + N_- \leq M_+ + M_- - N_+ - N_-$. This would result in a modified IBM total spdf boson number rule: $N = N_+ + N_-$. The usual IBM total boson number rule, in our notation for comparison, is: $N = \bar{N}$, where $\bar{N} = \bar{N}_+ + \bar{N}_-$ and $\bar{N}_\pm = \text{Min}(N_\pm, M_\pm - N_\pm)$. \bar{N} is the valence nucleon (or hole) pair number, usually determined by the nucleon (or hole) pairs above (or below) the nearest proton and neutron magic numbers. A restriction on the pf boson number: $n^1 + n^3 \leq M$, where $M = 4 \text{Min}(M_+, M_-)$, has to be imposed in cases $M < N$, and on the d boson number: $n^2 \leq \bar{N}$ if d bosons are assumed to follow the usual IBM behaviour.

2. Experiment

The $^{218}_{88}\text{Ra}_{130}$ level scheme has been experimentally studied in several publications ^{16-8/}, with which already comparisons have been made ^{12,5/}. Now, four of us have reinvestigated the excited states of $^{218}_{88}\text{Ra}$ via the $^{208}_{82}\text{Pb} (^{14}_6\text{C}, 4n)$ reaction in an experiment at the tandem accelerator of Strasbourg using the 4π array multi-detector called "Château de Cristal" (Crystal Castle) ^{19/}. The new essentially extended level scheme of $^{218}_{88}\text{Ra}$ is shown in the left hand side of figure 1.

Extended experimental details will be given in a forthcoming paper. Here we mention that spin assignments were established from the measured gamma ray anisotropies. Intensity balance was used to differentiate between M1 and E1 transitions. The previously known positive and negative parity bands were confirmed and extended up to $I^\pi = 30^+$ and $I^\pi = 31^-$. A different version of the second negative parity band is proposed in the present work. This band is observed up to $I^\pi = (24^-)$ and connected by M1 transitions to the first negative parity band. At high excitation energy it is

connected by E1 transitions to a possible second positive parity band.

So the main features of the new experimental results can be summarised as follows. 1) The level scheme is extended to much higher spins, and one can see that it preserves its collective vibrational type. The ground positive and negative parity π yrast bands are hybridised into one for all spins $I \geq 4$. One might say that this is the first pure case of hybridisation without signs of static deformations. 2) The most important feature of the new experiment is the discovery of peculiar side yrare bands with the same positive and negative space parities π as those of the ground yrast bands, but with spins I decreased by one spin unit. This means that their levels have opposite spin parity $(-)^I$. These levels have almost the same energies E . They are also hybridised into one band.

3. Interpretation

Let us now apply the spdf boson model for the interpretation of the experiment with all these features, without aiming at a detailed fit. The modification of the total boson number rule discussed at the end of the first section will be used. For our nucleus $N_+ = 3 + 2 = 5$. Indeed, its proton part for magic number 82 is $N_+ = (88 - 82)/2 = 3$. Its neutron part for magic number 126 is $N_+ = (130 - 126)/2 = 2$. Shell model reasons are consistent with the choice $N_- = M_- = 2 + 4 = 6$. In fact, for protons the lower valence subshell has parity $\pi_+ = -$ and is $2 f 7/2 + 3 p 3/2 + \dots$, and the upper full subshell of opposite parity $\pi_- = +$ can be assumed to be $2 d 3/2$ with the proton part of $N_- = M_- = 2$. For neutrons the lower valence subshell has parity $\pi_+ = +$ and is $2 g 9/2 + 3 d 5/2 + 4 s 1/2 + \dots$, and the upper full subshell of opposite parity $\pi_- = -$ can be assumed to be $3 p 1/2 + 2 f 5/2$ with the neutron part of $N_- = M_- = 4$. Then the total boson number will be $N = N_+ + N_- = 11$ instead of the usually accepted valence nucleon pair number $\bar{N} = \bar{N}_+ + \bar{N}_- = N_+ = 5$. The restriction on the pf boson number is not necessary here since for both nucleon type parts $M \geq N$, and on the d boson number it is $n^2 \leq \bar{N} = 5$.

