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**QUANTUM AND THERMODYNAMIC
FLUCTUATIONS
IN THE DEGENERATE SCHEMATIC MODEL
OF THE FINITE TEMPERATURE
RANDOM PHASE APPROXIMATION**

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Recently finite temperature nuclear dynamics has been the subject of much study^{/1-5/}, stimulated by the discovery of giant dipole resonances built on states above the yrast line of highly excited nuclei^{/6,7/}. To these systems the finite temperature Hartree-Fock-Bogoliubov (FT-HFB) formalism has been applied. Properties such as pair gaps and deformations have been determined as functions of spin and temperature^{/8/}. A variety of first and second order phase transitions have been identified^{/9-12/}. The FT-HFB theory ignores fluctuation effects of quantum and thermodynamic types. They have been studied in detail in^{/11,13,14/}. It has been shown in these studies that the thermodynamic fluctuations can wash out the second order phase transition, connected with the pairing collapse in finite nuclei. In our recent publication^{/5/} we have shown in the frame work of the finite temperature RPA(FT-RPA) how this collapse influences the centroids of the giant resonances in heated spherical nuclei. The aim of the present paper is to study the influence of quantum and thermodynamic fluctuations on the energy of collective states in heated spherical systems.

The quantum fluctuations we consider in the present paper are caused by violating the symmetries of the HFB Hamiltonian. In fact in the HFB method the conservation of particle number is taken into account approximately. Soloviev et al. and Mikhailov^{/15/} have shown that the quantum fluctuations of the particle number in the ground-state, the wave function of which is defined as the quasiparticle vacuum, renormalize the pair gap Δ as

$$\tilde{\Delta} = \left(1 + 1/(\Delta N)^2 \right) \Delta, \quad (1)$$

where the fluctuation of particle number with single-particle energy E_j and chemical potential λ is defined as

$$(\Delta N)^2 = \sum_j \left(j + \frac{1}{2} \right) \Delta^2 / (\Delta^2 + (E_j - \lambda)^2) \quad (2)$$

We shall use these results for our calculations at finite tem-



perature, where the BCS pair gap Δ is given by^{16,17/}

$$\Delta(T) = G \sum_j (j + \frac{1}{2}) u_j v_j (1 - 2n_j) \quad (3)$$

n_j is the occupation probability of the quasiparticle state with energy ϵ_j :

$$n_j = (1 + \exp(\epsilon_j/T))^{-1} \quad (4)$$

$$\epsilon_j = ((E_j - \lambda)^2 + \Delta^2(T))^{\frac{1}{2}} \quad (5)$$

u_j , v_j are Bogoliubov coefficients.

The critical temperature T_{crit} for which the collapse of $\Delta(T)$ takes place is given by

$$T_{crit} = 0.567 \Delta(T=0)$$

Using equation (3) to calculate the renormalized BCS pair gap $\tilde{\Delta}(T)$ from equation (1), we can determine the thermodynamic fluctuations through^{11,16/}

$$P(\tilde{\Delta}(T)) \sim \exp(-F(\tilde{\Delta}(T))/T) \quad (6)$$

A similar relation can be taken for $\Delta(T)$ from equation (3). The free energy F is

$$F = \mathcal{E} - \mathcal{S}T,$$

where the energy of the system and the entropy \mathcal{S} at temperature T are

$$\mathcal{E} = \sum_j (j + \frac{1}{2}) E_j (1 - \frac{E_j - \lambda}{\epsilon_j} (1 - 2n_j)) - \Delta^2(T)/G$$

$$\mathcal{S} = - \sum_j (n_j \ln n_j + (1 - n_j) \ln(1 - n_j)).$$

The average pair gap is now defined by^{11,16/}

$$\langle \Delta(T) \rangle = \int_0^\infty \Delta(T) P(\Delta(T)) d\Delta(T) / \int_0^\infty P(\Delta(T)) d\Delta(T) \quad (7)$$

for $\Delta(T)$ as well as for $\tilde{\Delta}(T)$.

The energies of collective excitations can be found by solving the set of FT-RPG equations which have been derived in our previous work^{5/} (see also^{1,17/}). For simplicity we use here the degenerate schematic model, in which N particles occupy N levels with the same quasiparticle energies $\epsilon_j = \epsilon_T$ interacting via a separable force^{18/}. In this case our set of FT-RPA equations can be solved immediately, from which we obtain

$$\omega_T = 2 \epsilon_T \left(1 - \frac{C_T (1 - 2n)}{2 \epsilon_T} \right)^{\frac{1}{2}}, \quad (8)$$

where $C_T = (2\lambda + 1)^{-\frac{1}{2}} \mathcal{X}^{(\lambda)} \sum_{jj'} (f_{jj'}^{(\lambda)})^2 f_{jj'}^{(\lambda)}$; $f_{jj'}^{(\lambda)}$ are the reduced matrix elements of the single-particle operators generating excitation of multipolarity λ ; $\mathcal{X}^{(\lambda)}$ is the effective constant of the separable interaction; n is obtained from equation (4) by substituting ϵ_T for ϵ_j ; $u_{jj'}^{(+)} = u_j v_{j'} + u_{j'} v_j$.

We investigate the behaviour of the dipole state in the lead -58 ($N = 58$, $G = 0.3$, $\Delta(T=0) = 1.4$ MeV) as function of temperature. Assuming the isovector interaction to be repulsive ($\mathcal{X}^{(1)} < 0$), we choose $\mathcal{E} = 4.5$ MeV at $T = 0$. C is adjusted to reproduce the dipole state at zero temperature $\omega_{exp}^{E1} \approx 17$ MeV^{19/}, from which we obtain $C_{T=0} = -23.11$ MeV.

