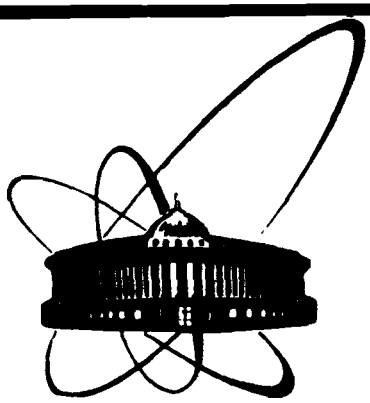

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FADDEEV EQUATIONS WITH AN EXTRA RESONANCE CHANNEL IN MUON CATALYSIS

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Introduction

Efficiency of the muon catalysis of the nuclear fusion

$$d + t + \mu \rightarrow {}^5He^*_{(3/2)^+} + \mu \rightarrow \begin{cases} {}^4He + n + \mu & 1-\mathring{\omega}_s \\ {}^4He\mu + n & \mathring{\omega}_s \end{cases} + Q \qquad (1)$$

as an energy source is determined by the sticking probability ω_s of the muon to helium[1]. First theoretical estimates of the probability ω_s have been made in refs.[2-4] by the so-called sudden-perturbation method[5,6] which is essentially a simplified version of the Born approximation. Later it has been found that the estimates are considerably higher than the experimental value of ω_s [7,8]. The obtained discrepancy between the theoretical and experimental data on ω_s upon the recalculation[9] of the observed ω_s (including the correction for stripping of the muon with ^{4}He) to the bare $\mathring{\omega}_s$ shows the necessity of a more accurate computation of the wave function of the mesic molecule in the initial channel and consideration of the influence of a nearthreshold resonance ${}^5He^*(3/2)^+$. The use of more accurate wave functions of the mesic molecule $dt\mu$ in refs.[10-13] has diminished the value of $\mathring{\omega}_s$ by about 25% as compared with[2-4], which allowed one to approach experimental data more closely[7,8]. What is the accuracy of the Born approximation itself in the problem (1) is still an open problem.

The problem (1) is multiparticle, with long-range interactions. Therefore, a complete answer to the question concerning the accuracy of the Born approximation can be found by considering all the system of six particles (the muon and five nucleons) as a unique object. At modern computers it is difficult to perform sufficiently accurate calculations of a six-body problem, especially as it is necessary to solve the scattering problem with two and more clusters in the final state when the sticking probability is determined. At present, numerical calculations with a sufficient accuracy are being made for systems consisting of no more than three particles. Therefore, for reaction (1) it makes sense to employ the model in which only three-particle systems are solved numerically. Just this model is investigated in this paper. Based on that model, we propose an exact definition of the sticking probability $\mathring{\omega}_s$ and formulae more accurate than in[4] and[9] for its computation.

In the considered model, reaction (1) is described by the Hamiltonian with three bound channels corresponding to certain stages of the reaction. The Hamiltonian parameters may be fitted from the data on the reaction $d+t \to {}^4He+n$ occurring without muon. If from the three-channel Schrödinger equation we eliminate the components corresponding to the initial ($dt\mu$) and intermediate (${}^5He^*\mu$) channels, the final channel (${}^4He, n, \mu$) will be described by a three-particle equation with energy-dependent potentials. On its basis we construct the Faddeev equations and study the form of asymptotic boundary conditions for the wave function of the final channel and its Faddeev components /14/. The sticking probability $\mathring{\omega}_S$ is expressed in terms of the amplitudes of spherical waves in the coordinate asymptotics of the wave function. For the amplitudes, the integral representations are found in terms of the wave functions describing the scattering in the system ${}^4He+n+\mu$. Approximate relations for calcucation of $\mathring{\omega}_S$ without numerical solution of the Faddeev equations follow from the exact formula $\mathring{\omega}_S$ upon substitution into the integral representations, including the scattering-wave-functions, Born approximations instead of the functions themselves. The final formulae follow from the expansion of the wave function of $dt\mu$ of the initial channel that is considered to be known /15/ by means of the muon Coulomb functions of the ${}^5He^*\mu$ intermediate channel. The obtained formulae may be compared with the expressions for $\mathring{\omega}_S$ proposed in /9/ and /13/. Thus, the integral representations we have constructed for the amplitudes allow us in principle to estimate the inaccuracy of the Born approximation for the sticking probability $\mathring{\omega}_S$.

The paper is organised as follows.

In the first and second sections, we introduce the notation and formulate the three-channel model for reaction (1).

In the third sect., we consider the fit of the model parameters from the data on the resonance ${}^5He^*(3/2)^+$. With this aim the resonance parameters are calculated within the corresponding three-channel model of the reaction $d+t \to {}^4He+n$.

In sect.4 the decay of the mesic molecule $dt\mu$ is analysed. The resonance shift of the energy and decay width are calculated in the first- and second-order perturbation theory in the channel coupling constant.

In sect.5, integral and differential Faddeev equations with energy-dependent potentials are formulated for the exit channel.

In sect.6, the Fredholm nature of the integral equations is proved for the generalized Green function of the final channel and some coordinate asymptotics are described.

In sect.7, asymptotic boundary conditions are formulated for the final-channel wave function and its components. In terms of these asymptotic conditions, exact definition of the probability $\mathring{\omega}_S$ is given.

In sect.8, the Born approximation for the sticking probability $\mathring{\omega}_S$ is studied on the basis of the integral representations for scattering amplitudes.

1. Three-Channel Hamiltonian

Reaction (1) will be described by the energy operator

$$H = \begin{pmatrix} H_1 & B_{12} & 0 \\ B_{21} & H_2 & B_{23} \\ 0 & B_{32} & H_3 \end{pmatrix} \qquad (2)$$

with three coupled channels. The Hamiltonian of the first channel H_1 describes the system $(dt\mu)$; that of the second channel H_2, the system $({}^5He^*\mu)$; and the third-channel Hamiltonian H_3 is the energy operator for the final stage $({}^4He, n, \mu)$ of reaction (1). The operators B_{12}, $B_{12}=B_{12}^*$, and B_{23}, $B_{23}=B_{23}^*$ do coupling between the channels. Now let us describe the operators H_i, $i=1,2,3$ in a greater detail.

In the first (initial) channel the clusters d and t will be considered structureless particles with internal energy $\varepsilon_d=-2.225$ MeV and $\varepsilon_t=-8.482$ MeV, resp. The Hamiltonian H_1 is defined by the equality

$$H_1 = H_o^{(1)} + V_{dt\mu}^c + V_{dt}^n + \lambda_1 , \quad V_{dt\mu}^c = V_{d\mu}^c + V_{t\mu}^c + V_{dt}^c ,$$

where $H_o^{(1)}$ is the kinetic-energy operator of the relative motion in the system $dt\mu$; V_{dt}^c, $V_{d\mu}^c$, and $V_{t\mu}^c$ are Coulomb potentials of the particle interaction; and V_{dt}^n is the nuclear potential of the deuteron-tritium interaction. The quantity λ_1 denotes the main threshold of the first channel, $\lambda_1=\varepsilon_d+\varepsilon_t$.

In the second channel, the compound nucleus ${}^5He^*(3/2)^+$ will be considered a structureless particle with internal energy $\lambda_2=\lambda_1+64$ KeV /16/. The Hamiltonian $H_o^{(2)}$ will be taken in the form

$$H_2 = H_o^{(2)} + V^c_{5He\mu} + \lambda_2 , \qquad (3)$$

where $H_o^{(2)}$ is the kinetic-energy operator in the system $^5He^*_\mu$ and $V^c_{5He\mu}$ is the Coulomb potential of the interaction between the muon and $^5He^*$ nucleus.

The third channel will be treated analogously. We shall consider the ^4He nucleus to be a particle with a fixed internal energy $\lambda_3 = -28.297$ MeV, and the Hamiltonian of the system $^4He, n, \mu$ is as follows

$$H_3 = H_o^{(3)} + V^c_{4Hen} + \lambda_3 ,$$

where $H_o^{(3)}$ is the kinetic-energy operator, $V^c_{4He\mu}$ and V^n_{4Hen} are resp. the Coulomb and nuclear potentials of the interaction of the ^4He nucleus with the muon and neutron.

Relative disposition of the main theresholds λ_1, λ_2, and λ_3 of the channels of reaction (1) are drawn in Fig.1.

2. Coordinates

We shall use the reduced Jacobi coordinates /14/ to describe the considered system of particles. The choice of coordinates will be dictated by the channel number.

In all the three channels the muon will be a particle of the first number. Numeration of other particles will be as follows (see Fig.2): in the first channel the deuteron will be particle 2 and triton, particle 3; in the second channel containing only two particles, the nucleus $^5He^*$ will be particle 2; in the third channel the neuteron has number 2 and the nucleus ^4He, number 3. The number of a pair in the initial and final channels coincides with the number of an extra particle.

For channels 1 and 3 the reduced relative coordinates x_1 and y_1 are given by the formulae

$$x_1 = \sqrt{\frac{2 m_2 m_3}{m_2 + m_3}}\,(r_2 - r_3) , \quad y_1 = \sqrt{\frac{2 m_1 (m_2 + m_3)}{m_1 + m_2 + m_3}}\left(r_1 - \frac{m_2 r_2 + m_3 r_3}{m_2 + m_3}\right)$$

the remaining coordinates x_α, y_α, $\alpha = 2,3$ follow from x_1 and y_1 by cyclic permutation of indices.

The only reduced relative coordinate in channel 2 will be denoted by y_1, $y_1 = \sqrt{\frac{2 m_1 m_2}{m_1 + m_2}}(r_1 - r_2)$. Note that the physical meaning of the coordinates $|y_1|$ is the same for all the three channels

Fig.1. The thresholds λ_i, $i = 1,2,3$ of the initial 1, intermediate 2, and final 3 channels of reaction (1).

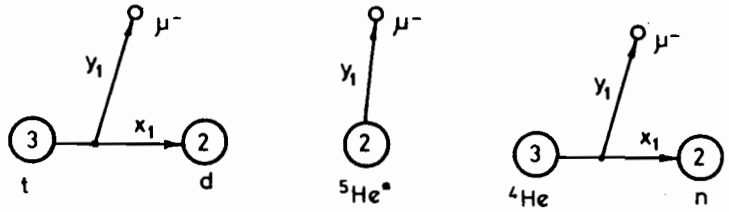

Fig.2. Reduced Jacobian coordinates of initial 1 intermediate 2 and final 3 channels of reaction (1).