We consider the vibrational limit with the simple hamiltonians (2) or (3). Let us choose both their parameters to fit the lowest and highest spin part of the yrast $\pi = +$ band, obtaining for (2) $\epsilon = 585$ keV and $\beta_7 = 3.4$ keV, or for (3) $\epsilon = 579$ keV and $\beta_3 = 0.596$ keV. All the three remaining bands: yrast $\pi = -$,

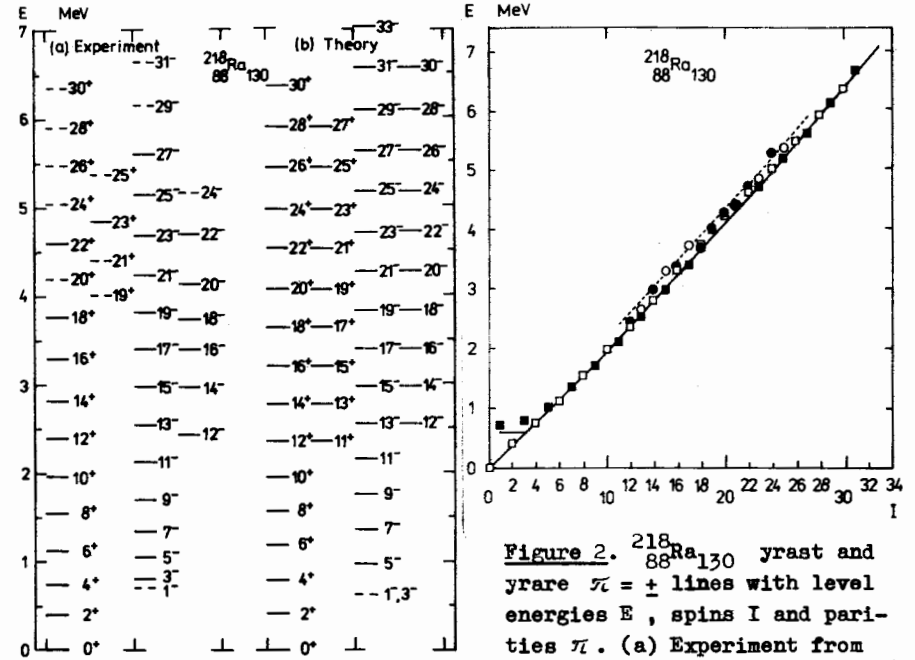


Figure 1. ^{218}Ra level scheme with level energies E , level spins and parities I^π . (a) This experiment except an isolated $(10)^+$ level with the 1^- , 3^- levels from [7]. (b) Theory (2) in the vibrational limit with correction (4) for $I < 4$ in the intermediate limit, two parameters: given in the text.

yrare $\pi = +$ and yrare $\pi = -$ are obtained automatically for $I \geq 4$. For the remaining levels $I < 4$, and in fact for the 1^- level only, we have to accept a transition to the intermediate limit. This is natural since our nucleus, according to deformation systematics, should be transitional in its ground state $I = 0$ with quite a small quadrupole deformation. On the other hand the yrast states have spins $I = n^1 + 2n^2 + 3n^3$ which means alignment of all the boson angular momenta. This might destroy the deformation at an accordingly low pdf boson number $n^1 + n^2 + n^3 = N - n^0 \approx 2$ or spin $I \approx 2 - 6$. It might be achieved e.g. by adding (4) multiplied by the step

Figure 2. $^{218}\text{Ra}_{130}$ yrast and yrare $\pi = \pm$ lines with level energies E , spins I and parities π . (a) Experiment from figure 1 with tentative 13^+ , 15^+ , 17^+ yrare levels from previous work; yrast $\pi = +$: empty squares \square , yrast $\pi = -$: filled squares \blacksquare , yrare $\pi = +$: empty circles \circ , yrare $\pi = -$: filled circles \bullet . (b) Theory from figure 1; yrast: solid line —, yrare: dashed line - - -.