The figure 1 compares the average pair gaps $\langle \Delta \rangle$ and $\langle \tilde{\Delta} \rangle$ with the BCS Δ and $\tilde{\Delta}$. Whereas the BCS Δ and $\tilde{\Delta}$ exhibit a sharp second order phase transition from superfluid to normal, the average $\langle \Delta \rangle$ and $\langle \tilde{\Delta} \rangle$ remain large even at very high temperature^{11/}. Moreover, the quantum fluctuations lead to a larger Δ

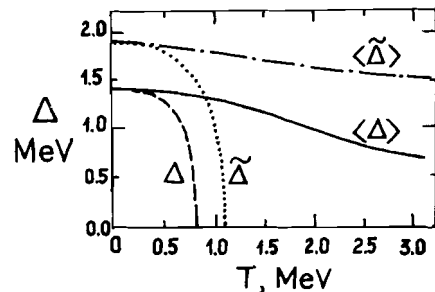


Fig. 1. The pair gap Δ of the lead-58 versus the temperature T . The dashed curve denotes the BCS Δ whereas the dotted curve gives the BCS Δ renormalized by the particle number fluctuations (the $\tilde{\Delta}$). The solid curve represents the average pair gap $\langle\Delta\rangle$ and the dashed-dotted curve is the result for $\langle\tilde{\Delta}\rangle$ including the particle number fluctuations (the $\langle\tilde{\Delta}\rangle$).

(the renormalized $\tilde{\Delta}$). At $T = 0$ the renormalized pair gap $\tilde{\Delta}$ is about 30% greater than Δ (Cf. ^{15/}). Consequently, the critical temperature of the phase transition, where the collapse of $\tilde{\Delta}$ takes place, $T_{crit} \approx 1$ MeV, is higher than $T_{crit} = 0.79$ MeV for the collapse of Δ . By the same reason the average $\langle\tilde{\Delta}\rangle$ is larger than $\langle\Delta\rangle$ and for increasing values of T from 0 to 3 MeV this difference increases from 30% to 50% roughly. It is also evident from figure 1 that the renormalized average $\langle\tilde{\Delta}\rangle$ depends more weakly on the temperature than the average $\langle\Delta\rangle$. Therefore we can conclude that the thermodynamic fluctuations in Δ wash out the second order phase transition (Cf. ^{11,13,14/}) and thanks to the quantum fluctuations this effect becomes stronger.

The energies of the dipole state in the lead-58, calculated from equation (8) as function of temperature T with the pair gaps $\Delta(T)$, $\tilde{\Delta}(T)$, $\langle\Delta(T)\rangle$ and $\langle\tilde{\Delta}(T)\rangle$ respectively, are shown in figure 2. The temperature dependences of the dipole state energy, obtained with the BCS Δ and $\tilde{\Delta}$, undergo a discontinuity at the critical points T_{crit} and \tilde{T}_{crit} as has been shown by us earlier ^{15/}. However as the thermodynamic fluctuations wash out the second order phase transition, such discontinuity disappears in the temperature dependence of the dipole state energy, calculated with the average $\langle\Delta\rangle$ and $\langle\tilde{\Delta}\rangle$, and for them we have smooth curves. The quantum fluctuations in $\langle\tilde{\Delta}\rangle$ increase the dipole state energy by 400-600 keV roughly as compared with this energy, calculated with $\langle\Delta\rangle$. Both the temperature dependences of the dipole state energy in the system, for which the quantum and thermodynamic fluctuations are taken into account, show the same characteristic feature to decrease for increasing values of temperature T , ^{2,4/} which is in agreement with the conclusion reached by other authors ^{2,4/} and the experimental observations ^{20/}. For $T \approx 2.0-2.4$ MeV the dipole state energies

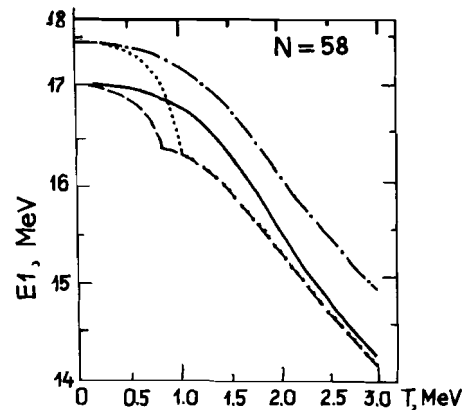


Fig. 2. The dipole state energy $E1$ of the lead-58 versus the temperature T . As in figure 1 the dashed, dotted, solid and dashed-dotted curves represent the values of $E1$ calculated from equation (8) with the pair gap Δ , $\tilde{\Delta}$, $\langle\Delta\rangle$ and $\langle\tilde{\Delta}\rangle$ respectively.

in figure 2 are 1.5 to 2.0 MeV smaller than at $T = 0$ when the measured spectra in medium and heavy nuclei at these values of T give the giant dipole resonance energies lower by 1-1.5 MeV as compared with those at $T = 0$ ^{7/}. Because of the simplicity of the schematic model used here we cannot pretend to a more quantitative description. However our results are enough to conclude that the quantum and thermodynamic fluctuations are undoubtedly important in the excitations at finite temperature. Moreover for finite systems such as nuclei, thermodynamic fluctuations in the order parameter as the pair gap Δ are large even for states far from critical point.

They lead to the dramatical effect which wash out the second order phase transition from the superfluid state to the normal one, while the number particle fluctuations intensify this result. Therefore such phase transition cannot be observed neither by the temperature dependence of the pair gap nor by the one of the collective state energies in heated spherical systems. It should be very desirable to investigate the effects of the quantum and thermodynamical fluctuations on the giant multipole resonance energy centroids in realistic hot nuclei, where we shall observe the concrete quantitative results. These investigations of such kind are now in progress and we will publish their results elsewhere.

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