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M.Kh.Khankhasayev, N.S.Topilskaya*

AN IMPROVED OPTICAL POTENTIAL FOR LOW-ENERGY PION-NUCLEUS SCATTERING

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* Institute for Nuclear Research
of the USSR Academy of Sciences

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In recent years much material has been accumulated on the elastic scattering of low-energy pions by a considerable number of nuclei (see, for example, the review /1/). Further information on the pion-nucleus interaction is provided by the data from pionic atom spectroscopy getting more and more accurate. Thus, the problem of unified description of both the scattering and pionic atom data becomes actual.

A semiphenomenological approach to the solution of this problem has been developed in /2-4/. Its main idea consists in extrapolating the Kisslinger-like optical potential /5/ describing pionic atoms to the region of finite scattering energies. The SMC-potential obtained in /2,3/ fits the scattering data quite well up to 30MeV but not at 50 MeV. In /3/ it has been shown that the agreement between the theory and experiment for 50 MeV pions can be restored if an essential energy dependence in the imaginary parts of the absorptive parameters is supposed.

On the other hand, the analysis of the pion-nucleus scattering in the framework of the unitary approach /6,7/ indicates an approximate constancy of the absorptive parameters in the energy range from 0 to 50 MeV. It is in agreement with the presupposed dominance of the two-nucleon mechanism of the pion absorption. This assumption is used, in particular, to obtain the absorption correction in the approximation of a local density.

In the present paper we show that a good description of the low-energy pion-nucleus scattering up to 50 MeV can be obtained by using the SMC-potential with the absorptive parameters fitted to the pionic atom data but changing essentially the extrapolating procedure for the single-particle parameters.

The SMC-potential for nuclei with zero spin and isospin has the form

$$2\omega\, U_{opt}(r) = -4\pi[b(r) + B(r)] + 4\pi\vec{\nabla}\{L(r)[c(r) + \mathcal{C}(r)]\}\vec{\nabla} - 4\pi\left\{\frac{P_1 - 1}{2}\vec{\nabla}^2 c(r) + \frac{P_2 - 1}{2}\vec{\nabla}^2 \mathcal{C}(r)\right\}, \quad (1)$$

where

$$L(r) = \left\{1 + \frac{4\pi}{3}\lambda[c(r) + \mathcal{C}(r)]\right\}^{-1} \quad (2)$$

is the well-known /5/ Lorentz-Lorenz correction (LLEE-effect), $b(r) = P_1 \bar{b}_0 \rho(r)$ and $c(r) = c_0 \rho(r)/P_1$ are the terms linear in the nuclear density $\rho(r)$, $B(r) = P_2 B_0 \rho^2(r)$ and $\mathcal{C}(r) = \mathcal{C}_0 \rho^2(r)/P_2$ are the terms quadratic in ρ, $P_1 = 1 + \omega/M$ and $P_2 = 1 + \omega/2M$ are kinematic factors, ω is the pion energy, and M is the nucleon mass.

The parameter $\bar{b}_0$ takes into account the rescattering of the pion on a pair of nuclear nucleons

$$\bar{b}_0 = b_0 + b^{(2)}, \quad b^{(2)} = -(b_0^2 + 2b_1^2)\langle \tfrac{1}{r}\rangle . \quad (3)$$

The reciprocal correlation length is represented /2-4/ by its expression in the Fermi-gas model: $\langle 1/r\rangle = 3K_F/2\pi$, where $K_F = 1.4\ F^{-1}$.

The single-nucleon parameters $b_{0,1}$ and $c_{0,1}$ can be calculated by using the data of the phase-shift analysis of the πN -scattering.