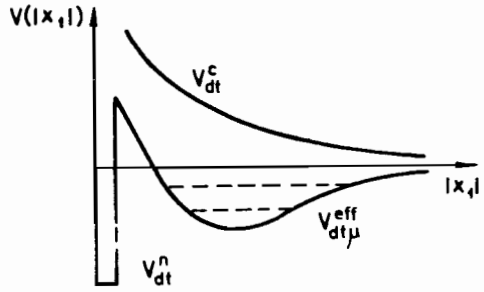

Fig.3. The effective Coulomb potential $V^{eff}_{dt\mu}$ created by the muon in the initial channel (1) of reaction (1) replaces the repulsion Coulomb potential V^c_{dt} in reaction (2). The nuclear potential V^n_{dt} in the same channel of reactions (1) and (2). The dashed lines represent nondegenerated energy levels of the mesic molecule $dt\mu$.

(see Fig.2); the distance from the muon to the centre of the complecs $3p+2n$, with almost the same coefficient:

$$\sqrt{\frac{2m_\mu(m_t+m_d)}{m_\mu+m_t+m_d}} \approx \sqrt{\frac{2m_\mu m_{5He^*}}{m_\mu+m_{5He^*}}} \approx \sqrt{\frac{2m_\mu(m_n+m_{4He})}{m_\mu+m_n+m_{4He}}}.$$

In channels 1 and 3 the relative coordinates x_α and y_α will be combined into vectors $X=\{x_\alpha, y_\alpha\}, X\in\mathbb{R}^6$. All the particles participating in the reaction will be considered spinless.

The energy operators of the channels H_1 and H_3 act in the Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_3$, resp., $\mathcal{H}_1=\mathcal{H}_3=L_2(\mathbb{R}^6)$ whereas H_2 in $\mathcal{H}_2=L_2(\mathbb{R}^3_{y_1})$. The total Hamiltonian H acts in the sum $\mathcal{H}=\bigoplus_{i=1}^{3}\mathcal{H}_i$ of channel spaces $\mathcal{H}_i$, $i=1,2,3$. The scalar product in $\mathcal{H}_i$ will be denoted by $(.,.)_i$. The kinetic energy operators $H_0^{(1)}$ and $H_0^{(3)}$ form a six-dimensional Laplacian $-\Delta_X$, the operator $H_0^{(2)}$ is a three-dimensional Laplacian $-\Delta_{y_1}$. All the potentials in (3) are two-body potentials and act on the corresponding variables x_α. The Coulomb potential in (3) are of the form

$$V_\alpha^C(x_\alpha) = n_\alpha |x_\alpha|^{-1}.$$

Here $n_\alpha = q_i q_j [2m_i m_j/(m_i+m_j)]^{1/2}$, where i and j are numbers of particles in a pair α, q_i and q_j are their charges. The mesic atomic units are chosen. The Coulomb potential of interaction between the muon and $^5He^*$ in (3) has the form

$$V^C_{5He\mu}(y_1) = \frac{q_{5He}\,q_\mu}{|y_1|}\sqrt{\frac{2m_{5He}m_\mu}{m_{5He}+m_\mu}}. \qquad (4)$$

3. Fit of the operators of coupling of channels

As a matter of fact, the muon in reaction (1) creates an effective potential[17] in which the deuteron and triton are under the barrier approaching each other at a sufficiently short distance (see Fig. 3).

In the subsequent fusion

$$d+t \to {}^5He^*(3/2)^+ \to {}^4He+n \qquad (5)$$

the muon as an electromagnetic particle does not participate. In view of the above-said, it is natural to assume that the operators of coupling of channels B_{12} and B_{23} are determined by the interaction in the system 2p+3n, and the influence of the muon Coulomb interaction of the parameters of these operators is small[16]. Therefore we shall describe reaction (5) with the Hamiltonian

$$h = \begin{pmatrix} h_1 & B_{12} & 0 \\ B_{12}^* & h_2 & B_{32}^* \\ 0 & B_{32} & h_3 \end{pmatrix}, \qquad (6)$$

where the operators $h_1 = -\Delta_{x_1} + V^C_{dt} + V^n_{dt} + \lambda_1$ and $h_3 = -\Delta_{x_1} + V^n_{4Hen} + \lambda_3$ act in the channel Hilbert spaces $\hat{\mathcal{H}}_1$ and $\hat{\mathcal{H}}_3$, resp., with $\hat{\mathcal{H}}_1 = \hat{\mathcal{H}}_3 = L_2(\mathbb{R}^3_{x_1})$. The operator h_2 of multiplication by the constant λ_2 operates in $\hat{\mathcal{H}}_2=\mathbb{C}$. The scalar product in the spaces $\hat{\mathcal{H}}_i$ will be denote by $\langle\cdot,\cdot\rangle_i$, $i=1,2,3$. The operators of channel coupling B_{12} and B_{23} are chosen as in (2).

Thus, the parameters of the operators B may be fitted from the experimental data of reaction (5). Obviously, the analysis of the Hamiltonian (6) is much more easy than (2) because in this case the operators h_1 and h_2 are two-particle operators. Note that a Hamiltonian like (6) was first considered in[16] where fit was made of its parameters.

The most simple but quite reasonable is to take $B_{12}, B_{12}: \hat{\mathcal{H}}_2 \to \hat{\mathcal{H}}_1$ and B_{32}. $B_{32}: \hat{\mathcal{H}}_2 \to \hat{\mathcal{H}}_3$ as separable operators of rank I operating on a complex number $u_2\in\hat{\mathcal{H}}_2$ by formulae

$$(B_{12}u_2)(x_1) = \beta u_2 \varphi(x_1),$$
$$(B_{32}u_2)(x_1) = \gamma u_2 \chi(x_1),$$

where φ and χ are fixed functions, $\varphi\in\hat{\mathcal{H}}_1$ and $\chi\in\hat{\mathcal{H}}_3$ with compact supports $\mathcal{D}_\varphi$ and $\mathcal{D}_\chi$. The supports $\mathcal{D}_\varphi$ and $\mathcal{D}_\chi$ are regions of the "phase transition" (i.e. the regions of the reaction $d+t\to{}^5He^*$ in $\mathbb{R}^3_{x_1}$ of the system dt into a compound state $^5He^*(3/2)^+$ and the analogous region for the system ^4He+n. We put $\|\varphi\|=\|\chi\|=1$. The parameters β and γ represent constants of coupling of the channels. The dependence of the moduli squared $|\varphi(x_1)|^2$ and $|\chi(x_1)|^2$ of the functions φ and χ on the coordinates x_1 specifies, as a matter of fact, the relative probability of reactions $d+t\to{}^5He^*$ and ${}^5He\to{}^4He+n$, resp. at a point x_1, of the configuration spaces $\mathbb{R}^3_{x_1}$ of the first and third channels. The rate of the considered reactions will be determined by the constants β and γ.

We will now apply the model (6) to compute the parameters of the resonance $^5He^*(3/2)^+$, the energy E_R and half-width $\Gamma/2$, as functions of the bare energy λ_2 and coupling constants β and γ, which allows us to determine the values of β and γ from experimental data.

From a mathematical point of view, the resonances are poles

of the Green function $g(z)=(h-z)^{-1}$ of the operator h on the second sheet of the Riemannian surface of energy z. Consider the starting equation $(h-z)g(z)=I$ for the Green function $g(z)$ written for components $g_{ij}(z)$:

$$\begin{pmatrix} h_1-z & \beta\varphi & 0 \\ \langle\cdot,\varphi\rangle_1 & \lambda_2-z & \langle\cdot,\chi\rangle_3 \\ 0 & \gamma\chi & h_3-z \end{pmatrix} \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{pmatrix} = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{pmatrix}$$

with I_i, an identical operator in an i-th channel. Upon eliminating the components g_{2j}, $j=1,2,3$, this equation reduces to the system of integral equations with a degenerated kernel with respect to the unknowns g_{1j} and g_{3j}:

$$g_{1j}+\frac{\beta g_1(z)\varphi}{z-\lambda_2}\left[\beta\langle g_{1j}\cdot,\varphi\rangle_1+\gamma\langle g_{3j}\cdot,\chi\rangle_3\right]=\delta_{1j}g_1(z)+\delta_{2j}\frac{\beta g_1(z)\varphi}{z-\lambda_2},$$

$$g_{3j}+\frac{\gamma g_3(z)\chi}{z-\lambda_2}\left[\beta\langle g_{1j}\cdot,\varphi\rangle_1+\gamma\langle g_{3j}\cdot,\chi\rangle_3\right]=\delta_{3j}g_3(z)+\delta_{2j}\frac{\gamma g_3(z)\chi}{z-\lambda_2}, \quad (7)$$

where $g_1(z)=(h_1-z)^{-1}$ and $g_3(z)=(h_3-z)^{-1}$ are the Green functions of operators h_1 and h_3, resp. We make also use of the notation:

$$\langle g_{1j}\cdot,\varphi\rangle_1=\int dx\, g_{1j}(x,\cdot,z)\,\bar\varphi(x),\quad \langle g_{3j}\cdot,\chi\rangle_3=\int dx\, g_{3j}(x,\cdot,z)\,\bar\chi(x).$$

From the system it is seen that the unknowns g_{ij}, $i=1,2$, are expressed in terms of the functions $\mathcal{X}_1=\beta\langle g_{1j}\cdot,\varphi\rangle_1$ and $\mathcal{X}_3=\gamma\langle g_{3j}\cdot,\chi\rangle_3$ (we consider j fixed in computations). The problem of finding the latter is purely algebraic. To construct equations for $\mathcal{X}_1$ and $\mathcal{X}_3$, it suffices to multiply (7) with $\beta\varphi$ and $\beta\chi$ as scalars. Poles of the Green function $g(z)$ are roots of the determinant $\mathcal{D}(z)$ of the resulting system of equations for $\mathcal{X}_1$ and $\mathcal{X}_3$:

$$\mathcal{D}(z)=1+\frac{1}{z-\lambda_2}\left[\beta^2\langle g_1(z)\varphi,\varphi\rangle_1+\gamma^2\langle g_3(z)\chi,\chi\rangle_3\right].$$

