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PARTIAL RESTORATION OF THE CHIRAL SYMMETRY IN THE GENERALIZED SKYRME MODEL

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In the low-energy region, where the hadronic mass spectrum is formed, quantum chromodynamics is characterized by spontaneous breaking of the chiral and conformal symmetry. This leads to generation of quark and gluon condensates. The effective lagrangian describing strong interactions in this region must express these pecularities of QCD in terms of the effective degrees of freedom. The latter are bosonic fields. In this approach the observable mesons are treated as small fluctuations above chirally nonsymmetric vacuum while the topologically nontrivial solitons correspond to baryons.

The aim of our work is to investigate some static nucleon properties in the framework of the model obtained by the simultaneous conformal and chiral bosonization method [1]. The model we consider is a generalization of the well-known Skyrme model [2] which takes into account the scalar field $\sigma(x)$ interacting with the chiral field $U(x)$.

The model is defined by following expression:

$$\mathcal{L}_{eff}(U,\sigma)= \frac{F_\pi^2}{4}\cdot e^{-2\sigma}\cdot \mathrm{Tr}\,\partial_\mu U\,\partial^\mu U^+ + N_f\,\frac{F_\pi^2}{4}\,e^{-2\sigma}(\partial_\mu\sigma)^2 \quad (1)$$

$$+ \frac{1}{128\pi^2}\,\mathrm{Tr}\left[\partial_\mu U\cdot U^+,\ \partial_\nu U\cdot U^+\right]^2$$

$$- C_g\,\frac{N_f}{48}\left[e^{-4\sigma}-1+\frac{4}{\varepsilon}\left(1-e^{-\varepsilon\sigma}\right)\right],$$

where F_π is the pion decay constant (93 MeV) and N_f is the number of flavours. The magnitude of the gluon condensate parameter $C_g=\langle G_{\mu\nu}^2\rangle$ lies within the interval of admissible values: $[(300-400)\ \mathrm{MeV}]^4$. The first and second terms are kinetic energies of the chiral and scalar fields, the third corresponds to the Skyrme self-interaction. The effective potential for a scalar field given by the last term of the Lagrangian is the extrapolation of the low-energy potential [3] into the high energy region. In this extrapolation, the one-loop approximation to the Gell-Mann-Low QCD β-function is used. The parameter ε depends on the flavour number: $\varepsilon = 8N_f/(33-2N_f)$.

Passing to baryonic sector we have to make some assumption about the form of the static chiral and scalar fields. In particular, for the chiral field we choose the Skyrme-Witten ansatz $U(x) = \exp\{i\vec{\tau}\vec{n}F(r)\}$ (where $\vec{n} = \vec{r}/r$) and propose spherical symmetry for the scalar field $\sigma(x)$. Then for the mass functional we have

$$\mathcal{M} = \mathcal{M}_2 + \mathcal{M}_4 + V. \tag{2}$$

Here

$$\mathcal{M}_2 = 4\pi(F_\pi/e)\cdot\int_0^\infty dx\left\{\frac{N_f}{4}x^2(\rho')^2+\rho^2\left[\frac{x^2(F')^2}{2}+\sin^2F\right]\right\}, \tag{3}$$

$$\mathcal{M}_4 = 4\pi(F_\pi/e)\cdot\int_0^\infty dx\left\{\frac{\sin^4F}{2x^2}+\sin^2F\cdot(F')^2\right\}, \tag{4}$$

$$V = 4\pi(F_\pi/e)\cdot D_{eff}\cdot\int_0^\infty dx\cdot x^2\left[\rho^4-1+\frac{4}{\varepsilon}(1-\rho^\varepsilon)\right],$$

$$\rho = e^{-\sigma(x)}. \tag{5}$$

In eqs. (3)-(5), the dimensionless variable $x = F_\pi\cdot e\cdot r$ includes the Skyrme parameter e equal to 2π in accordance with eq.(I). For the potential factor D_{eff} we have $D_{eff} = C_g N_f/48e^2F_\pi^4$. The mass functional leads to the following system of equations for F(x) and ρ(x):

$$F''\left[\rho^2x^2+2\sin^2F\right]+2F'x\left[x\rho\rho'+\rho^2\right]+\sin(2F)(F')^2 - \rho^2\sin(2F)-\frac{\sin^2F}{x^2}\sin(2F)=0, \tag{6}$$