Table 1. Optical parameter sets for 50 MeV scattering

	Set A	Extrapolated set A	Set I	Set II
$\bar{b}_0$ (fm)	-0.046+i0.006	-0.057+i0.006	-0.057-i0.003	-0.053-i0.003
b_1 (fm)	-0.134-i0.002	-0.134-i0.002	-0.134-i0.003	-0.130-i0.003
c_0 (fm^3)	0.66+i0.029	0.75+i0.029	0.75+i0.030	0.75+i0.030
c_1 (fm^3)	0.428+i0.014	0.428+i0.014	0.428+i0.014	0.45+i0.014
λ	1.4	1.4	1.4	1.4
B_0 (fm^4)	0.007+i0.19	-0.02 +i0.25	0.007+i0.19	0.007+i0.19
$\mathcal{C}_0$ (fm^6)	0.287+i0.93	0.36+i1.2	0.287+o.93	0.287+i0.93
$\bar{Q}_p$	0.31	0.31	-	-
Δ (MeV)	-	-	25	25

The absorption parameters B_0 and C_0, and also λ, are usually regarded as free and are obtained by fitting to the data on pionic atoms. Actually, the adjustable parameters also include $\bar{b}_0$ since a better description of the pionic atom data is achieved at values of it much larger than suggested by (3). In the first two columns of Table 1 we present two sets of parameters of the SMC-potential from ref. /3/ for the scattering at 50 MeV. In Set A the absorptive parameters are fitted to the pionic atom data and Extr. set A shows their extrapolated values to 50 MeV in accordance with /9/. It is seen from Fig. 1 that these sets do not provide good description of the data. The imaginary parts of the single-nucleon parameters $b_{0,1}$ and $C_{0,1}$ have been calculated in /2-4/ by multiplying the imaginary parts of the πN -amplitude by the Pauli factor $\bar{Q}_p(k)$. This factor for pions has been calculated in /10/ for infinite nuclear matter. It is obviously a rough approximation to a real situation of finite nuclei. Indeed, the Pauli factor for infinite nuclear matter

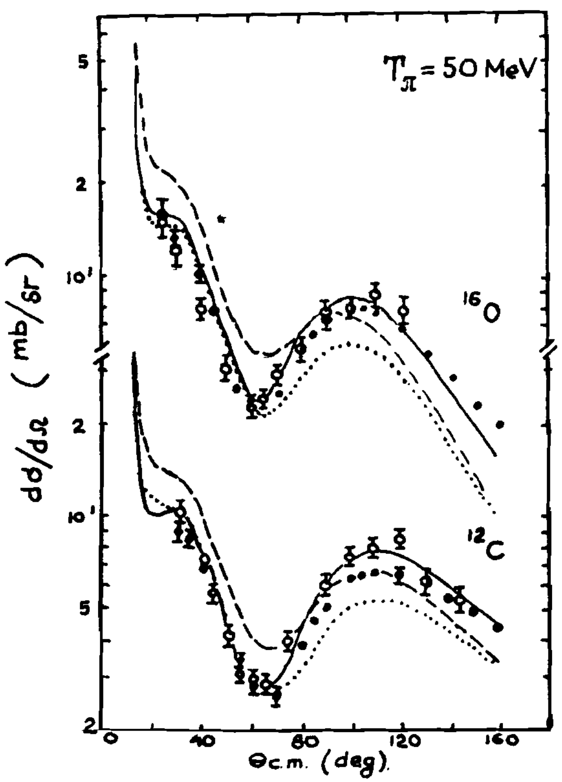

Fig. 1. Elastic scattering of 50 MeV π^+ from ^{12}C and ^{16}O. The curves are calculated using the parameters given in Table II. The solid curve is for set I, the dashed is for Extr. set A and the dotted is for set A. The data are from Refs. /13,14/.

strongly suppresses the forward angle scattering of a pion by a nucleon. To eliminate this unphysical behaviour, angle averaging has been performed in /10/. Further, the Pauli factor $\bar{Q}_p$ does not take into account the excitation spectrum of real nuclei. Thus, it does not provide a correct quasi-two-particle limit of the theory, i.e. the imaginary part of the optical potential does not vanish at pion energies below the threshold for excitation of the nuclear system.

In this paper, to allow for nuclear medium effects in pion--nucleus scattering we use the πN -amplitudes modified in the following way

$$f^{\alpha}_{\pi N}(k) \to \tilde{f}^{\alpha}_{\pi N}(k) = \frac{1}{2ik\,\xi_\alpha(k,k_\Delta)}\cdot\left(e^{2i\delta_\alpha(k)\,\xi_\alpha(k,k_\Delta)} - 1\right), \quad (4)$$

where the factor

$$\xi_\alpha(k,k_\Delta) = \frac{\mathcal{E}(k_\Delta)}{\mathcal{E}(k)}\left[\frac{g_\alpha(k_\Delta)}{g_\alpha(k)}\right]^2. \quad (5)$$