function $\theta(1-\hat{n})$ to (2) or (3) after multiplying their first terms by $1 - \theta(1-\hat{n})$, where $\hat{n} = \hat{n}^1 + \hat{n}^2 + \hat{n}^3 = \hat{N} - \hat{n}^0$, $\theta(x) = 0$ if $x < 0$, $\theta(x) = 1$ if $x \geq 0$. In order to introduce no new parameter, we conserve the 2^+ level position to get $\beta_5 = 97.5$ keV with (2) and $\beta_5 = 96.5$ keV with (3). Thus we obtain the levels $I < 4$ too.

The result for (2) with (4) is shown in the right hand side of figure 1 to be compared with the experiment in its left hand side. Another presentation of the comparison of theory with experiment, showing directly the hybridised yrast and yrare lines, can be seen in figure 2. Its normalised χ value:

$$\chi = \left[\frac{\sum_k}{1-1} (E_1^t - E_1^e)^2 / (k-2) \right]^{1/2} \quad (5)$$

providing a measure for the deviation of the individual theoretical E^t from experimental E^e level energy, k being the number of compared levels, is $\chi = 80.67$ (66.87) keV with all the levels in figures 1,2 included and $k = 45$ (respectively with dashed-line levels in figure 1 as well as the additional three levels in figure 2 excluded and $k = 29$). The result for (3) with (4) is not shown since it is rather similar to the previous one. Its normalised χ value is $\chi = 91.44$ (89.21) keV. So we can say that there is no essential difference in accuracy between both results, the first one being slightly better.

One can see that all the features of the experiment mentioned in the previous section are described, and the level scheme itself is reproduced. The peculiarities of the model allowing to achieve the description of the corresponding features of the experiment are as follows. 1) The smooth vibrational hybridised ground yrast line $4^+, 5^-, 6^+, 7^-, \dots$ is due to the first term of (2) or (3). The same result will be obtained even without d bosons ($n^2 = 0$) due to the ratio 3 of octupole \mathcal{E} to dipole $\mathcal{E}/3$ boson parameters. The collectivity of the levels up to a spin higher than the doubled spin allowed by IBM is understood by the modified IMBM total boson number rule (the end of the first section). 2) The existence of the $11^+, 12^-, 13^+, 14^-, \dots$ side yrare line of levels with almost the same energies as those of the $12^+, 13^-, 14^+, 15^-, \dots$ ground yrast line, in case (2) if $n^2 = 0$ or in case (3) if $n^2 = 0$ and $n^3 = \min$, is due to the vector addition of p and f boson angular momenta not only to maximal values as for the yrast line, but also to maximal values minus one for the yrare line. Let us notice that the last values do not appear with one type of bosons only. So the lack

Table 1

Numbers: n^1 of p bosons, n^2 of d bosons, and n^3 of f bosons in the vibrational ${}_{88}^{218}\text{Ra}_{130}$ yrast (yt) and yrare (ye) states (the latter as they appear for hamiltonian (2) with $n^2 = 0$) of parity $\pi = +$ or $-$ and spin I . For each I^π , above: hamiltonian (2) values, below: hamiltonian (3) values. Below n^3 changes from min to max with step 1 and $n^1 + 2n^2$ changes simultaneously from max to min with step -3. For fixed $n^1 + 2n^2$ above and below n^2 changes with step 1 and n^1 changes simultaneously with step -2 in limits determined by $n^1 \leq 11 - n^3$ and $n^2 \leq 5$.