$$\frac{N_f}{2}\cdot x\left[x\rho''+2\rho'\right]-2\rho\left[\frac{x^2(F')^2}{2}+\sin^2F\right]-4D_{eff}\left[\rho^3-\rho^{\varepsilon-1}\right]x^2=0,$$

where prime corresponds to the derivative with respect to x. According to the virial theorem, the solutions should satisfy the following condition: $\mathcal{M}_4 - \mathcal{M}_2 - 3V = 0$, that we use to control the accuracy of the numerical calculations. For small x we have $F(x)\sim\pi N-\alpha x$, $\rho(x)\sim\rho(0)+\beta x^2$ and for asymptotically large x: $F(x)\sim a/x^2$, $\rho(x)\sim 1-b/x^6$. There are nontrivial relations between the numerical coefficients α and β, a and b. The boundary conditions ensure a finite mass functional for a given value of the topological charge = N.

We may quantize the rotational degrees of freedom by means of collective variable method[4]. As a result, we get the following expression for the nucleon mass M_N

$$M_N = \mathcal{M} + \frac{3}{8I},$$

where the rotational moment of inertia is now

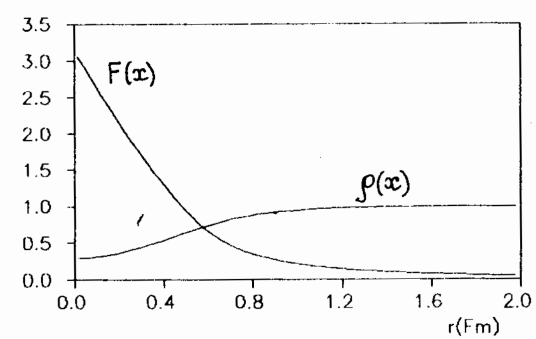
$$I = \frac{8\pi}{3}(F_\pi e^3)^{-1}\int_0^\infty dx\,\sin^2F\left[\rho^2x^2+(F')^2x^2+\sin^2F\right].$$

Some numerical results are given in the table, where also the mean square root radius of the baryon charge distribution $\langle r_b^2\rangle^{1/2}$ is shown.

Table. Our numerical results in the generalized Skyrme model. In the last column the results obtained in the original Skyrme model are given for the same values of F_π, e.

	(300 MeV)^4	(300 MeV)^4	Skyrme
N_f	3	2	2
F_π	93 MeV	93 MeV	93 MeV
e	2π	2π	2π
$\rho(0)$	0.29	0.22	1
$\mathcal{M}$	867 MeV	827 MeV	1087 MeV
$\langle r_b^2\rangle^{1/2}$	0.37 Fm	0.38 Fm	0.36 Fm
M_N	1072 MeV	1033 MeV	1260 MeV

One can see a partial restoration of the chiral symmetry which appears as the big deviation of $\rho(0)$ from its asymptotical value 1. (See the Figure).



The solution of the system (6) for chiral angle F(x) and scalar field ρ (x).

We should like to point out that the classical and rotational component of the nucleon mass are much smaller here as compared to the original Skyrme model.

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Николаев В.А., Новожилов В.Ю., Ткачев О.Г. E4-88-536
Частичное восстановление киральной симметрии
в обобщенной модели Скирма

Рассмотрена обобщенная модель Скирма, которая учитывает основные характерные свойства КХД /образование кваркового и глюонного конденсатов, благодаря нарушению киральной и конформной симметрий/. Показано, что эффективный потенциал для скалярных мезонов, полученный методом одновременной киральной и конформной бозонизации в КХД, существенно определяет статические свойства топологических солитонов. Теоретические значения барионных наблюдаемых значительно улучшаются при экспериментальном значении пионной постоянной F_π в сравнении с оригинальной моделью Скирма.

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Partial Restoration of the Chiral Symmetry
in the Generalized Skyrme Model

The improved Skyrme model that takes into account the main features of QCD (the formation of quark and gluon condensates due to the breaking of chiral and conformal symmetries) is considered. The effective potential for scalar mesons that was derived within the framework of the joint chiral and conformal bosonization method in QCD crucially defines the static properties of the topological solitons. The baryon observables improve considerably for the experimental value of the pion decay constant F_π in comparison with the original Skyrme model results.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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