The complex roots of $\mathcal{D}(z)$ are resonances. They can be calculated by the perturbation theory if β and γ are small (more exactly, small should be the products $\beta^2\langle g_1(z)\varphi,\varphi\rangle_1$ and $\gamma^2\langle g_3(z)\chi,\chi\rangle_3$). We are interested in the roots in the vicinity of λ_2. The existence of roots of that type at a distance of an order of β^2 or γ^2 is ensured by the Rushet principle. To determine those roots with an accuracy up to $O(\beta^2)$ and $\cdot O(\gamma^2)$, we rewrite the equation $\mathcal{D}(z)=0$ in an equivalent form

$$z=\lambda_2-\beta^2\langle g_1(z)\varphi,\varphi\rangle_1-\gamma^2\langle g_3(z)\chi,\chi\rangle_3 \quad (8)$$

and in the right-hand side replace z by $\lambda_2\pm i0$. As a result, we obtain two resonances $z^+(\lambda_2)$ and $z^-(\lambda_2)$,

$$z^\pm(\lambda_2)=\lambda_2+\Delta_{el}\pm i\,\Gamma_{el}/2+\Delta_{in}\pm i\,\Gamma_{in}/2, \quad (9)$$

where the shifts $\Delta_{el}+i\Gamma_{el}/2=-\beta^2\langle g_1(\lambda_2-i0)\varphi,\varphi\rangle_1$ and $\Delta_{in}+i\Gamma_{in}/2=-\gamma^2\langle g_3(\lambda_2-i0)\chi,\chi\rangle_3$ correspond resp. to elastic and inelastic channels. Here $\Delta_{el}=-\mathrm{Re}\,\beta^2\langle g_1(\lambda_2+i0)\varphi,\varphi\rangle_1$ and $\Gamma_{el}=2\,\mathrm{Im}\,\beta^2\langle g_1(\lambda_2+i0)\varphi,\varphi\rangle_1$. The quantities Δ_{in} and $\Gamma_{in}/2$ are determined analogously. Both the half-widths $\Gamma_{el}/2$ and $\Gamma_{in}/2$ are expressed through the jumps $g_i(\lambda_2+i0)-g_i(\lambda_2-i0)$ of the corresponding resolvents $g_i(z)$, $i=1,3$, when passing across the cut. These jumps are explicitly expressed in terms of the wave functions $\psi_{10}(k,x)$ and $\psi_{30}(k,x)$ of the continuous spectrum of operators h_1 and h_3, resp. And finally, we have for $\Gamma/2$:

$$\frac{\Gamma}{2}=\frac{\Gamma_{el}+\Gamma_{in}}{2}=\beta^2\frac{\sqrt{\lambda_2-\lambda_1}}{16\pi^2}\int_{S^2}d\hat k\,\left|\langle\psi_{10}(\sqrt{\lambda_2-\lambda_1}\,\hat k),\varphi\rangle_1\right|^2+$$
$$+\gamma^2\frac{\sqrt{\lambda_2-\lambda_3}}{16\pi^2}\int_{S^2}d\hat k\,\left|\langle\psi_{30}(\sqrt{\lambda_2-\lambda_3}\,\hat k),\chi\rangle_3\right|^2.$$

In reality, the coupling of channels 1 and 2 is not very small[16], and in (9) terms of the order β^4 are to be taken into account. For this aim equation (8) should be written in the form

$$z=\lambda_2-\beta^2\langle g_1(\lambda_2\pm i0)\varphi,\varphi\rangle_1-$$
$$-\beta^2\frac{d}{d\lambda}\langle g_1(\lambda\pm i0)\varphi,\varphi\rangle_1\Big|_{\lambda=\lambda_2}(z-\lambda_2)+O(\beta^6)- \quad (10)$$
$$-\gamma^2\langle g_3(\lambda_2\pm i0)\chi,\chi\rangle_3+O(\gamma^4)$$

and account is to be made of the fact that the expression $\mathrm{Im}\langle g_1(z^\pm(\lambda_2))\varphi,\varphi\rangle_1$ is of an order of $O(\beta^2)$. Solving (10) for and taking terms of an order up to $O(\beta^4)$ into account we get for $z^\pm(\lambda_2)$:

$$z^\pm(\lambda_2)=\lambda_2+\tilde\Delta_{el}\pm\frac{i\tilde\Gamma_{el}}{2}+\tilde\Delta_{in}\pm\frac{i\tilde\Gamma_{in}}{2},$$

where

$$\tilde\Delta_{el}=\frac{\Delta_{el}(\lambda_2)}{1-d},\quad \tilde\Gamma_{el}=\frac{\Gamma_{el}}{1-d},\quad \tilde\Delta_{in}=\frac{\Delta_{in}}{1-d},$$
$$\tilde\Gamma_{in}=\frac{\Gamma_{in}}{1-d},\quad d=\left[1-\frac{d}{d\lambda}\Delta_{el}(\lambda)\right]_{\lambda=\lambda_2}. \quad (11)$$

Formulae (8)-(11) allow us to fit the parameters β and γ from the experimental data.

4. Decay of the mesic molecule $dt\mu$ (perturbation theory)

The energy levels of the mesic molecule $dt\mu$, even the lowest ones, are close to the threshold λ_1 of the initial channel. Their shift relatively the threshold is of an order of several KeV. When the coupling between channels is switched on, i.e. when $B_{12} \neq 0$ and $B_{32} \neq 0$ in (2), the discrete spectrum of the operator H_1 transforms into resonances. If the discrete spectrum is multiple, the transformation into resonances may be only partial. This should be carefully verified[18].

We shall describe the transformation of the discrete spectrum into resonances using as an example a nondegenerate state $\Phi(X)$ of the mesic molecule $dt\mu$ with energy $\mathcal{E}$, $\mathcal{E} < \lambda_1$. In what follows we shall consider Φ, $\Phi \in \mathcal{H}_1$ at two states with $J=0$, $v=0$ or $J=0$, $v=1$ which are nondegenerated[15]. The case of degenerated states may be considered following[18].

We now do not study, as it has been made in sect. 3, the Green function of the operator H and analyse its behaviour on the second sheet because dispersion relations of the type (8) determining the poles of the resolvent on the unphysical sheet can be obtained immediately from the spectral problem $H\mathcal{U} = z\mathcal{U}$. Analysis of this problem within the perturbation theory appears to be less complicated than the study of the Green function.

So, consider the equation $H\mathcal{U} = z\mathcal{U}$ for eigenvalues of the operator H. In the representation of elements $\mathcal{U} \in \mathcal{H}$, $\mathcal{U} = \{u_1 u_2 u_3\}$ as vector-columns this equation reduces to the system

$$\begin{aligned} (H_1 - z)u_1 + B_{12}u_2 &= 0, \\ B_{21}u_1 + (H_2 - z)u_2 + B_{23}u_3 &= 0, \\ B_{32}u_2 + (H_3 - z)u_3 &= 0. \end{aligned} \qquad (12)$$

With the second and third equations we express u_2 and u_3 in terms of u_1 and eliminate u_2 from the first equation of (12). The result is the equation for the only component u_1 equivalent to the whole system (12):

$$\left[H_1 - z + W_{11}(z) + W_{131}(z) \right] u_1 = 0. \qquad (13)$$

Hereafter we make use of the notation $W_{ij}(z)$, $W_{ij}: \mathcal{H}_i \to \mathcal{H}_j$, $i,j = 1,3$ for energy-dependent potentials of the form

$$W_{ij}(z) = - B_{i2} G_2(z) B_{2j},$$

where $G_2(z) = (H_2 - z)^{-1}$ is the Green function of the operator H_2. The energy-dependent potential $W_{131}(z)$ is given by

$$W_{131}(z) = - W_{13}(z) R(z) W_{31}(z),$$

where $R(z)$ is the generalized Green function of the third channel,

$$R(z) = \left[H_3 - z + W_{33}(z) \right]^{-1}.$$

The potentials $W_{ij}(z)$ are integral operators with the kernels

$$W_{ij}(x, x', z) = - b_i(x_1)\, \overline{b_j(x_1)}\, G_2(y_1, y_1', z - \lambda_2), \qquad (14)$$

where $b_1 \equiv \varphi$ and $b_3 \equiv \chi$ are functions of the coupling of channels, G_2 is the Coulomb Green function of the second channels, and $X = \{x_1, y_1\}$, $X' = \{x_1', y_1'\}$. As the function $b_k(x_1)$ has compact supports $\mathfrak{D}_{b_k}$ (see sect.3) $k = 1,3$, the potential W_{ij} as a function of the variable X is concentrated in the cylinder $\Omega_1^{(i)} = \mathfrak{D}_{b_i} \times \mathbb{R}^3_{y_1}$ and as a function of the variable X', in the cylinder $\Omega_1^{(j)} = \mathfrak{D}_{b_j} \times \mathbb{R}^3_{y_1}$. That potential is separable in the variable x_1 and is an operator of the type of the contraction over the variable y_1. At $i = j = 1$ it may be considered as an extra two-body potential in channel 1 belonging to the subsystem dt. Analogously, the operator $W_{33}(z)$ may be considered an extra rapidly decreasing potential in the system $^4He, n, \mu$ belonging to the pair $^4He, n$. The dependence on the variable y_1 of the relative motion of the third particle in (14) signifies that these potentials are threeparticle potentials. Potentials of that sort always appear when the intrinsic structure of particles is taken into account in many-body problems[18] and[19].

When the coupling between channels is assumed to be weak $(\rho^2, \gamma^2) \ll 1$, the second potential in (13), W_{131}, is of a higher order of smallness $(\sim \rho^2 \gamma^2)$ than the first one $W_{11}(z)$, $(\sim \rho^2)$, therefore, it is just the potential $W_{11}(z)$ that mainly contributes to the level shift $\mathcal{E}$. We shall look for that shift within the perturbation theory.