I	n^1+2n^2	n^3	I	n^1+2n^2	n^3	I	n^1+2n^2	n^3
yt	ye		yt	ye		yt	ye	
0^+	0	0	11^-	11	0	22^+	21^+	4
	0-0	0-0		11-2	0-3		13-1	3-7
1^-	1	0	12^+	11	1	23^-	22^-	5
	1-1	0-0		12-0	0-4		11-2	4-7
2^+	2	0	13^-	12	1	24^+	23^+	3
	2-2	0-0		13-1	0-4		12-0	4-8
3^-	3	0	14^+	13	2	25^-	24^-	4
	3-0	0-1		14-2	0-4		10-1	5-8
4^+	4	0	15^-	14	2	26^+	25^+	2
	4-1	0-1		15-0	0-5		11-2	5-8
5^-	5	0	16^+	15	3	27^-	26^-	3
	5-2	0-1		16-1	0-5		9-0	6-9
6^+	6	0	17^-	16	3	28^+	27^+	1
	6-0	0-2		17-2	1-5		10-1	6-9
7^-	7	0	18^+	17	4	29^-	28^-	2
	7-1	0-2		18-0	1-6		8-2	7-9
8^+	8	0	19^-	18	4	30^+		0
	8-2	0-2		19-1	2-6		6-0	8-10
9^-	9	0	20^+	19	5	31^-	30^-	1
	9-0	0-3		20-2	2-6		4-1	9-10
10^+	10	0	21^-	20	5	32^+		-
	10-1	0-3		21-0	3-7		2-2	10-10
							0	11
						33^-	0-0	11-11

of lower side band spins, except for experimental difficulties to populate them, may be due to the fact that in the above mentioned cases their states contain p boson predominant configurations, whereas combined pf boson configurations appear at spins higher than about 1/3 of the maximal one: see the next section and table 1.

In this experiment there is an additional $(10)^+$ level, not shown in figure 1, a little bit higher than the yrast 10^+ /9/. It could be a double yrare level to the yrast 12^+ with the angular

momentum decreased not by one, but by two spin units. Levels of that type are also predicted in the model.

4. Dipole octupole correlations

In table 1 we present all spdf boson configurations allowed by our fits. For the fit with hamiltonian (2) only fixed n^3 values (above I^π) are allowed. For the fit with hamiltonian (3) all the n^3 values between a minimal first one and a maximal second one (below I^π) are allowed. Both for (2) and (3) all the n^1 and n^2 values, obeying the relation $n^1 + 2n^2 = \text{constant}$, for (3) limited by $n^1 \leq 11 - n^3$ and $n^2 \leq 5$, are allowed. Everywhere $n^0 = 11 - n^1 - n^2 - n^3$. Thus the f boson number n^3 is fixed for fit (2), but limited to an interval for fit (3) with maximal width 5 bosons at intermediate spins. The competition between d bosons with number n^2 and p bosons with number n^1 is not cleared up by the above mentioned, n^2 maximum extending from 0 to 5 bosons, n^1 from 0 to 11 bosons, both at intermediate spins.

This means that the pdf boson hamiltonian can be constructed in such ways that it fits the level energies with almost the same accuracy, but predicts different pdf boson participation in the states in the limits shown in table 1. Even without the table it is clear that any one p or d or f boson type description is impossible since it will miss many observed levels. From the two boson types, pd without f bosons should be rejected since it will miss the levels with spins $I > N + \bar{N} = 16$. df is nearer to the classical sdf model with up to one f boson at low spin and more f bosons at high spin, but without p bosons it should be rejected due to: 1) lack of the 1^- low energy level, 2) necessity of p bosons in the rotational limit observed in adjacent actinides /5/; moreover: 3) it gives no natural explanation of the allowed or not allowed side yrare levels, 4) many levels will disappear for hamiltonian (2). pf will explain all ground yrast band levels and in a natural way the allowed or not allowed side band levels discussed at the end of the previous section; but without d bosons at all it will miss some of the observed E2 at low spin and E1 at high spin transitions.

