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THERMO FIELD BOSON REALIZATIONS FOR OPERATORS OF THE FINITE TEMPERATURE RANDOM PHASE APPROXIMATION

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Recently, one of the most powerful tools in the description of nuclear collective motion in the space of many-nucleon (or many-quasiparticle) states is the boson mapping. Various kinds of this technique such as the Dyson (DR), Holstein - Primakoff (HPR) and Schwinger (SR) boson realizations are quite popular in relating two famous nuclear models: the microscopic shell model and the phenomenological Interacting Boson Model (IBM)[1]. These boson realizations are also an important cornerstone for many nuclear algebraic models based on dynamical symmetry. On the other hand, in recent studies of the compound nuclei formed in heavy ion reactions[2] the decay of highly excited states above the yrast line has been interpreted on the basis of mean field theories as collective excitations with given temperature. Applying the boson expansion method to the finite temperature case, Providência and Fiolhais have recently obtained within the scope of the two level Lipkin model the thermal HPR for the SU(2) operators $J_{\pm}$ and J_z [3]. Due to the oversimplified character of their schematic model this boson realization is not universal while microscopic theories based on the boson expansion at finite temperature should require a general thermal boson realization for bifermion operators.

The purpose of the present letter is therefore to construct the boson realizations for bifermion (two-quasiparticle) operators at finite temperature in the general case. A transparent way to realize this aim is to exploite the formalism of Thermo Field Dynamics (TFD)[4] which provides an elegant method of calculating in many-body theories at finite temperature. By constructing the thermal microscopic operators of the Finite Temperature Random Phase Approximation (FT-RPA) and formulating the conditions under which these FT-RPA operators have an appropriate unitary symmetry, we shall obtain for them the DR, HPR and SR at finite temperature. It will be shown also that the form of the thermal boson realization given by Providência and Fiolhais can be directly derived as a special case of our thermal HPR.

We start with the quasiparticle creation α^+_{jm} and annihilation α_{jm} operators satisfying the usual commutation relations for fermions. In the TFD formalism the transformation from α^+_{jm} and α_{jm} to the thermal quasiparticle operators $\alpha^+_{jm}(T)$ and $\alpha_{jm}(T)$ at finite temperature T read /4/

$$\begin{aligned} \alpha^+_{jm}(T) &= \sqrt{1-n_j}\,\alpha^+_{jm} - \sqrt{n_j}\,\alpha_{j\tilde m} \\ \alpha_{j\tilde m}(T) &= \sqrt{1-n_j}\,\alpha_{j\tilde m} + \sqrt{n_j}\,\alpha^+_{jm}\,, \end{aligned} \tag{1}$$

where the usual time-reversed notation ($\sim$) is used for the annihilation operator, $\alpha_{j\tilde m} = (-)^{j-m}\alpha_{j-m}$ and n_j is the occupation number of quasiparticle with energy ε_j at temperature T

$$n_j = \left[\exp(\varepsilon_j/T) + 1\right]^{-1}. \tag{2}$$

The two-quasiparticle operators we consider are the well-known

$$A^+_{JM}(ab) = \sum_{m_a m_b} \langle j_a m_a\, j_b m_b | JM\rangle\, \alpha^+_{j_a m_a}\alpha^+_{j_b m_b} \tag{3}$$

$$A_{JM}(ab) = \sum_{m_a m_b} \langle j_a m_a\, j_b m_b | JM\rangle\, \alpha_{j_b m_b}\alpha_{j_a m_a} \tag{4}$$

$$\begin{aligned} B_{JM}(ab) &= -\sum_{m_a m_b} \langle j_a m_a\, j_b m_b | JM\rangle\, \alpha^+_{j_a m_a}\alpha_{j_b \tilde m_b} \\ B^+_{J\tilde M}(ab) &= -(-)^{j_a+j_b-J} B_{JM}(ba)\,. \end{aligned} \tag{5}$$