We pass from (13) to the integral equation

$$u_1 = - G_1(z) \left[W_{11}(z) + W_{131}(z) \right] u_1, \qquad (15)$$

where $G_1(z)$ is the resolvent of the operator H_1, $G_1(z) = (H_1 - z)^{-1}$. To apply the perturbation theory to the leading order in small parameters ρ^2 and γ^2, in the vicinity of $z = \mathcal{E}$ in the kernel $G_1(z)$ it suffices to take into account only one pole term $(\cdot, \Phi)_1 \Phi / (\mathcal{E} - z)$. This is justified if the levels of the mesic molecule $dt\mu$ are

well separated, i.e. the parameter β^2 is sufficiently small as compared with the distances between levels. So, instead of (15) we will consider the approximate equation

$$u_1 = -(\varepsilon - z)^{-1}\left([W_{11}(z) + W_{131}(z)]u_1, \Phi\right)_1 \Phi \qquad (16)$$

from which it immediately follows that $u_1 = \mathcal{X}\Phi$ where $\mathcal{X}$ is a number. In accordance with (16) the equation for $\mathcal{X}$ is as follows:

$$\left[z - \varepsilon - (W_{11}(z)\Phi, \Phi)_1 - (W_{131}(z)\Phi, \Phi)_1\right]\mathcal{X} = 0. \qquad (17)$$

For equation (17) to have a nontrivial solution it is necessary that the function $\mathcal{D}(z)$:

$$\mathcal{D}(z) = z - \varepsilon - (W_1(z)\Phi, \Phi)_1 - (W_{131}(z)\Phi, \Phi)_1$$

turns into zero. With the explicit form of the operators B_{ij}, we shall write the equation $\mathcal{D}(z) = 0$ in the following form

$$z = \varepsilon - \beta^2 (G_2(z)\tilde{\Phi}, \tilde{\Phi})_2 - \beta^2\gamma^2 (G_2(z)\langle R(z)\chi, \chi\rangle_3 G_2(z)\tilde{\Phi}, \tilde{\Phi})_2, \qquad (18)$$

where $\tilde{\Phi} = \langle \Phi, \varphi \rangle_1$ is a result of averaging of the wave function Φ of the mesic molecule $dt\mu$ over the function φ that realises the coupling between channels 1 and 2. Retaining in the r.h.s. of (18) only the terms of $O(\beta^4)$ and $O(\beta^2\gamma^2)$ we obtain (see sect. 3) the resonances

$$z^{\pm} = \varepsilon + \Delta_{dt\mu} \pm \frac{i\Gamma_{dt\mu}}{2}$$

(the plus stands for the upper halfplane; the minus, for the lower halfplane), where

$$\Delta_{dt\mu} = -\beta^2 (G_2(z)\tilde{\Phi}, \tilde{\Phi})_2 \left[1 - \beta^2 \frac{d}{dz}(G_2(z)\tilde{\Phi}, \tilde{\Phi})_2\Big|_{z=\varepsilon}\right] -$$

$$- \beta^2\gamma^2 \operatorname{Re}\left(G_2(\varepsilon)\langle R(\varepsilon \pm i0)\chi, \chi\rangle_3 G_2(\varepsilon)\tilde{\Phi}, \tilde{\Phi}\right)_2 + O(\beta^6, \beta^4\gamma^2),$$

$$\Gamma_{dt\mu} = \beta^2\gamma^2 \left(G_2(\varepsilon)\langle \operatorname{Im} R(\varepsilon + i0)\chi, \chi\rangle_3 G_2(\varepsilon)\tilde{\Phi}, \tilde{\Phi}\right)_2.$$

Note that the fourth order of smallness ($\sim\beta^2\gamma^2$) of the width $\Gamma_{dt\mu}$ is caused by the absence of direct coupling of the initial and exit channels. The coupling of these channels is realized only through the intermediate resonance channel 2.

5. Inhomogeneous Faddeev equations for the final channel

The sticking probability for the muon to helium in the reaction (1) can be obtained by solving the spectral problem $H\mathcal{U} = z\mathcal{U}$

on the second sheet of the Riemann surface of the parameter z and integrating the coordinate asymptotics of the component $\mathcal{U}_3$ of the solution $\mathcal{U}$ corresponding to the resonances $z^{\pm}$. Here it may be assumed that the coefficients of different asymptotic terms of the type of spherical waves in $\mathbb{R}^6$ and $\mathbb{R}^3$ are the amplitudes of processes, resp., with three ($^4He + n + \mu$) and two ($^4He\mu + n$) particles in the final state. The "spherical waves" are exponentially growing functions. The procedure for studying equations for functions of that sort is not yet developed. To avoid this difficulty, we make a natural assumption that the component $\mathcal{U}_1$ of the solution $\mathcal{U} = \{u_1 u_2 u_3\}$ of system (12) does not much differ from the initial wave function Φ of the mesic molecule $dt\mu$ [16]. The function Φ can be found by solving the Schrödinger equation $(H_1 - \varepsilon)\Phi = 0$ for channel 1. We assume Φ to be known for instance, calculated in the adiabatic approximation[15]. Instead of the complex energy of the resonances $z^{\pm}$ we take initial real-valued energy ε. Recall that in this approximation we admit inaccuracy $O(\beta^2)$ (see sect.4). In this way, we leave only two equations of the system (12):

$$(H_2 - z)u_2 + B_{23}u_3 = -B_{21}\Phi, \qquad (19)$$

$$B_{32}u_2 + (H_3 - z)u_3 = 0. \qquad (20)$$

Eliminating the unknown u_2 from (20) by means of (19) we obtain for the third channel one equation with the energy-dependent potential W_{33} and inhomogeneous term $F_{31} = W_{31}(z)\Phi$:

$$(H_3 + W_{33}(z) - z)u_3 = -W_{31}(z)\Phi. \qquad (21)$$

The inhomogeneous term $F_{31}(X, z)$ is localized in the cylinder $\Omega^{(3)}$ (see (14)) and is factorized owing to the factor χ in the potential W_{31}. The function F_{31} can be written in the term $F_{31}(X, z) = \chi(x_1)\mathcal{F}(y_1)$, where $\mathcal{F}(y_1) = -\int dx_1' dy_1' \overline{\varphi(x_1)}\, G_2(y_1, y_1', z - \lambda_2)\Phi(X')$.

As the function $\Phi(X)$ of the mesic molecule $dt\mu$ is exponentially decreasing, the asymptotics in y_1 of $\mathcal{F}(y_1)$ is easily calculated and to the leading order represents a spherical wave in $\mathbb{R}^3_{y_1}$:

$$\mathcal{F}(y_1) \approx \frac{f(z, \hat{y}_1)}{4\pi|y_1|} \exp\left\{i\left[\sqrt{z - \lambda_2}|y_1| - \frac{n_2}{2\sqrt{z - \lambda_2}} \ln\left(2\sqrt{z - \lambda_2}|y_1|\right)\right]\right\},$$

where $f(z, \hat{k}) = -\int dx_1 dy_1\, \overline{\varphi}(x_1)\Phi(X)\Psi_c(z - \lambda_2, \hat{k}, y_1)$, and Ψ_c is the eigenfunction of the continuous spectrum of the operator H_2. For $z = \varepsilon$ and $\varepsilon < \lambda_2$ the function $\mathcal{F}$ is exponentially decreasing.

As the binding energy ε of the mesic molecule $dt\mu$ lies above

the disintegration threshold λ_3 into three particle in the final channel, it makes sense to use in the numerical calculations, instead of (21), modified differential Faddeev equations for which asymptotic boundary conditions are more simple in form.

To derive these equations, we split the Coulomb potential $V_2^c \equiv V_{^4He\mu}^c$ in (21) into two terms[14], $V_2^c(x_2) = \hat{V}_2(X) + V_2^{as}(X)$. The first $\hat{V}_2$ describes the interaction of the muon and ^4He nucleus when they are not very far from each other, i.e. in the region Ω^c, where $|x_2| < a(1+|y_2|)^\nu$, $\nu < 1/2$. The second term V_2^{as} represents the Coulomb background and in the region Ω^c vanishes. The terms $\hat{V}_2$ and V_2^{as} are given by the relations $\hat{V}_2(X) = \eta(X)\, V_2^c(X)$ and $V_2^{as}(X) = V_2^c(x_2) - \hat{V}_2(X)$, where $\eta(X)$ is a smooth function equal to unity in Ω^2 and zero outside that region (see Fig.4)

$$\eta(X) = \begin{cases} 1, & |x_2| < a(1+|y_2|)^\nu \\ 0, & |x_2| > a'(1+|y_2|)^{\nu'}, \ \nu < \nu' < 1/2, \ a < a'. \end{cases}$$

Introducing the notation $V_1(z)$ for the total potential of interaction between ^4He and n,

$$V_1(z) = V_{^4Hen}^n + W_{33}(z),$$

we rewrite eq. (21) in the form

$$[\lambda_3 - \Delta_X + V_2^c + V_1(z) - z]\, u_3 = -W_{31}(z)\Phi. \qquad (22)$$

Then we transfer the short-range potentials $\hat{V}_2$ and V_1 into the r.h.s. of (22)

$$[\lambda_3 - \Delta_X + V_2^{as} - z]\, u_3 = [\hat{V}_2 + V_1(z)]\, u_3 - W_{31}\Phi$$

and invert the differential operator $H^{as} - z$, $H^{as} = \lambda_3 - \Delta_X + V_2^{as}$,

$$u_3 = -R^{as}(z)[\hat{V}_2 + V_1(z)]\, u_3 - R^{as} W_{31}(z)\Phi,$$

where $R^{as}(z) = (H^{as} - z)^{-1}$. We consider the parameter z arbitrary and $\mathrm{Im}\, z \neq 0$; at the end we take the limit $z \to \varepsilon \pm i0$..