Here $f^{\alpha}_{\pi N}$ is the free πN - amplitude, $\alpha \equiv (\ell, j, I)$ labels the quantum numbers of the πN -system; $\mathcal{E}(k) = k^2/[2\pi^2 dE_0(k)/dk]$ is the level density; $E_0(k)$ is the collision energy; the form factor $g_\alpha(k)$ of the rank-one separable potential

$$v_\alpha(k,k') = \sigma_\alpha\, g_\alpha(k)\, g_\alpha(k') \quad (6)$$

takes into account the off-shell behaviour of the πN -scattering amplitude. In the calculations, we have used

$$g_\alpha(k) = k^{\ell}/[k^2 + \beta^2]^{\ell+1} \quad (7)$$

with $\beta = 2.1$ fm^{-1} corresponding to the pion-nucleon phase-shift data from ref. /11/ (RSL- πN).

The shift of the πN-amplitude from the energy shell is estimated by the parameter Δ which is a certain mean excitation energy of the nuclear system. It has entered into the theory (see below) due to the use of the completeness approximation in deriving (4). The momentum k_Δ is determined by the equation

$$E_0(k) - E_0(k_\Delta) - \Delta = 0\,. \quad (8)$$

The parameter Δ depends, in general, on the energy of the incident pion. As it will be shown below (see, Fig. 2) the value of $\Delta \sim 20$ MeV will provide the best description of the differential cross sections in the whole energy region 0-50 MeV. The same values for Δ have been got in /7/ from the analysis of the scattering data in the framework of the unitary approach.

Expression (4) provides a correct threshold behaviour of the pion-nucleon amplitude in nuclear medium. At low energies, as $\kappa_\Delta \to 0$ ($\kappa < \kappa_0 \approx \sqrt{2m\Delta}$, m is the reduced mass of the π-nucleus system) the parameter $\xi_\alpha \to 0$ and $\tilde{f}_\alpha$ becomes real

$$\tilde{f}_\alpha \simeq \frac{1}{\kappa}\,\delta_\alpha(\kappa) \; .$$

Then, if $\Delta \to 0$ or $\kappa \gg \kappa_0$, the factor $\xi_\alpha \to 1$, and $\tilde{f}_\alpha$ coincides with the free πN-amplitude. Unlike the Pauli factor Q_P , the factor ξ_α has a nontrivial dependence on quantum numbers of the πN-channel

$$\xi_\alpha(\kappa, \kappa_\Delta) \sim (\kappa_\Delta/\kappa)^{2\ell+1} .$$

The second change concerns the S-wave parameter $\bar{b}_0$ in (3). In [2-4] its imaginary part is formed only by the imaginary parts of the single-nucleon parameters. As it has been shown by Thies[12], some additional contribution comes from the imaginary part of the pion propagator in $b^{(2)}$. Following [12], but taking into account the influence of the nuclear medium on the pion rescattering, one can get at low energies that

$$\langle \tfrac{1}{r} \rangle \;\to\; \langle \tfrac{1}{r} \rangle + i\kappa\,\xi_0(\kappa, \kappa_\Delta) , \qquad (9)$$

where $\langle 1/r \rangle = 3k_F/2\pi$ and ξ_0 is defined in (5) for $\ell = 0$.

Expressions (4) and (9) are the main results of the present paper. Both the modifications concern mainly the imaginary part of the optical potential (1). The factor ξ_α provides a correct threshold behaviour of the potential part of (1), which in the low-energy limit becomes Hermitian.

In Table 1 we represent two sets (I and II) of the parameters for 50 MeV pions. In both the sets the absorption parameters are the same as in the pionic set A, and the imaginary parts of the single-nucleon parameters $\bar{b}_0$ and C_0 have been calculated by using Eqs. (4) and (9) for $\Delta = 25$ MeV. The real parts of $\bar{b}_0$ and C_0 in set I are the same as in the extrapolated set A. In set II we also represent the calculated values for $Re\,\bar{b}_0$ and $Re\,C_0$ which are very close to those in the extrapolated set A. Thus, set II represents the pure theoretical calculations of $\bar{b}_0$ and C_0 . In the calculations, the RSL-πN-phase shifts [11] have been used.