Applying the ansatz (1), we find for the thermal analogues of the operators (3)-(5)

$$\begin{aligned} A^+_{JM}(ab)_T = {} & \sqrt{(1-n_{j_a})(1-n_{j_b})}\,A^+_{JM}(ab) - \sqrt{n_{j_a}n_{j_b}}\,A_{J\tilde M}(ab) + \\ & + \sqrt{(1-n_{j_a})n_{j_b}}\,B_{JM}(ab) + \sqrt{(1-n_{j_b})n_{j_a}}\,B^+_{J\tilde M}(ab) \\ & - \sqrt{2j_a+1}\sqrt{(1-n_{j_a})n_{j_a}}\,\delta_{j_a j_b}\delta_{J0} \end{aligned} \tag{6}$$

$$\begin{aligned} A_{J\tilde M}(ab)_T = {} & \sqrt{(1-n_{j_a})(1-n_{j_b})}\,A_{J\tilde M}(ab) - \sqrt{n_{j_a}n_{j_b}}\,A^+_{JM}(ab) + \\ & + \sqrt{(1-n_{j_a})n_{j_b}}\,B^+_{JM}(ab) + \sqrt{(1-n_{j_b})n_{j_a}}\,B_{J\tilde M}(ab) \\ & - \sqrt{2j_a+1}\sqrt{(1-n_{j_a})n_{j_a}}\,\delta_{j_a j_b}\delta_{J0} \end{aligned} \tag{7}$$

$$\begin{aligned} B_{JM}(ab)_T = {} & \sqrt{(1-n_{j_a})(1-n_{j_b})}\,B_{JM}(ab) - \sqrt{n_{j_a}n_{j_b}}\,B^+_{J\tilde M}(ab) - \\ & - \sqrt{(1-n_{j_a})n_{j_b}}\,A^+_{JM}(ab) - \sqrt{(1-n_{j_b})n_{j_a}}\,A_{J\tilde M}(ab) \\ & + \sqrt{2j_a+1}\,n_{j_a}\,\delta_{j_a j_b}\delta_{J0}\,, \end{aligned} \tag{8}$$

where the subscript T denotes the thermal bifermion operators. Employing the well-known DR for the operators (3)-(5) /5,6/, by a simple insertion of these DR into the r.h.s. of Eqs. (6)-(8) we obtain the DR for the thermal two-quasiparticle operators in the l.h.s. of Eqs. (6)-(8). It reads

$$\begin{aligned} \left[A^+_{JM}(ab)_T\right]^{DR} = {} & \sqrt{(1-n_{j_a})(1-n_{j_b})}\Big\{ b^+_{JM}(ab) - \\ & - \sum_{J_1J_2J_3J_4}\sum_{j_c j_d} C^J_{J_1J_2J_3J_4}(abcd)\big[[b^+_{J_1}(ac)\otimes b^+_{J_2}(bd)]_{J_4}\otimes \tilde b_{J_3}(cd)\big]_{JM}\Big\} + \\ & + \sum_{J_1J_2j_c} D_{J_1J_2}(abc)\Big\{\sqrt{(1-n_{j_a})n_{j_b}}\,[b^+_{J_1}(ac)\otimes \tilde b_{J_2}(bc)]_{JM} + \\ & \qquad + \sqrt{(1-n_{j_b})n_{j_a}}\,[b^+_{J_2}(bc)\otimes \tilde b_{J_1}(ac)]_{J\tilde M}\Big\} \\ & - \sqrt{n_{j_a}n_{j_b}}\,\tilde b_{JM}(ab) - \sqrt{2j_a+1}\sqrt{(1-n_{j_a})n_{j_a}}\,\delta_{j_a j_b}\delta_{J0}\,, \end{aligned} \tag{9}$$

