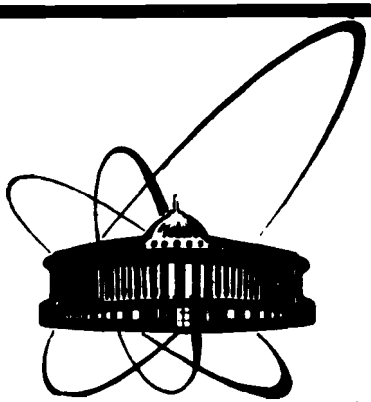

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ON THE EFFECT OF STRONG INTERACTION IN pdμ-MESIC MOLECULES

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Introduction

The effect of strong interactions in different mesic molecules of the hydrogen isotops has been discussed in a number of papers[1]. Strong interaction is treated as the effective potentials between nuclei. In this description, Coulomb levels of mesomolecules acquire some shifts and widths.

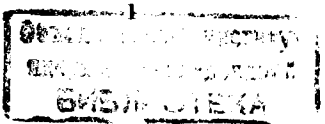
In this paper, it will be shown that the explicit inclusion of the exchange mechanism of interaction between mesic molecular nuclei leads not only to shifts but also to the splitting of the levels.

To demonstrate the effect, we consider the $pd\mu$ mesic molecular ion. In this system there is a finite probability that the neutron bounded in the deuteron could penetrate to a neighbouring proton (as to form again a deuteron). A picture like that corresponds to the motion of a quantum-mechanical particle in the field formed by two potential wells separated by some barrier. The wave function of the system in this case describes the presence of the particle (neutron) simultaneously in both potentials. There are two functions of that sort corresponding to two states of the system distinguished in parity. The difference of the energies of these states gives the value of splitting of the degenerated level in the potential well (i.e. in the deuteron).

The $pd\mu$ molecule considered in this paper is in fact a four body system. It is a hard task to solve the corresponding 4-body equations exactly. There are many ways to simplify this exact equations.

For example one can in a way eliminate the muonic degree of freedom[2]. As a result, one obtains three-body equations with the effective interactions which determine the dynamics of heavy particles (nucleons) in the muonic molecule.

The paper consists essentially of two parts. In the first part we briefly explain the computational method. In the second part we report the numerical results for splitting of levels and the relevant discussion. In particular, possible experiments for detection of the effect are proposed.



I. The method of calculation

The effective hamiltonian which describes the motion of heavy particles in the considered $pd\mu$ -molecule could be written in the form:

$$H = T_{\vec R} + T_{\vec r} + V_a\left(|\vec R - \tfrac{\vec r}{2}|\right) + V_b(r) + V^s_{ab}\left(|\vec R + \tfrac{\vec r}{2}|\right) + W(\vec R) + \Delta(\vec R, \vec r), \tag{1}$$

where $\vec R$ and $\vec r$ are the Jacobi coordinates of heavy particles pictured in fig. 1a; $T_{\vec R}$ and $T_{\vec r}$ represent the corresponding kinetic energies; V_a and V_b are potentials acting between the neutron and protons placed at points a and b respectively; V^s_{ab} is the short-range strong interaction potentials of the proton . The function $W(\vec R)$ describes the molecular potential that creates the Coulomb energy spectrum of the $pd\mu$ mesic molecula[3]. The term $\Delta(\vec R, \vec r)$ appears due to internal structure of deuteron. It may be represented in an approximate form:

$$\Delta(\vec R, \vec r) = \left\langle \chi_\mu \left| \frac{1}{S} - \frac{1}{|\vec S + \frac{\vec r}{2}|} \right| \chi_\mu \right\rangle, \qquad \vec S = \vec\rho - \frac{\vec R}{3} \tag{2}$$

where $\vec\rho$ is the coordinate of the μ -meson (see fig.1a). The wave function χ_μ depends on this variable. The explicit form of this function is not required in estimating this term and its contribution to the energy splitting.

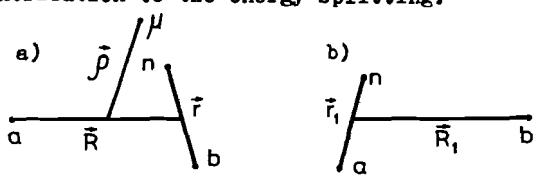


Fig.1.