The Faddeev components are introduced by the formulae

$$u^1 = -R^{as}(z) V_1(z) u_3 - R^{as}(z) W_{31}(z)\Phi, \qquad (23)$$

$$u^2 = -R^{as}(z)\hat{V}_2\, u_3. \qquad (24)$$

Considering that $u_3 = u^1 + u^2$, we apply the operator $H^{as} - z$ to both the sides of (23) and (24) obtain the sought-for differential Faddeev equations

$$[-\Delta_X + V_2^{as} + V_1(z) + \lambda_3 - z]\, u^1 = -V_1(z) u^2 - W_{31}(z)\Phi, \qquad (25)$$

$$[-\Delta_X + V_2^c + \lambda_3 - z]\, u^2 = -\hat{V}_2\, u^1. \qquad (26)$$

The integral Faddeev equations for u^1 and u^2 follow from (25) and (26) if we invert the operators in the left-hand side

$$u^1 = -R_1(z) V_1(z) u^2 - R_1(z) W_{31}(z)\Phi,$$

$$u^2 = -R_2(z)\hat{V}_2\, u^1,$$

where $R_1(z) = [-\Delta_X + V_2^{as} + V_1(z) + \lambda_3 - z]^{-1}$, $R_2(z) = [-\Delta_X + V_2^c + \lambda_3 - z]^{-1}$.

To determine the physical solutions to the Schrödinger equation (22) and the Faddeev equations (25), (26), it is necessary to formulate the asymptotic boundary conditions for the wave function u_3 and its Faddeev components u^1 and u^2. The amplitudes of spherical waves in the asymptotics define the sticking probability of the muon to helium (see sect.7). The asymptotics of u_3 will be obtained by using eq. (21) in the integral form

$$u_3 = -R(z) W_{31}(z)\Phi. \qquad (27)$$

From this equation it is seen that the asymptotic behaviour of the functions u_3 is determined by the kernel $(RW_{31})(X, X', z)$. Consequently, it is first necessary to investigate the Green function and the product of its with the energy-dependent potential $R(z)W_{31}(z)$ this will be made in the next section.

6. Asymptotic properties of the kernel $RW_{31}(X, X', z)$

In this section, we will show that the asymptotic behaviour as $(X, X' \to \infty)$ of the product of the Green function with the potential, $(RW_{31})(X, X', z)$, is analogous to that of the same product in a three-body problem with conventional energy-independent potentials (see[14]). This problem is nontrivial as the analytic in energy z properties of the potential $W_{31}(z)$ are rather complicated being defined by the properties of the resolvent $G_2(z)$. Specifically, it will be shown that the continuous-spectrum branch of the operator H_2, $[\lambda_2, +\infty)$, influences neither the solutions to eqs. (21), (25,26), nor the product $RW_{31}(z)$. The spectrum branch $[\lambda_2, +\infty)$ of the total Hamiltonian H would yield extra slowly decreasing terms, unusual for the three-body problem, in the $-c$

-asymptotics of the wave function $\mathcal{U}_3$ and its components which should be taken into account (if the energy is above the threshold λ_2) in the numerical solution of the differential equations (21) and (25,26).

To investigate the kernel RW_{31} , we construct the integral Fredholm equations for the Green function $R(z)$ using the method given in [14]. We rewrite the initial equation $[H_3 - z + W_{33}(z)]R(z) = I$, for $R(z)$ so that only the "nonperturbed" Hamiltonian be kept in the l.h.s. and invert the operator $H^{as} - z$. Then we introduce the components $R^1 = R^{as} - R^{as} V_1(z) R$ and $R^2 = - R^{as} \hat{V}_2 R$ and write for them the following Faddeev equations

$$\begin{cases} R^1(z) = R_1(z) - R_1(z) V_1(z) R^2(z) \\ R^2(z) = - R_2(z) \hat{V}_2(z) R^1(z) . \end{cases} \qquad (28)$$

Asymptotic properties of the kernels of these equations are as follows: The properties of the kernel $R_2(z)\hat{V}_2$ are quite simple as the Green function $R_2(z)$ corresponds only to the Coulomb potential $V_2^c(x_2)$. The variables x_2 and y_2 in the operator , $\lambda_3 - \Delta_X + V_2^c$, are separated, and the Green function may be represented by the contour integral

$$R_2(X, X', z) = \frac{1}{2\pi i} \int_{\mathcal{L}} d\zeta \, g_3^c(x_2, x_2', \zeta) r_0(y_2, y_2', z - \lambda_2 - \zeta)$$

in terms of the two-body Green function $g_3^c(z) = [-\Delta_{x_2} + V_2^c - z]^{-1}$ corresponding to the Coulomb forces in the system $^4He\mu$. The free Green function in $\mathbb{R}^3$, is given by $r_0(y, y', z) = \exp\{i\sqrt{z}|y - y'|\}|y - y'|^{-1}$. The contour $\mathcal{L}$ embraces the spectrum of the operator $h_3^c = -\Delta_{x_2} + V_2^c$.

Considering each of the contributions of the discrete and continuous spectrum of the operator h_3^c separately, we write the Green function R_2 as a sum, $R_2 = R_2^d + R_2^c$. The term R_2^d ,

$$R_2^d(X, X', z) = \sum_j \psi_j(x_2) \bar{\psi}_j(x_2') r_0(y_2, y_2', z - \lambda_3 + \tilde{\varkappa}_j^2) \qquad (29)$$

described the discrete spectrum; ψ_j and $-\tilde{\varkappa}_j^2$ are resp. the Coulomb wave functions of the discrete spectrum of h_3^c and the corresponding energy levels. The term R_2^c describes the contribution to $R_2(z)$ from the continuous spectrum of the operator h_3^c . Its asymptotics when $x_2 \to \infty$ is computed by the stationary-phase method and is of the form

$$R_2^c(X, X', z) \simeq C_{z-\lambda_3} \frac{\exp\{i\sqrt{z-\lambda_3} L_{20} + i W_2(k_2, x_2)\}}{L_{20}^{5/2}} \psi_{3,c}^{(+)}(k_2, x_2'), \qquad (30)$$

where the eikonal L_{20} and the Coulomb phase W_2 are given by

$$L_{20} = \sqrt{|x_2|^2 - (y_2 - y_2')^2}, \quad W_2(k, x) = - \frac{n_2}{2|x_2|} \ln(2|k||x|),$$

where the "momentum" $k_2 = \sqrt{z - \lambda_3} \frac{|x_2|}{L_{20}} \hat{x}_2$; $\psi_{3,c}^{(+)}$ is the wave function of the continuous spectrum of the operator h_3^c , and C_z is as follows

$$C_z = \frac{1}{2\pi} \left(- \frac{i\sqrt{z}}{2\pi} \right)^{3/2} .$$

The second kernel $R_1 V_1$ of the system (28) is generated by the two-channel Hamiltonian

$$\hat{H} = \begin{pmatrix} H_2 & B_{23} \\ B_{32} & H_{as} + V_{^4He\mu}^n \end{pmatrix}$$

acting in the sum $\mathcal{H}_2 \oplus \mathcal{H}_3$ of the second and third channels. If we denote components of the resolvent $\mathcal{G}(z) = (\hat{H} - z)^{-1}$ operator $\hat{H}$ by $\mathcal{G}_{ab}$, $a, b = 2, 3$, the Green function $R_1(z)$ is obviously the component $\mathcal{G}_{33}$.

The c -asymptotics of the kernel $R_1(X, X', z)$ are studied by the same scheme as for the Green function $R_{as}(z)$ in [14], viz., on the basis of the locality principle a "bare" function is to be constructed that represents a good approximation $R_1^{(0)}(z)$ to $R_1(z)$ in the sense that the discrepancy $A(X, X', z)$ in the equation

$$R_1^{(0)} (H_{as} + V_1(z) - z) = I + A \qquad (31)$$

be a rapidly decreasing function of the variables X and X' . The degree of decrease of the kernel $A(X, X', z)$ should be such that the equation for R_1 , $R_1 = R_1^{(0)} - A R_1^{(0)}$, following from (31) be the Fredholm equations. The only difference from the consideration in [14] consists in that it is necessary to take into account the potential $W_{33}(z)$ entering into the potential V_1 being energy-dependent. It is important here that the Hamiltonian H for $\gamma \neq 0$ has no longer the continuous spectrum branch $[\lambda_2, \infty)$ that is present in the channel Hamiltonian H_2 . To prove this, we neglect the difference (about 0.4%) between the masses $m_{^5He^*}$ and $m_{^4He}$. So, in the potential (4) we change $m_{^5He^*}$ to $m_{^4He}$, and instead of (4) we use in (3) the Coulomb potential

$$\tilde{V}_{^5He\mu}^c(y_1) = \frac{q_{^5He^*} q_\mu}{|y_1|} \sqrt{\frac{2(m_{^4He} + m_n) m_\mu}{m_{^4He} + m_n + m_\mu}} . \qquad (32)$$

Owing to that change, the variables in the Hamiltonian $\tilde{H}$ are asym-

ptotically (for $y_1 \to \infty$, $|x_1| < a|y_1|^{\nu}$, $\nu < 1/2$, $a = const$) separated because $V_2^{as}(X) \underset{y_1\to\infty}{=\!=} \tilde{V}^c_{5He\mu}(y_1)$. Near the cylinder $\Omega_1^{(3)} = \mathcal{D}_x \times \mathbb{R}^3_{y_1}$, i.e. for $|x_1| \le a|y_1|^{\nu}$, $\nu < 1/2$, the operator $\tilde{H}$ can be represented by the sum

$$\tilde{H} \cong \tilde{H}_0 \equiv \tilde{h} \otimes I_{y_1} + I \otimes h_2^c , \qquad (33)$$

where $\tilde{h} = \begin{pmatrix} h_2 & B_{23} \\ B_{32} & h_3 \end{pmatrix}$, is the identity operator in $\tilde{\mathcal{H}}_2 \oplus \tilde{\mathcal{H}}_3$ and $h_2^c = -\Delta_{y_1} + \tilde{V}^c_{5He\mu}(y_1)$. As λ_2 is not the point of the discrete spectrum of the operator $\tilde{h}$ (see sect. 3), the operator $\tilde{H}_0$ has no continuous spectrum branch $[\lambda_2, \infty)$, and hence the operator $\tilde{H}$ as well. All other its properties coincide with the properties of the Green functions for energy-independent potentials.[14] The kernels $R_1 V_1(z)(X, X')$ and $R_1 W_{31}(z)(X, X')$, owing to the separation of variables (33), also have asymptotic properties analogous to the properties of similar kernels for energy-independent potentials (see[18], p.63). In particular, when $x_1 \to \infty$ the asymptotics of the kernel $R_1 V_1(X, X', z)$ to the leading order is described by the formula

$$R_1 V_1(X, X', z) \cong C_{z-\lambda_3} \frac{\exp\{i\sqrt{z-\lambda_3}\, L_{10} + i W_1\}}{L_{10}^{5/2}} F(X, X', z), \qquad (34)$$

where $F_1(X, X', z)$ is a slowly oscillating function, the eikonal $L_{10} = \sqrt{|x_1|^2 + (y_1 - y_1')^2}$ and the Coulomb phase W_1 is given by

$$W_1 = \frac{L_{10}}{2\sqrt{z-\lambda_3}} \frac{n_2}{|x_2 - S_{21} y_1'|} \ln\left\{ \frac{|x_2 - S_{21} y_1'||x_2| + (x_2 - S_{21} y_1', x_2)}{|x_2 - S_{21} y_1'||S_{21} y_1'| + (x_2 - S_{21} y_1', S_{21} y_1')} \right\}.$$

The coefficient S_{21} stands in the transformation from the coordinates x_1, y_1 to x_2, y_2 :

$$x_2 = C_{21} x_1 + S_{21} y_1 , \quad y_2 = -S_{21} x_1 + C_{21} y_1 .$$

The kernel $R_1 W_{31}(z)$ also possesses an analogous asymptotics out of special directions. We will not describe the asymptotics in special directions defined solely by the Coulomb interaction (see [14]).