$$\begin{aligned} \left[A_{J\tilde M}(ab)_T\right]^{DR} = {} & -\sqrt{n_{j_a}n_{j_b}}\Big\{ b^+_{JM}(ab) - \\ & - \sum_{J_1J_2J_3J_4}\sum_{j_c j_d} C^J_{J_1J_2J_3J_4}(abcd)\big[[b^+_{J_1}(ac)\otimes b^+_{J_2}(bd)]_{J_4}\otimes \tilde b_{J_3}(cd)\big]_{JM}\Big\} + \end{aligned} \tag{10}$$

$$
+\sum_{J_1J_2j_c} D_{J_1J_2}(abc)\Big\{\sqrt{(1-n_{j_b})n_{j_a}}\,[b^+_{J_1}(ac)\otimes\tilde{b}_{J_2}(bc)]_{JM} + \sqrt{(1-n_{j_a})n_{j_b}}\,[b^+_{J_2}(bc)\otimes\tilde{b}_{J_1}(ac)]_{J\tilde{M}}\Big\} + \sqrt{(1-n_{j_a})(1-n_{j_b})}\,\tilde{b}_{JM}(ab) - \sqrt{2j_a+1}\sqrt{(1-n_{j_a})n_{j_a}}\,\delta_{j_aj_b}\delta_{J0}\ ,
$$

$$
\begin{aligned}
[B_{JM}(ab)_T]^{DR} = -\sqrt{(1-n_{j_a})n_{j_b}}\,\Big\{ b^+_{JM}(ab) &- \sum_{J_1J_2J_3J_4}\sum_{j_cj_d} C^J_{J_1J_2J_3J_4}(abcd)[[b^+_{J_1}(ac)\otimes b^+_{J_2}(bd)]_{J_4}\otimes\tilde{b}_{J_3}(cd)]_{JM}\Big\} + \\
&+\sum_{J_1J_2j_c} D_{J_1J_2}(abc)\Big\{\sqrt{(1-n_{j_a})(1-n_{j_b})}\,[b^+_{J_1}(ac)\otimes\tilde{b}_{J_2}(bc)]_{JM} - \\
&- \sqrt{n_{j_a}n_{j_b}}\,[b^+_{J_2}(bc)\otimes\tilde{b}_{J_1}(ac)]_{J\tilde{M}}\Big\} - \\
&- \sqrt{(1-n_{j_b})n_{j_a}}\,\tilde{b}_{JM}(ab) + n_{j_a}\sqrt{2j_a+1}\,\delta_{j_aj_b}\delta_{J0}\ ,
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
[B^+_{J\tilde{M}}(ab)_T]^{DR} = -\sqrt{(1-n_{j_b})n_{j_a}}\,\Big\{ b^+_{JM}(ab) &- \sum_{J_1J_2J_3J_4}\sum_{j_cj_d} C^J_{J_1J_2J_3J_4}(abcd)[[b^+_{J_1}(ac)\otimes b^+_{J_2}(bd)]_{J_4}\otimes\tilde{b}_{J_3}(cd)]_{JM}\Big\} + \\
&+\sum_{J_1J_2j_c} D_{J_1J_2}(abc)\Big\{\sqrt{(1-n_{j_a})(1-n_{j_b})}\,[b^+_{J_2}(bc)\otimes\tilde{b}_{J_1}(ac)]_{J\tilde{M}} - \\
&- \sqrt{n_{j_a}n_{j_b}}\,[b^+_{J_1}(ac)\otimes\tilde{b}_{J_2}(bc)]_{JM}\Big\} - \\
&- \sqrt{(1-n_{j_a})n_{j_b}}\,\tilde{b}_{JM}(ab) + n_{j_a}\sqrt{2j_a+1}\,\delta_{j_aj_b}\delta_{J0}\ .
\end{aligned}
\tag{12}
$$