Thus all three pdf boson types are necessary. To explain the allowed or not allowed side yrare band levels, their configurations should be near to those with two pf boson types. This means that the pf boson participation is essential, and that the d boson participation might be considerably lower than previously accepted,

e.g. in numerical calculations with eight parameters /2/. Then the hypothesis $n^1 \approx \text{max}$, $\text{max} - 2$, $n^2 \approx 0, 1$, $n^3 \approx \text{hamiltonian}$ (2) fixed value (above I^π in table 1) is the best one to account for spin I intervals in which the side yrare band levels are allowed or possibly not. At the same time it permits the observed transitions to exist (see below). It will mean that the f boson number n^3 (octupole correlations) will be near to 0 at low spin I and increase from 0 at spin $I = N = 11$ up to $N = 11$ at spin $I = 3N = 33$. The d boson number n^2 (quadrupole correlations) will remain oscillating at a low level, most probably 0 - 1. The p boson number n^1 (dipole correlations) will increase from 0 at spin $I = 0$ up to 9 - 11 at spin $I = N = 11$ and decrease to 0 at spin $I = 3N = 33$.

Let us point out that our n^1, n^2, n^3 values in the cases of table 1, including the best choice mentioned above, will permit the observed E1, M1, E2 transitions to exist with the already introduced lowest order T^{E1}, T^{M1}, T^{E2} transition operators /4/, except for M1 $I \rightarrow I - 1$ transitions, first order T^{M1} being sufficient for $\pi(-)^I = \text{even}$, but possibly having to be completed by a second order term for $\pi(-)^I = \text{odd}$:

$$T^{M1} = \sum_{j=1}^3 m_j^1 [b^{j+} b^j] + m_{0,1,2,3}^1 [b^{2+} b^0 + [b^3 b^1]^2 + [b^{3+} b^1]^2 b^2 b^0] \quad (6)$$

We should pay attention that the hamiltonians (2) or (3) with the correction (4) are too simplified to account for all the details. Surely, one should include a boson interaction able to explain: 1) a smooth transition from the vibrational to the intermediate limit at lowest spin, 2) small level energy deviations from experiment to theory shown in figure 1, 3) scarce data on E1, M1, E2 transition rates.

In conclusion, the point about essential participation of both the p and f bosons is in our opinion sufficient to be viewed as evidence for the existence of combined dipole octupole correlations in nuclei. It supports the main idea of IMBM to include a definite combination of negative parity and odd spin bosons, in this case p and f bosons.

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Михайлов И.Н., Наджаков Е.Г.,
Аиш М., Бриансон Ш., Шульц Н., Ванен В.

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Дипольные октупольные корреляции в бозонной модели
и демонстрация их существования в ядрах

Модель многих взаимодействующих бозонов /ММББ/, введенная ранее, в ее случае $spdf$ бозонов, применена к новому эксперименту по вибрационному ядру $^{218}_{88}Ra_{130}$. Показано, что она описывает естественным образом, при существенном участии pf бозонов, как основные ирраст полосы положительной и отрицательной четности, так и своеобразные ирраст полосы одинаковой пространственной π и обратной спиновой $(-)^I$ четности. Таким образом этот эксперимент с его бозонной интерпретацией можно рассматривать как демонстрацию существования комбинированных дипольных октупольных корреляций в ядрах.

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Mikhailov I.N., Nadjakov E.G.,
Aïche M., Briançon Ch., Schulz N., Vanin V.

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Dipole Octupole Correlations In a Boson Model
and Evidence for Their Existence in Nuclei

The Interacting multi boson model /IMBM/ introduced earlier, in its $spdf$ boson case, is applied to a new experiment on the vibrational nucleus $^{218}_{88}Ra_{130}$. It is shown to describe in a natural way, with the essential participation of pf bosons, both the ground yrast bands with positive and negative parity, and the peculiar side yrast bands with the same space π and opposite spin $(-)^I$ parity. So this experiment together with its boson interpretation can be viewed as evidence for the existence of combined dipole octupole correlations in nuclei.

The Investigation has been performed at the 1/ Laboratory of Theoretical Physics, JINR, 2/ CRN, Strasbourg and 3/ CSNSM, Orsay, France.

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