From the relations (29), (30) and (34) it follow that the equations are Fredholm[14]. Indeed, it suffices to consider only one equation, $R^1 = R_1 + R_1 V_1(z) R_2 \hat{V}_2 R^1$, obtained upon eliminating the component R^2 from (28). From that equation it follows that the asymptotic properties of the product $R(z) W_{31}(z)$ are determined by the properties of the product $R_1 W_{31}$ and the kernel R_1.

Note that were the potential $V^c_{5He\mu}$ not changed by $\tilde{V}^c_{5He\mu}$,

the separation of variables (33) would not occur. In this case the scheme of studying the resolvent $R(z)$ is to be modified.

7. The coordinate-asymptotics of the wave function and the sticking probaoility of the muon to helium

Upon studying the system of equations (28) it may be shown that at $z = E + i0, E > \lambda_3$ the C-asymptotics of the wave function of the third channel is described by the expression

$$u_3 \underset{x\to\infty}{=\!=} f_0(\hat{x}) \exp\{i\sqrt{E-\lambda_3}|x| + i W_0(x, E)\}|x|^{-5/2} +$$
$$+ \sum_j A_j(\hat{y}_2)\, \psi_j(x_2) \exp\{i\sqrt{E-\lambda_3+\tilde{\varkappa}_j^2}\,|y_2|\}|y_2|^{-1}, \qquad (35)$$

where the Coulomb phase is of the form

$$W_0(x, E) = -\frac{|x|}{2\sqrt{E-\lambda_3}} \frac{n_2}{|x_2|} \ln\left(2\sqrt{E-\lambda_3}\,|x|\right).$$

The functions $f_0(\hat{x})$ and $A_j(\hat{y}_2)$ are scattering amplitudes: f_0 when the muon gets free, and A_j when the result of the reaction is the mesic atom $^4He\mu$ with binding energy $\tilde{\varkappa}_j^2$.

Analogous are asymptotics of the wave-function components u^{α}, $\alpha = 1, 2$:

$$u^{\alpha} \underset{x\to\infty}{=\!=} f_0^{\alpha}(\hat{x}) \exp\{i\sqrt{E-\lambda_3}|x| + i W_0(x, E)\}|x|^{-5/2} +$$
$$+ \delta_{\alpha 2} \cdot \sum_j A_j(\hat{y}_2)\, \psi_j(x_2) \exp\{i\sqrt{E-\lambda_3+\tilde{\varkappa}_j^2}\,|y_2|\}|y_2|^{-1}, \qquad (35)$$

where $f_0^1 + f_0^2 = f_0$.

Ø The **Schrödinger** equation (21) with the asymptotic conditions (35) has a unique solution, u_3.

Ø Ø The Faddeev equations (25) and (26) with the conditions (36) have unique solutions u^1 and u^2, and $u^1 + u^2 = u_3$.

To determine the sticking probability of the muon to helium, we shall calculate the total flux of the probability. We take the surface through which the flux J is passing to be the surface S, a cylinder σ_r, $|y_2| = r$ when $|x_2| < |y_2|^{\nu}$, $\nu < 1/2$ and a sphere S_R, $|x| = R$, $R = \sqrt{r^2 + r^{2\nu}}$, when $|x_2| \ge |y_2|^{\nu}$ (see Fig.4), i.e. $S = S_R \cup \sigma_r$. The flux J when $R \to \infty$ is determined by the expression

$$J = \lim_{R\to\infty} \frac{1}{i} \int_S ds \left[\bar{u}_3 \frac{\partial}{\partial n} u_3 - u_3 \frac{\partial}{\partial n} \bar{u}_3 \right] =$$
$$= -i \lim_{R\to\infty} \left[\int_{\sigma_r} ds\, u_3 \overleftrightarrow{\frac{\partial}{\partial r}} \bar{u}_3 + \int_{S_R} ds\, u_3 \overleftrightarrow{\frac{\partial}{\partial R}} \bar{u}_3 \right]. \qquad (37)$$

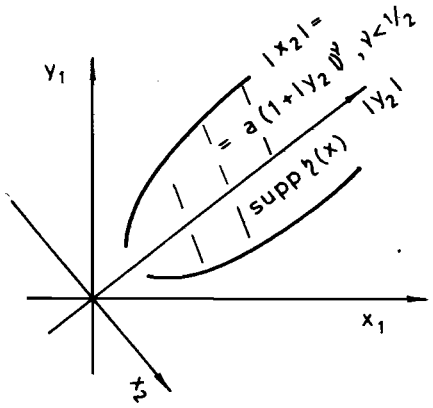

Fig.4. The region of definition of the short-range $\hat{V}_2$ and long-range V_2^{as} parts of the Coulomb potential $V_2^c(x_2)$.

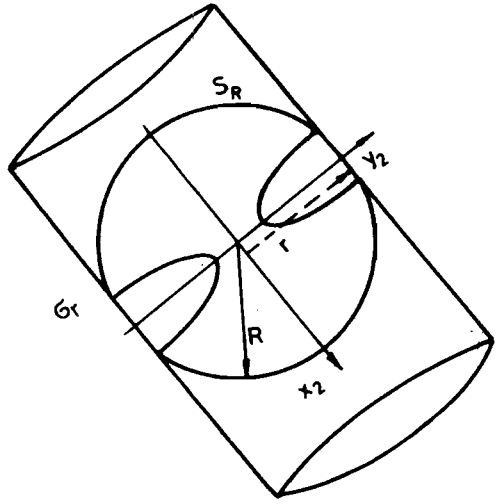

Fig.5. The surface S in $\mathbb{R}^6$ formed by a sphere S_R and a cylinder σ_r appearing in the determination of the probability flux J.

The change of the function U_3 in (37) by its asymptotics (35) leads to the expression

$$J = 2\sqrt{E-\lambda_3}\int_{S^5} d\hat{X}\,|A_0(\hat{X})|^2 + 2\sum_j \sqrt{E-\lambda_3+\tilde{\varkappa}_j^2}\int_{S^2} d\hat{y}_2\,|A_j(\hat{y}_2)|^2, \quad (38)$$

where S^5 and S^2 represent unit sphere in $\mathbb{R}^6$ and $\mathbb{R}^3$, respectively. From that equality it follows that the sticking probability $\mathring{\omega}_S$ of the muon to helium is given by the formula

$$\mathring{\omega}_S = J_S / J, \quad (38')$$

where $J_S = 2\sum_j \sqrt{E-\lambda_3+\tilde{\varkappa}_j^2}\int_{S^2} d\hat{y}_2\,|A_j(\hat{y}_2)|^2$.
The $\mathring{\omega}_S$ should be computed at $E=\varepsilon$.

To complete the section, we report some integral identities that allow us to estimate $\mathring{\omega}_S$. To this end we shall make use of the Green-function asymptotics for

$$R(X,X',E+i0) \doteq C_{E-\lambda_3}|X|^{-5/2}\exp\{i\sqrt{E-\lambda_3}|X| + iW_0(X,E)\}\,{}^0\Psi_0(P,X') + \frac{1}{4\pi}\sum_j |y_2|^{-1}\exp\{i\sqrt{E-\lambda_3+\tilde{\varkappa}_j^2}\,|y_2|\}\,\Psi_j(x_2)\,{}^0\Psi_j(q_j,X'). \quad (39)$$

Here ${}^0\Psi_0(P,X)$, $P=\sqrt{E-\lambda_3}\,\hat{X}$ is the wave function of the continuous spectrum of the system $^4He\mu$ describing scattering with three free particles in the initial state. The wave function of the continuous spectrum ${}^0\Psi_j(q_j,X)$, $q_j=\sqrt{E-\lambda_3+\tilde{\varkappa}_j^2}\,\hat{y}_2$ describes scattering in which the muon and helium initially constitute a mesic atom in the state Ψ_j whereas the neutron is free.

The substitution of the asymptotics (39) into the integral representation (27) for the wave function U_3 leads to the integral representations

$$A_0(\hat{X}) = C_{E-\lambda_3}\left(W_{31}(E+i0)\Phi, {}^0\Psi_0(P)\right)_3,$$
$$A_j(\hat{y}_2) = \frac{1}{4\pi}\left(W_{31}(E+i0)\Phi, {}^0\Psi_j(q_j)\right)_3, \quad (40)$$

for the amplitude A_0 and A_j.