The result of the elastic scattering calculations with set I (solid curve) is shown in Fig. 1. We see that the modified SMC-potential provides a good description of the data [13,14] with the pionic absorption parameters.

In Fig. 2 we demonstrate the sensitivity of the calculations to the second-order corrections and to the parameter Δ . The dashed line corresponds to $b^{(2)} = 0$ and $\Delta = 0$. The dotted and the solid curve show the full calculations for $\Delta = 0$ and $\Delta = 25$ MeV, respectively.

The mean excitation energy parameter $\Delta \simeq 20 \div 25$ MeV provides the best fit of the scattering data. The same value for Δ has been obtained in [7] from the analysis of the pion scattering on ^{12}C and ^{16}O at 50 MeV in the framework of the unitary approach [1]. It is natural to suppose (see also[7]) that the ~ 20 MeV value for Δ reflects the dominant role of the resonance mechanism in the formation of the pion-nucleus inelasticity parameters.

In Fig. 3 we also present the calculated result for the $\pi\,^{12}C$-differential cross section at lower energy (30 MeV). It should be noted that at energies below 30 MeV, the sensitivity to Δ becomes weaker because the imaginary part of the optical potential is formed mainly by the absorption channel.

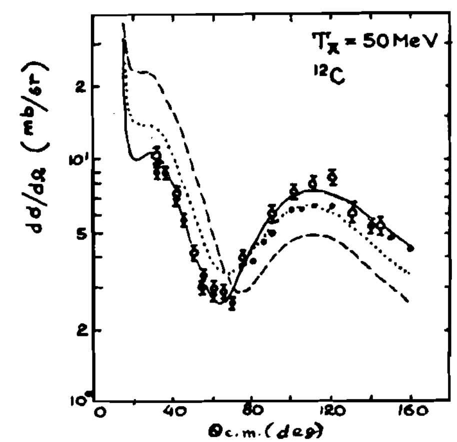

Fig. 2. Elastic scattering of 50 MeV π^+ from ^{12}C. The solid curve and the dotted one show the full calculations for Δ = =25 MeV (set II) and for Δ =0, respectively. The dashed line is for $b^{(2)}$ =0 and Δ =0. The data are from [13,14].

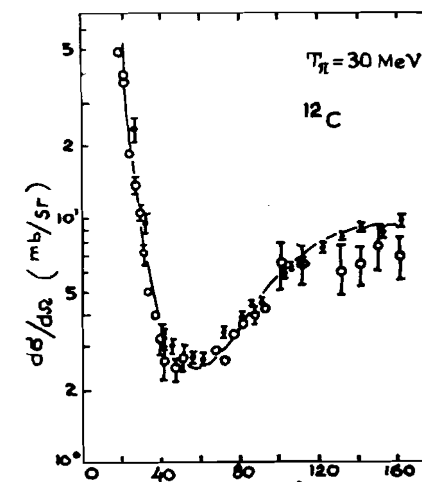

Fig. 3. Elastic scattering of 30 MeV π^+ from ^{12}C. The solid curve is calculated using (4) and (9) and the pionic absorption parameters from set A. The data are from [15,16].

Now we outline briefly the basic elements of the derivation of (4) and (9). The detailed derivation will be reported in the extended version of the paper. The starting point is the equation for the auxiliary operator T_i

$$T_i(E) = v_i + A\, v_i\, \hat{Q}\, G(E)\, T_i(E). \tag{10}$$

In terms of this operator the optical potential is

$$V_{opt}(E) = \sum_{i=1}^{A} T_i(E). \tag{11}$$

Here E is the collision energy; $G(E)=(E^{+}-h)^{-1}$ is the Green function, $h=K_\pi+H_A$, K_π is the pion kinetic energy operator, H_A is the nuclear Hamiltonian; $\hat{Q}=\sum_{n>0}|n\rangle\langle n|$ is the operator of projection onto excited state of nucleus.