The operators $b^+_{JM}(ab)$ and $\tilde{b}_{JM}(ab)$, etc.,in the r.h.s. of Eqs.(9)-(12) are the ideal boson operators satisfying the commutation relations

$$
\begin{aligned}
&[b_{JM}(ab), b^+_{J'M'}(a'b')] = \delta_{JJ'}\delta_{MM'}[\delta_{j_aj'_a}\delta_{j_bj'_b} - (-)^{j_a+j_b+J}\delta_{j_aj'_b}\delta_{j_bj'_a}] \\
&[b_{JM}(ab), b_{J'M'}(a'b')] = [b^+_{JM}(ab), b^+_{J'M'}(a'b')] = 0\ .
\end{aligned}
\tag{13}
$$

The notation $[b^+_{J_1}(ab)\otimes\tilde{b}_{J_2}(cd)]_{J_3M_3}$ denotes the tensor product

$$
[b^+_{J_1}(ab)\otimes\tilde{b}_{J_2}(cd)]_{J_3M_3} = \sum_{M_1M_2}\langle J_1M_1J_2M_2|J_3M_3\rangle\, b^+_{J_1M_1}(ab)\, b_{J_2\tilde{M}_2}(cd)\ . \tag{14}
$$

The coefficients $C^J_{J_1J_2J_3J_4}(abcd)$ and $D_{J_1J_2}(abc)$ in Eqs. (9)-(12) are

$$
\begin{aligned}
&C^J_{J_1J_2J_3J_4}(abcd) \equiv \hat{J}_1\hat{J}_2\hat{J}_3\hat{J}_4\begin{Bmatrix} j_a & j_c & J_1 \\ j_b & j_d & J_2 \\ J & J_3 & J_4 \end{Bmatrix}(-)^{J_3+J_4+J} \\
&D_{J_1J_2}(abc) \equiv \hat{J}_1\hat{J}_2(-)^{j_a+j_c+J+J_2}\begin{Bmatrix} j_a & j_c & J_1 \\ J_2 & J & j_b \end{Bmatrix} \\
&\hat{J} \equiv \sqrt{2J+1}\ .
\end{aligned}
\tag{15}
$$

Thus, we have already obtained the DR for the thermal two-quasi-particle operators $A^+_{JM}(ab)_T$, $A_{J\tilde{M}}(ab)_T$, $B_{JM}(ab)_T$ and $B^+_{J\tilde{M}}(ab)_T$ at finite temperature. It is clear from Eqs.(9)-(12) that in the zero temperature limit $T\rightarrow 0$ these formulae transform completely into the usual DR for the operators $A^+_{JM}(ab)$, $A_{J\tilde{M}}(ab)$, $B_{JM}(ab)$ and $B^+_{J\tilde{M}}(ab)$ given in /5,6/.

Let us now introduce the RPA operators at finite temperature T

$$
Q^+_{JMi}(T) = \frac{1}{2}\sum_{j_aj_b}[\psi^{Ji}_{ab}A^+_{JM}(ab)_T - \varphi^{Ji}_{ab}A_{J\tilde{M}}(ab)_T]\ . \tag{16}
$$

Using the boson realizations (9)-(12) we find for the operators (15) the boson realization

$$
\begin{aligned}
[Q^+_{JMi}(T)]^{DR} = \frac{1}{2}\sum_{j_aj_b}\Big\{\tilde{\psi}^{Ji}_{ab}\Big(b^+_{JM}(ab) &- \sum_{J_1J_2J_3J_4}\sum_{j_cj_d}C^J_{J_1J_2J_3J_4}(abcd)\times \\
&\times[[b^+_{J_1}(ac)\otimes b^+_{J_2}(bd)]_{J_4}\otimes\tilde{b}_{J_3}(cd)]_{JM}\Big) - \tilde{\varphi}^{Ji}_{ab}\,\tilde{b}_{JM}(ab) +
\end{aligned}
\tag{17}
$$

