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E4-88-471 e

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PROTON POLARIZABILITY CORRECTION TO THE PROTON-PROTON REACTION

Submitted to "Physics Letters B"

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1988

At present there exists a serious discrepancy [1] between theoretical predictions of the capture rate of solar neutrinos and the rate measured in the solar neutrino experiments. The main part of the solar neutrino flux is due to reactions of the proton-proton chain [2]. The capture rate of these neutrinos ($\sim S_{11}^{-5/2}$) is most sensitive [3] to the cross-section factor S_{11} of the initial $p+p \rightarrow d e^{+} \nu_e$ reaction. Therefore estimations of any correction to this factor are important.

The original statement [4] about the existence of a high maximum in the cross-section of the proton-proton reaction due to the proton electric polarizability effect disagrees with a recent examination [5] of this effect performed within a modified low-energy scattering theory for Coulomb plus long-range potentials [6]. Therefore an independent high-accuracy numerical calculation of the proton polarizability correction to the factor S_{11} is important both in view of theoretical discrepancies [4,5] and new planned experiments [7]. The way and results of that calculation are discussed in the present work.

The factor S_{11} is proportional to the square of the nuclear transition matrix element. In the impulse approximation the calculation of this element reduces to an evaluation of the overlap integral [8]

$$\Lambda = \left(\frac{\gamma^3}{8\pi \kappa^2 C_0^2} \right)^{1/2} \int_0^{\infty} d\tau \, u_d(\tau) \, u_{pp}(\tau) \, . \qquad (1)$$

Here $\gamma^{-1} = 4.316$ fm [9] is the deuteron radius, $C_0^2(\eta)$ is the Coulomb-barrier factor, $\eta = 1/2\kappa R$ with $R = 28.81$ fm [9] is the Bohr radius and κ is the proton-proton relative momentum corresponding to the energy $E = (\hbar\kappa)^2/m$ in the proton-proton c.m.s. To calculate the function u_d, i.e. the S-component of the deuteron wave function, we have used the $S_{3,1}$-spline interpolation [10] of tabulated Reid values [11] corresponding to the Reid soft-core potential. The S-wave proton-proton scattering function u_{pp} satisfies the Schrödinger equation written in units $\hbar = c = m = 1$ as follows

$$H u_{pp}(\tau) = (\partial_\tau^2 + \kappa^2 - V(\tau)) \, u_{pp}(\tau) = 0 \, . \qquad (2)$$

In our case the interaction V is the sum of Coulomb ($1/\tau R$), short-range (V_s) Reid soft-core [11] and polarization (V_p) potentials. The polarization potential in the short-range region is not yet determined exactly, therefore we limit ourselves to the representation

$$V_p = -(\alpha/r^4 R)\,\theta(r - r_p), \qquad (3)$$

where θ is the step function and the experimental value of the single-proton electric polarizability constant α is equal to 10^{-3} fm^3 [12]. In the low-energy limit the phase-shift of the scattering problem (2) is not sensitive [6,13] to variations of the radius r_p. We used $r_p = 4$ fm, which corresponds to the interaction range of the short-range potential V_s. According to our WKB-estimation [14] the proton polarizability contribution to the integral (I) is small (~ 10^{-4}%). Hence, a high-accuracy calculation of u_{pp} - function is necessary.

Now we describe the most significant numerical details. The problem (2) has been solved within the variable phase approach [15] described in detail in ref.[6]. The equations for the phase-functions $sn(r)$ and $cs(r)$ (see eqs. (3.3) of ref.[6]) have been integrated in the interval $r_\ell \le r \le r_u$. For an accurate consideration of the short-range singularity of the Reid soft-core potential we have chosen the lower limit r_ℓ sufficiently small, namely, $r_\ell = 10^{-3}$ fm. The upper limit r_u has been defined by the criterion (5) of ref.[13], namely so that the phase-function $cs(r)$ does not change in eight significant digits with increasing r in the region $r > r_u$. To obtain the physical long-range asymptotics for the u_{pp} - function, we multiplied it (see eq. (3.5) of ref.[6]) by the normalization factor

$$const = \kappa\, C_0(\eta)\cos\delta^{c,ps} / cs(r_u).$$

Further we calculated this function, its first and second derivatives with step 10^{-4} fm, and used the $S_{5,3}$- spline interpolation[10]. In this way we obtained the proton-proton wave function satisfying the inequality

$$|u_{pp}^{-1}(r)\, H\, u_{pp}(r)| \le 10^{-8},$$

where the H operator is defined by eq. (2), and $r_\ell \le r \le r_\Lambda$. The practical upper limit r_Λ of integral (1) was chosen equal to 80 fm, which ensured the calculation of Λ with relative accuracy $\sim 10^{-6}$.

Our result $\Lambda(E,\alpha) = 2.638$ for the zero energy and zero constant α in eq. (3) agrees with $\Lambda(0) = 2.633$ calculated in ref. [16] for the same Reid soft-core potential. To demonstrate the influence of the proton polarizability effect of the proton-proton

reaction, we have constructed the function

$$C(E,\alpha) = (\Lambda^2(E,\alpha)/\Lambda^2(E,0) - 1). \qquad (4)$$

For any fixed $\alpha \le 1$ fm^3 this function increases by a factor of 1.02 with increasing energy from 0 to 6 kev corresponding to the average temperature of the solar interior. The values (in per cent) of function (4) for zero energy and several constants $\alpha(fm^3)$ are presented in the table. Even if the constant α is thousand times as large as its experimental value, the contribution of the proton

Table . Proton-polarizability correction (4) to the factor S_{11} for zero energy

α (fm^3)	0.001	0.01	0.1	1.
$C(0,\alpha)$(%)	0.0002	0.002	0.02	0.2

polarizability to the factor S_{11} is much smaller than its well-known 9%-uncertainty [1] due to 2.5%-uncertainty from the nucleon-nucleon interaction, 2%-uncertainty from the exchange mesonic currents, and so on. Therefore the proton polarizability correction to the factor S_{11} like the vacuum polarization correction [17] and the electron screening one [8] is unimportant unless the nucleon-nucleon interaction is known with a sufficient accuracy. Our novel numerical results prove that polarizability of the proton has a negligible effect on the cross-section of the proton-proton reaction and thus confirm the conclusions of ref. [5].

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Received by Publishing Department
on June 30, 1988.

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Вклад эффекта поляризуемости протонов
в сечение протон-протонной реакции

В рамках метода фазовых функций установлено, что вклад эффекта электрической поляризуемости протонов в величину астрофизического S_{11}-фактора протон-протонной реакции не превышает $2\cdot 10^{-4}$%.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1988

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In the framework of the phase function approach the contribution from electric proton polarizability to the astrophysical factor S_{11} of the proton-proton reaction is found to be smaller than $2\cdot 10^{-4}$%.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1988