The goal is to get an appropriate approximation to the many-body operator $T_i(E)$ by some two-body scattering matrix $\tilde{t}_i$ which is defined in the space of plane waves. To do this, let us utilize the completeness approximation by setting in the second term of Eq. (10) $E_n = E_0 + \mathcal{E}_n \simeq E_0 + \Delta$, where $\mathcal{E}_n$ is the energy of the excited state of the nucleus measured from the ground-state energy, and Δ is a certain mean excitation energy of the nuclear system. This approximation makes it possible to get

$$A\hat{Q}G(E) \simeq g(\omega-\bar{\Delta}), \tag{12}$$

where $g(\omega)$ is the free Green function, ω is the energy of the πN-subsystem, and $\bar{\Delta}=(m/\mu)\Delta$, μ and m are the reduced masses of the πN and πA - systems, respectively.

The approximation (12) is valid if only the pion-nucleon interactions, which can effectively be considered as rescattering of the pion on the same nuclear nucleon by a given πN-potential (the diagrams a) and b) in Fig. 4), are taken into account. Thus, from eq. (10) we obtain the following approximate equation for the two-body $\tilde{t}_i$-matrix:

$$\tilde{t}_i(\omega) = v_i + v_i\, g(\omega-\bar{\Delta})\, \tilde{t}_i(\omega). \tag{13}$$

Hence, it follows that $\tilde{t}(\omega)$ is the off-shell extrapolation of the free πN-scattering matrix. Using the rank-one separable potential (6), one can show that

$$\tilde{t}_\alpha(\kappa,\kappa;\omega(\kappa)) = \frac{g_\alpha^2(\kappa)}{g_\alpha^2(\kappa_\Delta)}\, t_\alpha(\kappa_\Delta,\kappa_\Delta;\omega(\kappa_\Delta)) = \tag{14}$$

$$= -\frac{1}{2\pi i\, \mathcal{E}(\kappa)\, \xi_\alpha(\kappa,\kappa_\Delta)}\left[e^{2i\delta_\alpha(\kappa_\Delta)}-1\right],$$

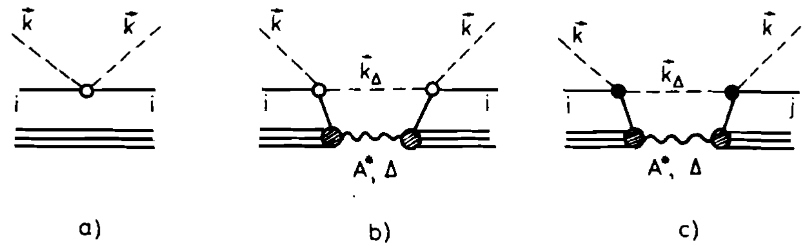

Fig. 4. Graphycal representation of the pion rescattering by the nuclear nucleons. The dashed line is for pion; the solid is for nucleon, the waved labels the excited nucleus (Δ); the open circle - $v^i_{\pi N}$; the black circle - $\tilde{t}^i_{\pi N}$; and the shaded circle - the nuclear wave function $|n\rangle$.

where $\omega(\kappa_\Delta)=\omega(\kappa)-\bar{\Delta}$, and the factor ξ_α is defined in (5). Taking into account that at low energies

$$\delta_\alpha(\kappa_\Delta) \simeq \delta_\alpha(\kappa)\, \xi_\alpha(\kappa,\kappa_\Delta)$$

we get expression (4).

The same consideration of the pion rescattering on the two nucleons (the diagram c) in Fig. 4) leads to the formula (9).

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Ханхасаев М.Х., Топильская Н.С. E4-88-36
Оптический потенциал для пион-ядерного рассеяния
при низких энергиях

Предлагается новая процедура расчета мнимой части известного SMC-потенциала. Основным ее элементом является приближенное выражение для амплитуды пион-нуклонного рассеяния, учитывающее эффекты ядерной структуры. Показано, что полученный таким образом потенциал с параметрами поглощения, определенными по пион-атомным данным, обеспечивает стабильное описание данных рассеяния при энергиях до ~ 50 МэВ.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Khankhasayev M.Kh., Topilskaya N.S. E4-88-36
An Improved Optical Potential
for Low-Energy Pion-Nucleus Scattering

A new procedure for calculating the imaginary part of the Stricker, McManus and Carr (SMC) optical potential is proposed. It is based on an approximate expression for the pion-nucleon scattering amplitude including nuclear structure effects. It is shown that the resulting potential with the absorption parameters fitted to the pionic atom data provides a good description of the scattering up to 50 MeV.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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