It is convenient to consider two sets of Jacobi coordinates (see figs. 1a and 1b) and one can introduce two corresponding "unperturbative" hamiltonians:

$$H_b = T_{\vec R} + T_{\vec r} + W(\vec R) + V_b(\vec r), \tag{3}$$

$$H_a = T_{\vec R_1} + T_{\vec r_1} + W(\vec R_1) + V_a(\vec r_1). \tag{4}$$

From the structure of these hamiltonians it follows that their eigenfunctions may be represented in a factorised form

$$\psi_a(\vec R_1, \vec r_1) = \varphi(\vec r_1)\, G(\vec R_1), \tag{5}$$

$$\psi_b(\vec R, \vec r) = \varphi(\vec r)\, G(\vec R), \tag{6}$$

where the functions $\varphi(\vec r)$ and $G(\vec R)$ obey the following equations

$$\left[T_{\vec r} + V_b(\vec r)\right]\varphi(\vec r) = \mathcal{E}\,\varphi(\vec r), \tag{7}$$

$$\left[T_{\vec R} + W(\vec R)\right] G(\vec R) = E\, G(\vec R). \tag{8}$$

Heaving in mind the natural boundary condition for the considered mesomolecule,it is obvious that the solution of equation (7) must be taken as the deuteron wave function $\varphi(\vec r) \approx \varphi_d(r)$ and $\mathcal{E} = \mathcal{E}_d$, where $\mathcal{E}_d$ is the binding energy of the deuteron. (We neglect the admixture of the D-wave in the deuteron). The function $G(\vec R)$ will be expanded in a partial waves $G(\vec R) \sim \sum_{LM} G_L(R)\, Y_{LM}(\vec R)$.

The components $G_L(R)$ satisfy the equation

$$\left[T^L_R + W(R)\right] G_L(R) = E_L\, G_L(R)$$

with $T^L_R = \frac{1}{2M}\left[-\frac{1}{R^2}\frac{d}{dR}\left(R^2\frac{d}{dR}\right) + \frac{L(L+1)}{R^2}\right]$, (9)

where M is the reduced mass of the pd system.

The eigenvalues E_L are the Coulomb spectrum of the mesomolecula. As will be seen later on, when the level splitting of the mesomolecule is estimated its value does not depend on E_L . The only thing required in this case is the value of the

$|G_0(0)|^2$ constant. For this quantity we shall take the known in literature values calculated by different methods. The primary "exact" hamiltonian (1) could be written in terms of non-perturbative ones H_a and H_b in two possible ways:

$$H = H_a + V_b + V_{ab} + \Delta = H_b + V_a + V_{ab} + \Delta . \quad (10)$$

Due to the short-range character of the strong potentials entering into the hamiltonian (1) or (10), the nonperturbative wave functions (5) and (6) coincide with exact one everywhere outside the ranges of these potentials. So, it is natural to look for the solution of the Schrödinger equation:

$$H\Psi = E\Psi \quad (11)$$

as a linear combination of the solutions (5) and (6). This approximation corresponds to the approximation known in the molecular physics as LCAO[4]. However, unlike the LCAO approximation the motion of "heavy" particles is taken into account through the functions $G_L(R)$. Since the dimension of the "atom" (deuteron) is negligible, as compared to the dimension of the meso-molecule, one can expect that this approximation will be much more reliable than the LCAO applied in the theory of the molecular hydrogen ion (where the corresponding dimensions are of the same order). So, the solution of eq. (11) with a given L has the form*)

$$\Psi^L = C_1 \Psi_a^L + C_2 \Psi_b^L , \quad (12)$$

where

$$\Psi_a^L = \varphi_d(r_1)\, G_{NL}(R_1)\, Y_{LM}(\hat{R}_1) , \quad (13)$$

$$\Psi_b^L = \varphi_d(r)\, G_{NL}(R)\, Y_{LM}(\hat{R}) . \quad (14)$$

*) One has to note that the total angular momentum of the 3-body system with the effective interaction in general is not equal to the total angular momentum of the primary four-body system. For the electronic molecules the equality is satisfied with high accuracy. However, for mesic molecules this approximation must be investigated.