8. The sticking probability in the Born approximation

For approximate estimation of $\mathring{\omega}_S$, the wave function Ψ in (40) can be replaced by a distorted Coulomb plane wave $U_0(P,X) = \Psi_{3,c}(x_2,k_2)\exp\{i(p_2y_2)\}$, $P=\{k_2,p_2\}$; and the continuous-spectrum wave functions ${}^0\Psi_j(q_j,X)$ by bound plane waves $U_j(q_j,X) =$

$\Psi_j(x_2)\exp\{i(q_j, y_2)\}$. To this approximation,

$$\mathfrak{A}_0(\hat{X}) = C_{E-\lambda_3}\ (W_{31}(E+i0)\Phi, U_0(P))_3\ , \qquad (41)$$

$$A_j(\hat{y}_2) = \frac{1}{4\pi}\ (W_{31}(E+i0)\Phi, U_j(q_j))_3\ , \qquad (42)$$

where E and Φ are resp. the energy and wave function of the bound state of the mesic molecule $dt\mu$.

Note that within that approximation the sticking probability ω_S is independent of the model parameters β and γ . Really, according to (14) $W_{31}(z) = -\beta\gamma\langle\cdot, \varphi\rangle\chi G_2(z)$. With this expression inserted into (41), (42) and then into (38), the factors $\beta^2\gamma^2$ appearing in the numerator and denominator of (38) cancel out.

We shall now analyse expressions (41) and (42) for $\mathfrak{A}_0$ and A_j in greater detail. To this end, using (14) we rewrite the amplitude A_j in the form

$$A_j(\hat{q}_j) = -\frac{\beta\gamma}{4\pi}\ (\Phi_\varphi(y_1)\chi G_2(E+i0), U_j(q_j))_3\ , \qquad (43)$$

where $E=\varepsilon$, $\Phi_\varphi(y_1)$ is the averaged wave function of the mesic molecule $dt\mu$ with the form factor φ : $\Phi_\varphi(y_1) = \langle\Phi(\cdot, y_1), \varphi\rangle_1$ and $G_2(z) = (H_2 - z)^{-1}$ is the Coulomb Green function of the intermediate channel $^5\mathrm{He}^*\mu$. Integration in $\langle\chi, U_j(q_j)\rangle_3$ runs only over Suppχ (see Fig. 6). To determine the behaviour of the factor $\langle\chi, U_j(q_j)\rangle_3$, when $y_1 \to \infty$, one should pass from the coordinate system of the final channel x_2, y_2 , corresponding to "sticking" of the muon to $^4\mathrm{He}$ and production of the mesic atom, to the system x_1, y_1 of the same channel, corresponding to "shaking-off" the muon $^4\mathrm{He}+n+\mu$ (see Fig.7):

$$\begin{aligned} U_j(q_j) &= \Psi_j(x_2)\, e^{i(q_j, y_2)} = \\ &= \Psi_j(C_{21}x_1 + S_{21}x_1)\, e^{i(q_j, -S_{21}x_1 + C_{21}y_1)} \approx \\ &\approx e^{-iS_{21}(q_j, x_1)}\, \Psi_j(S_{21}y_1)\, e^{i(q_j, C_{21}y_1)}, \\ &\qquad |x_1| < a = \mathrm{const}, \quad |y_1| \gg |x_1|\ , \end{aligned} \qquad (44)$$

where $C_{21} = -\left(\frac{m_1 m_2}{(m_1+m_3)(m_2+m_3)}\right)^{1/2}$, $S_{21}^2 = 1 - C_{21}^2$, $S_{21} > 0$, are coefficients of the transition from $x_2 y_2$ to $x_1 y_1$ (see Fig.7). Then, with (44) the amplitude $A_j(\hat{q}_j)$ assumes the form

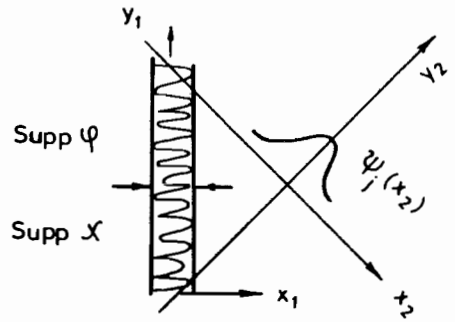

Fig.6. Distribution of the probability densities: the functions φ and χ connecting the initial $d+t+\mu$ and final $^4\mathrm{He}+n+\mu$ channels with the intermediate channel $^5\mathrm{He}^*\mu$ and the functions $\Psi_j(x_2)$ of the mesic atom $^4\mathrm{He}\mu$ in the final channel.

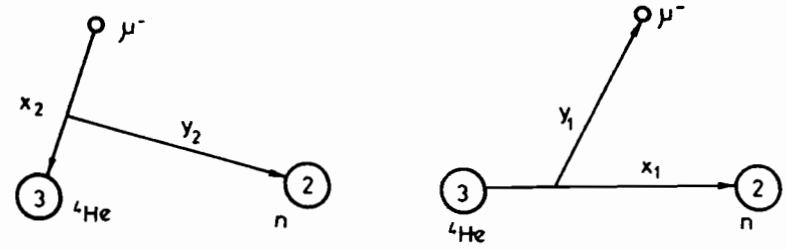

Fig.7. The Jacobi coordinates $x_1 y_1$ and $x_2 y_2$ of the final channel (3) in which shaking off and sticking of the muon occur, resp.

$$A_j(\hat{q}_j) = -\beta\gamma \frac{\langle \chi, u_0(q_j)\rangle_3}{4\pi} \int dy_1 dy_1' \overline{\psi_j(S_{21} y_1)} \times$$

$$\times \; e^{-i(q_j, C_{21} y_1)} G_2(y_1, y_1', E+i0) \Phi_\varphi(y_1'), \qquad (45)$$

where $u_0(q_j) = \exp\{-i S_{21}(q_j, x_1)\}$. We transform the integral in (45) using the spectral expansion of the Green function

$$G_2(z) = \sum_j \frac{\langle \cdot, \psi_j^{(2)}\rangle \psi_j^{(2)}}{-\varkappa_j^2 + \lambda_2 - z} + \frac{1}{(2\pi)^3}\int dp \frac{\langle \cdot, \psi_p^{(2)}\rangle}{p^2 + \lambda_2 - z}, \qquad (46)$$

where $\psi_j^{(2)}(y_1)$ are Coulomb eigenfunctions of the discrete spectrum $\sigma_d(\hat{H}_2) = \{-\varkappa_j^2\}$ of the Hamiltonian $\hat{H}_2$: $\hat{H}_2 = H_2^{(0)} + \hat{V}^c_{5He\mu} + \lambda_2$, $\hat{V}^c_{5He\mu} = |y_1|^{-1} q_{5He} q_\mu \sqrt{2 m_\mu (m_t + m_d)/(m_\mu + m_t + m_d)}$; $\psi_p^{(2)}$ are wave functions of the continuous spectrum $\sigma_c(\hat{H}) = [0, \infty)$.

We shall make also use of the wave function Φ of the mesic molecule $dt\mu$ in the adiabatic representation[20] associated with the Jacobi coordinates $\{x_1, y_1\}$;

$$\Phi(X) = \mathcal{F} F \cdot f \equiv \sum_n F_n(x_1, y_1) f_n(x_1) + \int dp \, F_p(x_1, y_1) f_p(x_1). \qquad (47)$$

Here $F = \begin{pmatrix} F_p \\ F_n \end{pmatrix}$ is a frame in the Hilbert fibre space with the basis $\mathcal{B} = \mathbb{R}^3_{x_1}$ and typical fibre $\mathcal{L} = L^2(\mathbb{R}^3_{y_1})$ and $f = \begin{pmatrix} f_p \\ f_n \end{pmatrix}$ are the expansion coefficients of the function Φ over a moving frame F whose dynamics is generated be the Hamiltonian

$$H(x_1) = -\Delta_{y_1} + \lambda_1 + V^c_{d\mu} + V^c_{t\mu}. \qquad (48)$$

The expansion (47) induces the corresponding expansion for Φ_φ:

$$\Phi_\varphi(y_1) = \mathcal{F} \langle F \cdot f, \varphi\rangle_1 \underset{|x_1|/|y_1| \to 0}{\sim} \mathcal{F} \psi^{(2)}(y_1) \langle f, \varphi\rangle_1 \overset{\cdot}{F}(x) \underset{|x_1|=0}{=} \psi^{(2)}(y_1), \qquad (49)$$

Here we made use of the known property: where $\psi^{(2)} = \begin{pmatrix} \psi_p^{(2)} \\ \psi_n^{(2)} \end{pmatrix}$. Inserting (46) and (49) into (45) we obtain the following expression for the amplitude:

$$A_j(\hat{q}_j) = \frac{S_{21}^{-3/2}}{4\pi} \int dy_1 \overline{\psi_j(y_1)} e^{-iC_{21}(q_j, y_1)} \times$$

$$\times \left\{ \sum_n T_n(q_j) \psi_j^{(2)}(y_1) + \frac{1}{(2\pi)^3} \int dp \, T_p(q_j) \psi_p^{(2)}(y_1) \right\}, \qquad (50)$$

where

$$T_n(q_j) = -\beta\gamma \frac{\langle \chi, u_0(q_j)\rangle_3 \langle f_n, \varphi\rangle_1}{-\varkappa_n^2 + \lambda_2 - E - i0}, \qquad (51)$$

$$T_p(q_j) = -\beta\gamma \frac{\langle \chi, u_0(q_j)\rangle_3 \langle f_p, \varphi\rangle_1}{p^2 + \lambda_2 - E - i0}. \qquad (52)$$

Here we took advantage of the functions $\psi_j(S_{21} y_1)$ in (44) being proportional to the wave functions $\psi_j(y_1)$: $\psi_j(S_{21} y_1) = S_{21}^{-3/2} \psi_j(y_1)$.