$$+\sum_{J_1 J_2 j_c} D_{J_1 J_2}(abc)\left(\tilde{\xi}^{Ji}_{ab}\,[b^+_{J_1}(ac)\otimes\tilde{b}_{J_2}(bc)]_{JM} - \tilde{\zeta}^{Ji}_{ab}\,[b^+_{J_2}(bc)\otimes\tilde{b}_{J_1}(ac)]_{J\tilde{M}}\right) - $$
$$-\frac{1}{2}\sqrt{2j_a+1}\left(\tilde{\xi}^{Ji}_{aa} - \tilde{\zeta}^{Ji}_{aa}\right)\delta_{j_a j_b}\delta_{J0}\Big\},$$

where we use the notation

$$\begin{aligned}
\tilde{\psi}^{Ji}_{ab} &\equiv \psi^{Ji}_{ab}\sqrt{(1-n_{j_a})(1-n_{j_b})} + \varphi^{Ji}_{ab}\sqrt{n_{j_a}n_{j_b}} \\
\tilde{\varphi}^{Ji}_{ab} &\equiv \varphi^{Ji}_{ab}\sqrt{(1-n_{j_a})(1-n_{j_b})} + \psi^{Ji}_{ab}\sqrt{n_{j_a}n_{j_b}} \\
\tilde{\xi}^{Ji}_{ab} &\equiv \psi^{Ji}_{ab}\sqrt{(1-n_{j_a})n_{j_b}} - \varphi^{Ji}_{ab}\sqrt{n_{j_a}(1-n_{j_b})} \\
\tilde{\zeta}^{Ji}_{ab} &\equiv \varphi^{Ji}_{ab}\sqrt{(1-n_{j_a})n_{j_b}} - \psi^{Ji}_{ab}\sqrt{n_{j_a}(1-n_{j_b})}\,.
\end{aligned} \tag{18}$$

Employing now once more the DR for operators $A^+_{JM}(ab)$, $A_{J\tilde{M}}(ab)$, $B_{JM}(ab)$ at zero temperature given in[5,6] we express in terms of them the operator (17)(one can also use the Eqs.(6),(7) to obtain the same result

$$Q^+_{JMi}(T) = \frac{1}{2}\sum_{j_a j_b}\left\{\tilde{\psi}^{Ji}_{ab}A^+_{JM}(ab) - \tilde{\varphi}_{ab}A_{J\tilde{M}}(ab) + \tilde{\xi}^{Ji}_{ab}B_{JM}(ab) - \tilde{\zeta}^{Ji}_{ab}B^+_{J\tilde{M}}(ab)\right\}. \tag{19}$$

We see from Eq.(19) that in contrast with the RPA operator introduced at zero temperature[7](Eq. (16) with T = 0) the FT-RPA operator (19) contains besides the operators A^+ and A, also the operator B and B^+ of scattering quasiparticles. The terms containing the operators B and B^+ in (19) appear only at finite temperature and lead to the ($p-p$) and ($h-h$) transitions whereas the ($p-h$) transitions are due to the terms of A and A^+. The operator (19) is just the thermal phonon operator, introduced by us earlier in the FT-RPA, based on somewhat different interpretation[8].

In the first version of the IBM-1, the scalar monopole s^+, s and quadrupole d^+, d bosons have been considered. In the microscopic foundation these bosons correspond to the collective nucleon pairs with angular momenta 0 and 2. In the quasiparticle representation this leads to the consideration of the two-quasiparticle operators of the types (3)-(6) with J = 2, while the momentum J = 0

is neglected to exclude the "spurious" components caused by the nucleon-number non-conservation[9]. In fact, all the monopole pairing interactions have already been included by introducing the Bogolubov quasiparticles[7]. Thus, considering only the momentum J = 2 in the collective approximation with the first (i = 1) solution of the RPA we have from Eq. (16) for the zero temperature case

$$\begin{aligned}
Q^+_{2M1} &= \frac{1}{2}\sum_{j_a j_b}\left[\psi^{21}_{ab}A^+_{2M}(ab) - \varphi^{21}_{ab}A_{2\tilde{M}}(ab)\right] \\
Q_{2\tilde{M}1} &= \frac{1}{2}\sum_{j_a j_b}\left[\psi^{21}_{ab}A_{2\tilde{M}}(ab) - \varphi^{21}_{ab}A^+_{2M}(ab)\right].
\end{aligned} \tag{20}$$