Inserting the trial function (12) into the Schrödinger equation (11) with Hamiltonian (1) for the coefficients one obtain two solutions:

$$C_2 = \pm C_1 . \quad (15)$$

So, from (15) and (12) for the new eigenfunctions one has:

$$\Psi_\pm^L = N_\pm^L (\Psi_a^L \pm \Psi_b^L) , \quad (16)$$

where $N_\pm^L$ are the corresponding normalisation factors.

The ordinary calculations bring us to the following expression for the energy difference in these states

$$\Delta E^{NL} = 2\, \frac{I_1^{NL} H_2^{NL} - H_1^{NL} I_2^{NL}}{(I_1^{NL})^2 - (I_2^{NL})^2} \approx 2\left[\frac{H_2^{NL}}{I_1^{NL}} - \frac{H_1^{NL}}{I_1^{NL}} \cdot \frac{I_2^{NL}}{I_1^{NL}}\right], \quad (17)$$

where

$$H_1^{NL} = \langle \Psi_a^L | H | \Psi_a^L \rangle , \quad (18)$$

$$I_1 = \frac{1}{4\pi} , \quad I_2^{NL} = \langle \Psi_a^L | \Psi_b^L \rangle , \quad (19)$$

$$H_2^{NL} = \langle \Psi_a^L | H | \Psi_b^L \rangle . \quad (20)$$

II. Numerical estimation of the energy splitting and discussion

We estimated the energy splitting of the ground state $(N=0, L=0)$ for the $pd\mu$ -molecular ion. Simple but rather cumbersome calculation results in the expression:

$$\Delta E^{00} = 8\pi |G_0(0)|^2 \left\{ \frac{4\varepsilon_d}{\varkappa_d^3} + 0.766\, \frac{V_0^S \mu_S}{\mu_S^3} - \frac{8\pi\, 3{,}135}{\mu_S^3 \varkappa_d^5} |G_0(0)|^2 V_0^S \mu_S \right\} . \quad (21)$$

The expression (21) follows from formulae (17)-(20) where the deuteron wave function has been taken of the Yukawa type, where $\varphi_d(r) = \sqrt{\frac{\varkappa_d}{\pi}}\, \frac{1}{r}\, e^{-\varkappa_d r}$, $\varkappa_d^2 = m|\varepsilon_d|$, ($m$ -is the nucleon mass). The singlet and triplet NN potentials are also

taken in the Yukawa form with parameters fixed in reference [5].

The main contribution to the energy splitting comes from the first term in the bracet of formula (21). This term originates from the exchange part of pd-interaction. Its dominance is understandable since this interaction has the largest range due to the small value of the dueteron binding energy.

So,

$$\Delta E^{oo} \approx \frac{32\pi \varepsilon_d}{\varkappa_d^3} |G_e(0)|^2. \qquad (22)$$

The contributions of the last two terms in (21) are at least one order of magnitude as small as (22).

The ground-state-energy splitting calculated by formula (22) for two values of $|G_e(0)|^2$ existing in literature is presented in the table. In deriving formula (22), for the sake of simplicity, we ignore the spin-isospin structure of the pd wave function in an explicit form. The inclusion of these degrees of freedom changes the r.h.s. (22) by a factor of about 1. We drop it since only the order of magnitude of the splitting is of interect. What is important nevertheless is that the levels created due to the splitting of the ground state ($N=0$, $L=0$) possess definite total nuclear spins, $S=\frac{1}{2}$ and $S=\frac{3}{2}$. The structure of splitting of the excited mesomolecular states ($L>0$) is much more complicated.

Table

	[6]	[3]
$\|G_0(0)\|^2$ [10^{-12} fm^{-3}]	1.4	0.93
ΔE^{00} [10^{-2} eV]	2.00	1.33

It is known [7] that the radiative pd capture at very low energies from the quartet state is much slower than from the doublet one. By varying the temperature of the hydrogen, one can change the population of these levels, and thus obtain the temperature dependence of the yield of γ-quant and μ-mesons from the following reactions

$$pd\mu \rightarrow \mu He^3 + \gamma$$
$$pd\mu \rightarrow He^3 + \mu$$

occuring at temperatures $kT \sim \Delta E^{oo}$. Obviously, a similar phenomenon exists also in the $dt\mu$ system. But due to the corresponding $|G_{dt\mu}(0)|^2$ [8] constant, being small the value of the splitting in this case is two orders of magnitude as small. The strong interaction drastically changes the behaviour of the $G_o(R)$ function at small distances, thus the last conclusion may be changed.

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