Now we will represent the amplitude A_0 in a form analogous to (50). To this end we write the wave function $U_0(X, P)$ in terms of the coordinates $\{x_1, y_1\}$. For $|x_1| < a = \text{const}$ and $|y_1| \gg |x_1|$ we get

$$U_0(X, P) \simeq S_{21}^{-3/2} \psi_{k_2}(y_1) \exp\{i C_{21}(p_2, y_2)\} u_0(x_1, \hat{y}_1, P), \qquad (53)$$

where

$$u_0(x_1, \hat{y}_1, P) = \exp\{i S_{21}(C_{21}|k_2|\hat{y}_1 - p_2, x_1)\}. \qquad (54)$$

In this representation, the amplitude acquires the form

$$A_0(\hat{P}) = S_{21}^{-3/2} C_{E-\lambda_3} \int dy_1 dy_1' \langle \chi, u_0(\hat{y}_1, P)\rangle_3 \times \qquad (55)$$

$$\times \overline{\psi_{k_2}(y_1)} e^{-iC_{21}(p_2, y_1)} G_2(y_1, y_1', E+i0) \Phi_\varphi(y_1'),$$

where the numerical factor $S_{21}^{-3/2}$: $\psi_{k_2}(S_{21} y_1) = S_{21}^{-3/2} \psi_{k_2}(y_1)$. Utilizing the expressions (46) and (49) we obtain

$$A_0(\hat{P}) = S_{21}^{-3/2} C_{E-\lambda_3} \int dy_1 \overline{\psi_{k_2}(y_1)} e^{-iC_{21}(p_2 y_1)} \times \qquad (56)$$

$$\times \left[\sum_n T_n(\hat{y}_1, \hat{P}) \psi_n^{(2)}(y_1) + \frac{1}{(2\pi)^3} \int dp \, T_p(\hat{y}_1, \hat{P}) \psi_p^{(2)}(y_1) \right],$$

where

$$T_n(\hat{y}_1, \hat{P}) = -\beta\gamma \frac{\langle \chi, u_0(\hat{y}_1, P)\rangle_3 \langle f_n, \varphi\rangle_1}{-\varkappa_n^2 + \lambda_2 - E - i0} \qquad (57)$$

$$T_p(\hat{y}_1, \hat{P}) = -\beta\gamma \frac{\langle \chi, u_0(\hat{y}_1, P)\rangle_3 \langle f_p, \varphi\rangle_1}{p^2 + \lambda_2 - E - i0}. \qquad (58)$$

The Born approximation for ω_S in terms of the amplitude A_j and A_0 will be obtained by substituting (50) and (56) into (38).

In conclusion, we would like to note the following.

1. The factor $\exp\{-iC_{21}(q, y)\}$ in (50) and (56) corresponds to the known Migdal notion of a "rider"[5]. So, for instance, $C_{21}(q_j, y_1) = (\tilde{q}_j, \tilde{y}_1)$ where

$$\tilde{q}_j = \frac{m_\mu}{m_\mu + m_{4He}} \tilde{p}_j, \quad \tilde{p}_j = \left\{ \frac{2(Q-E_j)\, m_n (m_\mu + m_{4He})}{(m_\mu + m_n + m_{4He}} \right\}^{1/2},$$

$$\tilde{y}_i = \left\{ \frac{m_\mu + m_n + m_{4He}}{2\, m_\mu (m_n + m_{4He})} \right\}^{1/2} y_i ,$$

$\tilde{p}_j$ is the relative momentum of the mesic atom $^4He\mu$ and neutron n in the c.m.s., $E_j = -\tilde{\varkappa}_j^2$ is the energy of the mesic atom $^4He\mu$ and $Q = E - \lambda_3$ is the energy release in the reaction under consideration.

2. Singularities of the amplitudes T_n and T_p in energy are on the real axis and have no characteristic Breit-Wigner structure. This is due to the Born approximations used for Ψ_j and Ψ_0. The Breit-Wigner structure with a nontrivial resonance width may only be obtained by calculating the amplitudes A_j and A_0 on the exact three-body wave functions Ψ_j and Ψ_0. Therefore the schemes of computation[9,21] based on the Born approximation to any order for Ψ_j and Ψ_0 with a simultaneous change of the energy S-function by a resonance Breit-Wigner factor like $A/(E - E_R - i\Gamma/2)$ are not in agreement in the accuracy of approximation and no hold true. However, for qualitative estimates the use of the Born approximation may be useful.

3. For the mathematically correct computation of the sticking probability ω_s with the inclusion of the resonance channel $^5He^*$ within the proposed model one should calculate the wave function Φ, of the $dt\mu$ bound state, fit parameters of the model $(\beta, \gamma, \varphi, \chi)$ from the data on the reaction, and then calculate the amplitudes A_j and A_0 on the basis of the solution of the Faddeev equations (25) and (26) with the boundary conditions (36). Solution of the latter problem may be realized with a required accuracy be the methods developed in[22] and[23].

4. The problem of applicability in the considered problem of the scheme (2) of the coupling of Hamiltonians H_j and the choice of a separable approximation of rank I for the operators of the channel coupling $W_{ij}(z)$ require special consideration. This investigation can be made within the methods of local adiabatic expansions[20] for a 6-body problem with the use of the Feshbach projection method[24]. That approach will allow us to obtain exact representations in particular clusterizations $(dt\mu, {}^4He\, n\mu)$ in a 6-th-body system for the energy-dependent potentials $W_{ij}(z)$, to estimate the contribution of polarization effects of the clusters $\{d, t, {}^4He\}$ and

specifically, to solve the problem of making the potentials $W_{ij}(z)$ separable. These problems will be dealt with in a separate publication.

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References

1. Yu.V.Petrov. Nature 285 (1980) 466;
Muon Catalysed Fusion 1 (1987) 351.
2. J.D.Jackson. Phys.Rev. 106 (1957) 330.
3. L.Bracci and G.P.Fiorentini. Nucl.Phys. A364 (1981) 383.
4. S.S.Gerstein et al. Zh.Eksp.Teor.Fiz., 80 (1981) 1990.
(Sov.Phys. - JETP 53 (1981) 383).
5. A.B.Migdal. Zh.Eksp.Teor. Fiz., 9 (1939) 1163.
A.M.Dexne, G.L.Yudin. Usp.Fiz.Nauk (1978) 377.
6. A.Messia. Quantum Mechanics. Mosk., Nauka, 1978, v.2 (in Russian)
7. S.E.Jones et al. Phys.Rev.Lett., 56 (1986) 588.
8. W.H.Breenlich. Muon Catalysed Fusion 1 (1987) 29.
9. L.N.Bogdanova et al. Nucl.Phys. A454 (1986) 653.
10. D.Ceperley, B.J.Alder. Phys.Rev. A31 (1985) 1999.
11. Chi-Yu Hu. Phys.Rev. A34 (1986) 2536; A36 (1987) 4135.
12. S.E.Haywood, H.J.Monkhorst, K.Szalewic. Phys.Rev. A36 (1988) 4256.
13. V.E.Markushin. Preprint 336, Moscow, FIAN, 1987.
Muon Catalysed Fusion 2 (1988) 256.
14. S.P.Merkuriev, L.D.Faddeev. Quantum Theory of Scattering for Few Body System, Moscow, Nauka, 1985 (in Russian).
15. S.I.Vinitsky et al. Zh.Eksp. Teor.Fiz., 79 (1980) 698.
16. L.N.Bogdanova, V.E.Markushin and V.S.Melezhik, Zh. Eksp. Teor. Fiz., 81 (1981) 829. (Sov.Phys.JETP 54 (1981) 174).
17. L.N.Bogdanova et al. Yad.Fiz., 34 (1981) 1191.
(Sov.J.Nucl.Phys. 34 (1981) 662).
18. Yu.A.Kuperin et al. In: Printed matter: 3 All-Union School on Few Body and Quark-Adronic Systems, Palanga 30.9-9.10.86, p.28 - AN USSR, Vilnious, 1986, in Russian; Proc. of Intern'l Conf. on the Theory of Few Body and Quark-Hadronic Systems, Dubna, 16-20 June, 1987, p.40.

19. Yu.A.Kuperin et al. In: Printed Matter: Theory of Quantum Systems with Strong Interaction, p.4, Kalinin State University, Kalinin, 1987 (in Russian); B.S.Pavlov. Uspekhi Mat.Nauk, 42, (1987) 99.
20. K.A.Makarov, Yu.A.Kuperin, B.S.Pavlov, V.M.Dubovik, B.L.Markowski, S.I.Vinitsky. A Local Adiabatic Representation in the Few-Body Quantum Scattering Problem. Preprint FUB-HEP/87-11, Berlin (Wests) 1987.
21. M.Danos, B.Muller, J.Rafelski. Detuning of Resonant μ-Catalyzed d-t Fusion by Muon Sticking, Preprint University of Cape Town, UCT-TP 31/1985.
22. Yu.A.Kuperin, S.P.Merkuriev, A.A.Kvitsinsky. Yad.Fiz. 37 (1983) 1440; Preprint ITP-Budapest-Report N 441, Budapest, Univ.,1986.
23. A.A.Kvitsinsky et al. Phys.Elem.Part. At Nucl., 17, (1986), 267, Moscow, Energoatomizdat, 1986, v.17, p.267-317.
24. H.Feshbach. Ann.Phys. 5 (1958) 357; ibid. 19 (1962) 287.

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Мотовилов А.К. и др. E4-88-291

Уравнение Фаддеева с дополнительным резонансным каналом в реакции мюонного катализа

В рамках трехканальной модели получены неоднородные интегральные и дифференциальные уравнения Фаддеева с энергозависимыми потенциалами. Показана фредгольмовость интегральных уравнений. Дано точное определение коэффициента прилипания $\mathring{\omega}_s$ в терминах амплитуд сферических волн в асимптотике полной волновой функции выходного канала. Приведено соответствующее интегральное представление для $\mathring{\omega}_s$ через волновые функции непрерывного спектра. В первом борновском приближении для волновых функций это представление дает явное выражение для $\mathring{\omega}_s$ через коэффициенты разложения волновой функции $(dt\mu)^+$ входного канала.

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Motovilov A.K. et al. E4-88-291

Faddeev Equations with an Extra Resonance Channel in Muon Catalysis

A three-body model is applied to derive inhomogeneous integral and differential Faddeev equations with energy-dependent potentials. The integral equations are shown to be Fredholm equations. The stricking probability $\mathring{\omega}_s$ is determined in terms of the amplitudes of spherical waves in the asymptotics of the exit-channel wave function. The integral representation for $\mathring{\omega}_s$ is given in terms of the continuum wave functions. To the first Born approximation for the wave function, this representation yields an explicit expression for $\mathring{\omega}_s$ through the expansion coefficients of the wave function of $dt\mu$ of the initial channel.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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