In[10] it has been shown that the operators (20) together with their commutators close an SU(6) algebra under some conditions on the amplitudes ψ^{21}_{ab} , φ^{21}_{ab} . In this way, a microscopic interpretation on the Hamiltonian level for the IBM and the Quadrupole Phonon Model (QPM)[11] has been proposed within the framework of the Quasiparticle Phonon Nuclear Model (QPNM)[12]. The extension to the case of SU(m) symmetry and SU(m/n) supersymmetry has already been done in[13]. In the present case at finite temperature we can without toil show that the quadrupole FT-RPA operators (16) together with their commutation relations will form an SU(6) algebra if the following conditions hold:

$$\mathscr{C}_{L=0,2,4} = \tilde{C}\quad ;\quad \mathscr{C}_{L=1,3} = 0 \tag{21}$$

$$\mathscr{D}_{L=1,2,3,4} = 0\,, \tag{22}$$

where

$$\mathscr{C}_L = \frac{25}{2}\sum_{j_a j_b j_c j_d}\left(\psi_{ab}\psi_{cd}\psi_{ac}\psi_{bd} - \varphi_{ab}\varphi_{cd}\varphi_{ac}\varphi_{bd}\right)P_{abcd}\begin{Bmatrix} j_a & j_b & 2 \\ j_c & j_d & 2 \\ 2 & 2 & L\end{Bmatrix} \tag{23}$$

$$\mathscr{D}_L = \frac{25}{2}\sum_{j_a j_b j_c j_d}\left(\psi_{ab}\psi_{cd}\psi_{ac}\varphi_{bd} - \varphi_{ab}\varphi_{cd}\varphi_{ac}\psi_{bd}\right)P_{abcd}\begin{Bmatrix} j_a & j_b & 2 \\ j_c & j_d & 2 \\ 2 & 2 & L\end{Bmatrix} \tag{24}$$

with all superscripts J = 2, i = 1 in ψ , φ omitted for simplicity. In comparison with the zero temperature case[10] the thermal factor

$$P_{abcd} = (1-n_{j_a})(1-n_{j_b})(1-n_{j_c})(1-n_{j_d}) - n_{j_a}n_{j_b}n_{j_c}n_{j_d} \tag{25}$$

appears in Eqs. (23), (24). It reaches the value 1 in the limit $T \to 0$ and 0 when $T \to \infty$.

Under the conditions (21), (22) the DR, HPR and SR for the quadrupole thermal operator (16) ($J = 2$; $i = 1$) can be now readily obtained by the usual procedure[10]

$$\left\{ \begin{array}{l} [Q^+_M(T)]^{DR} = b^+_M \left(1 - \tilde{C} \sum_\tau b^+_\tau \tilde{b}_\tau\right) \\ [Q_M(T)]^{DR} = b_M \\ [Q_M(T), Q^+_{M'}(T)]^{DR} = \delta_{MM'} \left(1 - \tilde{C} \sum_\tau b^+_\tau \tilde{b}_\tau\right) - \tilde{C} b^+_{M'} \tilde{b}_M \end{array} \right. \tag{26}$$

$$\left\{ \begin{array}{l} [Q^+_M(T)]^{HPR} = b^+_M (1 - \hat{N}/N)^{1/2} \qquad \hat{N} = \sum_\tau b^+_\tau \tilde{b}_\tau \\ [Q_M(T)]^{HPR} = (1 - \hat{N}/N)^{1/2} b_M \\ [Q_M(T), Q^+_{M'}(T)]^{HPR} = \delta_{MM'} (N - \hat{N})/N - b^+_{M'} \tilde{b}_M / N \end{array} \right. \tag{27}$$

$$\left\{ \begin{array}{l} [Q^+_M(T)]^{SR} = d^+_M s \, N^{-1/2} \\ [Q_M(T)]^{SR} = s^+ \tilde{d}_M N^{-1/2} \\ [Q_M(T), Q^+_{M'}(T)]^{SR} = \delta_{MM'} s^+ s/N - d^+_{M'} \tilde{d}_M / N \end{array} \right. \tag{28}$$

where for simplicity, we write only the z- projection M in the laboratory system. In Eqs. (26)-(28) b^+_M , b_M are the ideal quadrupole boson operators satisfying the commutation relations for bosons

$$\begin{array}{l} [b_M, b^+_{M'}] = \delta_{MM'} \qquad (J=2\,;\ i=1) \\ [b_M, b_{M'}] = [b^+_M, b^+_{M'}] = 0 \end{array} \tag{29}$$

is the maximal boson number at finite temperature which is expressed microscopically in terms of the FT-RPA amplitudes , as the nearest interger of $\tilde{C}^{-1}$:

$$N = \left[\frac{1}{\tilde{C}}\right]. \tag{30}$$

We note that the extension to the case of an arbitrary J can also be done based on[13]. Using the HP thermal boson realization (27) we

now consider an oversimplified case with degenerate two-quasiparticle levels:

$$\varepsilon_{j_a} = \varepsilon_{j'_a} = \ldots = \varepsilon_A \ ; \ \varepsilon_{j_b} = \varepsilon_{j'_b} = \ldots = \varepsilon_B \ ; \ \varepsilon_{j_c} = \varepsilon_{j_d} = 0 \, .$$

In this case it is easy to find that

$$\tilde{C} = CP \tag{31}$$

where
$$P = 1 - n_A - n_B$$
$$C = \mathscr{C}_{L=0,2,4}(T=0) \ [10].$$

The inverse realization is now derived directly from Eq. (27) as

$$\begin{array}{l} [Q^+_M]^{HPR} = b^+_M \left(1 - \frac{\hat{N}}{NP}\right)^{1/2} \\ [Q_M]^{HPR} = \left(1 - \frac{\hat{N}}{NP}\right)^{1/2} b_M \\ [Q_M, Q^+_{M'}]^{HPR} = \frac{NP - \hat{N}}{NP} \delta_{MM'} - \frac{1}{NP} b^+_{M'} \tilde{b}_M , \end{array} \tag{32}$$

where $\bar{N}$ is the zero temperature boson number[10]

$$\bar{N} = NP = \left[\frac{1}{C}\right]. \tag{33}$$

It is not difficult to recognize from Eqs.(32) in the independent particle model the same form of the thermal HPR obtained by Providência and Fiolhais[3].

In conclusion, we would like to remark that as any boson mapping our thermal boson realizations do not avoid the problem of "spurious" states attributed to the orthogonality of the boson basis states. Recently, it has been shown by Sheikh[6] that while carrying out the calculations of the matrix elements with a physical Hamiltonian in the ideal boson basis state one can already separate the physical and unphysical components and need not worry about the "spurious" states. In this sense it should be very interesting to study the boson structure of a microscopic nucleon Hamiltonian at finite temperature using the above obtained thermal boson realizations. Since at $T = 0$ the conditions (21) and (22) calculated with RPA amplitudes ψ , φ are not satisfied, it is also highly desirable to investigate quantiatively the fulfillment of these conditions at finite temperature based on the FT-RPA amplitudes ψ and φ .The restoration of the symmetry and supersymmetry at finite temperature,studied recently in[14],provides some hope that as compared to the zero temperature case[10], the thermal conditions (21), (23) of the enforced SU(6) symmetry will perhaps turn out to be good at $T \neq 0$. The investigations of this kind are now in progress and their results will be displayed in a forthcoming paper.

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